Action Principle for Relativistic Extended Magnetohydrodynamics

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Whereas extended MHD (XMHD) has drawn attention recently, there are few studies of the relativistic version of XMHD [1]. Here we derive the relativistic XMHD from two kinds of Eulerian action principles [2]. The first action formalism uses Lin's method [3] with constraints of density, entropy, and Lagrangian label conservation. This gives Clebsch representations of a generalized momentum vector and a generalized vector potential. Second, a transformation of the Clebsch variables to the vector field Eulerian variables gives a covariant Poisson bracket action principle [4, 5], in which the constraints are implemented in degeneracy of the Poisson bracket. These action principles encompasses other magnetohydrodynamic models (*e.g.* relativistic Hall MHD, inertial MHD, and usual ideal MHD).

Relativistic Hall MHD, which is proposed for the first time in our study, has intriguing features. For example, Ohm's law for homentropic electrons is written as

$$\left(u_\nu - d_{\rm i}\frac{J_\nu}{n}\right)F^{\star\mu\nu} = 0,$$

where $F^{\star\mu\nu}$ is a generalized Faraday tensor given by a generalized vector potential $A^{\star\nu} = A^\nu + d_{\rm i}\,(\Delta h)\,u^\nu - {d_{\rm i}}^2(\Delta h)J^\nu/n$, $d_{\rm i}$ is a normalized ion skin depth, and Δh is the difference of the relativistic thermal inertiae between ions and electrons. The conserved 2-form field is not the standard Faraday tensor but $F^{\star\mu\nu}$ when $\Delta h \neq 0$. Therefore, the magnetic flux-frozen in condition is violated. This Δh effect is similar to the role played by electron inertia on the electron skin depth scale. In relativistic Hall MHD, therefore, a collisionless magnetic reconnection may occur on ion skin depth scale without electron inertia.

References

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