Solving kinetic equations using a dynamic low-rank projector-splitting integrator

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Many problems encountered in plasma physics require a kinetic description. The associated partial differential equations are posed in an up to six-dimensional phase space. A direct discretization of this phase space, often called the Eulerian approach, has many advantages but is extremely expensive from a computational point of view.

In this talk we propose a dynamical low-rank approximation to the Vlasov equation. This approximation is derived by constraining the dynamics to a manifold of low-rank functions via a tangent space projection. Then the projection is split into the sub-projections from which it is built. This reduces a time step for the six- (or four-) dimensional Vlasov--Poisson equation to solving two systems of three- (or two-) dimensional advection equations. This projector-splitting approach also enables us to dynamically adjust the rank during the simulation.

The resulting systems of advection equations can then be solved by standard techniques such as semi-Lagrangian or spectral methods. We highlight the favorable behavior of the proposed numerical method by presenting a number of numerical simulation. These simulation show that that the proposed algorithm is able to drastically reduce the required computational effort.

This talk is based on `https://arxiv.org/abs/1801.01103` (to appear in SISC) and `https://arxiv.org/abs/1807.02338`.