Conservative finite-element method for the relativistic Coulomb collision operator

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This work documents new developments regarding finite-element discretizations of the relativistic Beliaev-Budker Coulomb collision operator and the nonrelativistic Landau operator. Where energy conservation in a finite-element approximation of the relativistic collision operator was previously thought to be elusive, it is now achieved even with linear elements. The same result applies to the nonrelativistic Landau operator for which the energy conservation was thought to require at least quadratic elements. In both cases, the momentum and density conservation are guaranteed as previously. The new outcomes benefit from the findings reported in a finitedifference-scheme paper [Shiroto & Sentoku, arXiv:1902.07866] which we generalize to the finite-element method. This work focuses solely on the direct discretization of the collision operator, leaving the discretization of the underlying metriplectic formulation of the relativistic collision operator to near future.

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