

# Semi-implicit time stepping for finite element particle-in-cell methods

Katharina Kormann<sup>1,2</sup>, Eric Sonnendrücker<sup>1,2</sup>, Michael Kraus<sup>1,2</sup>, Philip J. Morrison<sup>3</sup>,  
Benedikt Perse<sup>2</sup>

<sup>1</sup>*Numerical Methods in Plasma Physics  
Max-Planck-Institut für Plasmaphysik  
Boltzmannstr. 2, 85747 Garching, Germany*

<sup>2</sup>*Technical University of Munich  
Department of Mathematics*

*Boltzmannstr. 3, 85748 Garching, Germany*

<sup>3</sup>*Department of Physics and Institute for Fusion Studies  
The University of Texas at Austin  
Austin, TX, 78712, USA*

Numerical schemes that preserve the structure of the underlying kinetic equations can provide new insights into the long time behavior of fusion plasmas. The geometric electromagnetic particle-in-cell framework [Kraus et al., *Journal of Plasma Physics* 83, 2017] provides a semi-discretization of the Poisson bracket of the Vlasov-Maxwell system based on a compatible spline finite element discretization of the fields and a standard particle-in-cell representation of the distribution function. The equations of motion derived from the semi-discrete Poisson bracket preserves on the non-canonical Hamiltonian structure of the Vlasov-Maxwell equations. The explicit time-stepping based on a Hamiltonian splitting proposed in the paper preserves Gauss law, however, not the total energy. Implicit time propagation is an alternative that can preserve total energy and also allows for larger time steps. Semi-implicit schemes can be achieved by an antisymmetric splitting of the Poisson matrix instead of splitting the Hamiltonian. Energy conserving schemes can then be devised using a discrete gradient approximation of the subsystems. In the talk, we will compare various temporal discretizations for some benchmark problems in 1d2v phase space.