Quasi Monte Carlo inverse transform sampling for phase space conserving Lagrangian particle methods

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Low-discrepancy sequences, also called quasi-random sequences, provide convergence rates close to $\mathcal{O}(N^{-1})$ which are far superior to random numbers settling in at only $\mathcal{O}(N^{-0.5})$. It remains the question whether Lagrangian particle methods such as PIC which rely on Monte Carlo integration can benefit from such quasi-random-sequences, since the particle distributions are nonlinearly perturbed by the forward flow following the characteristics. Any nonlinear measure preserving map does not affect the low-discrepancy of a QMC sequence such that the order of convergence remains. The forward flow of phase space conserving geometric particle methods induces naturally such a measure preserving map underlying their importance.
On the other hand Eulerian and Lagrangian solvers have different strengths and weaknesses, such that we present a way of transiting from a spectral discretization of the Vlasov–Poisson system to a PIC simulation. This is achieved by higher dimensional inverse transform sampling, which circumvents Markov Chain Monte Carlo techniques thus allowing for the used of pseudo and quasi-random numbers yielding better convergence rates in the latter case. This can also be observed in the nonlinear phase.