Conservative and positivity-preserving semi-Lagrangian kinetic schemes with spectrally accurate phase-space resolution

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The Convected Scheme (CS) is a family of semi-Lagrangian algorithms, most usually applied to the solution of Boltzmann's equation, which uses a method of characteristics in an integral form to project a moving cell (MC) forward to a group of mesh cells. In earlier work [1], a 4th-order version of the cell-centered CS was presented, which was based on applying an a-priori correction to the position of the MC after the ballistic move and prior to remapping to the mesh. Such corrections were calculated by means of a modified equation analysis applied to the continuity equation with a prescribed flow field. The resulting 4th-order CS showed a drastically reduced numerical diffusion, while it retained the desirable properties of the original scheme (i.e. mass conservation, positivity preservation, and simplicity).

Here we describe recent work [2] on the construction of arbitrarily high-order versions of the CS, which are suited to the accurate solution of the Vlasov-Poisson system with minimum computational resources. For sufficiently smooth profiles on a periodic domain, this new scheme shows spectral convergence to the exact solution. For non-smooth profiles, filtering in Fourier space mitigates spurious Gibbs oscillations and stabilizes the solution. We show the scheme's behavior in classical 1D-1V test cases for the Vlasov–Poisson system, in both the linear and non-linear regimes, where it enables unprecedented phase-space resolution with virtually no numerical dissipation. While being strictly mass conservative, the proposed algorithm does not exactly conserve the total energy, the sum of kinetic and potential energy. Hence, we study the effect of inexact energy conservation on the solution, and we investigate the role of the time-splitting error by comparing traditional Strang splitting with higher order symplectic schemes, as in [3, 4].

As part of our current work, we then discuss the introduction of elastic and inelastic collisions in our model, which results in the 1D-2V Boltzmann-Poisson system. Specifically, we aim at investigating the formation of a planar plasma presheath in a weakly collisional plasma, and at comparing the new high-order results with simulation performed with early versions of the CS [5]. In order to properly model an absorbing wall, two alternative implementations are investigated: a periodic extension of the domain, which allows us to use FFTs, and a hybrid scheme which uses lower order polynomial interpolation in configuration space. In both cases, the solution shows a very sharp gradient in phase-space, at the wall and for low velocities; such a gradient becomes a discontinuity in the limit of vanishing collisionality, and it requires careful treatment.

References

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