Deriving Numerical Models from Action Principles*

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Routinely numerical models are obtained by discretizing, either through means of a grid or a basis expansion, the relevant equations of motion. The primarily difficulty with this approach has to do with the effects of discretization on the system's conservation laws. When the underlying dynamics satisfies an action principle, there appears to be a significantly better alternative: performing the discretization directly in the action. This preserves the link between symmetries and conservation laws and, empirically, leads to better behaviour compared with models obtained from an ad-hoc discretization. Since this approach yield a finite degree-of-freedom Lagrangian systems, in general, it is possible to perform a Legendre transform to obtain an (canonical) Hamiltonian system. Further, it is possible to incorporate arbitrary boundary conditions into the variational formulation. Boundary conditions often destroy symmetries leading to loss of the associated conservation law. Nonetheless the variational approach is still relevant as it readily yields flux-balance equations for the conservation laws altered by boudnary conditions. I will discuss various examples of this technique relevant to laser-plasma interactions and examine some interesting open questions.

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