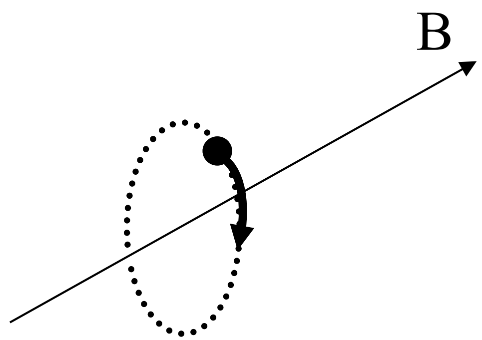
Magnetic confinement of plasmas

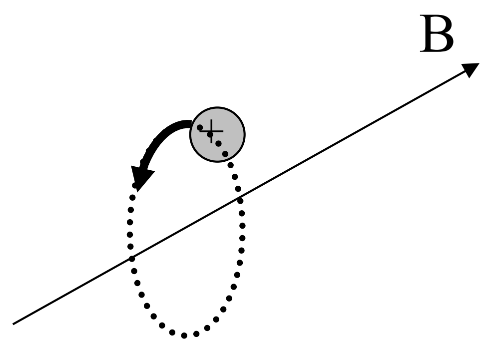
Particle motion in electric and magnetic fields

Electric field: $\vec{F} = q \cdot \vec{E} \quad ; \quad q = \{-e, +e, +Z \cdot e\}$

Magnetic field: $\vec{F} = q \cdot \left[\vec{\mathrm{v}} \times \vec{B}\right]$ Lorentz force



Electron



Ion

in field direction:

to the right

to the left

Gyration in magnetic field

Gyration frequency: $$\omega_{ce} = \frac{e \cdot B}{m_e} \qquad \omega_{ci} = \frac{Z \cdot e \cdot B}{m_i}$$

Fusion plasmas: Ions: 30 … 60 MHz
Electrons: 100 … 150 GHz (mm-waves)

Technical plasmas (0.1 T)
Electrons: 2.5 GHz (Micro wave)

Gyration in magnetic field

Gyration radius:

$$r_{ge} = \frac{m_e \mathrm{V}_\perp}{e \cdot B} \qquad r_{gi} = \frac{m_i \mathrm{V}_\perp}{Z \cdot e \cdot B}$$

- 10 keV electron in earth magnetic field ($5\ 10^{-5}$ T): 6.75 m
- proton in solar wind, v=300 km/s, $5\ 10^{-9}$ T: 626 km
- 1 keV He^+ ion in solar atmosphere ($5\ 10^{-2}$ T): 0.183 m
- 3.5 MeV He^{2+} in 8T fusion reactor: 3.38 cm
- today's fusion experiments (2T, 1 keV): electrons: 53 μm
  ions: 2.2 mm

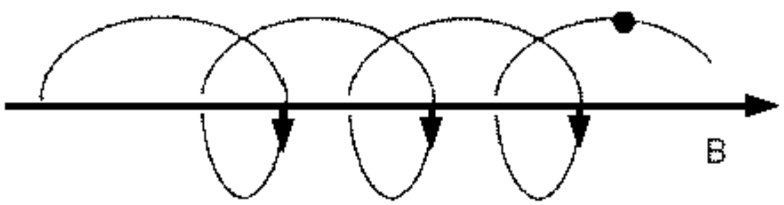
Homogeneous magnetic field

Gyration frequency: $\omega_{ce} = \dfrac{e \cdot B}{m_e}$ $\qquad \omega_{ci} = \dfrac{Z \cdot e \cdot B}{m_i}$

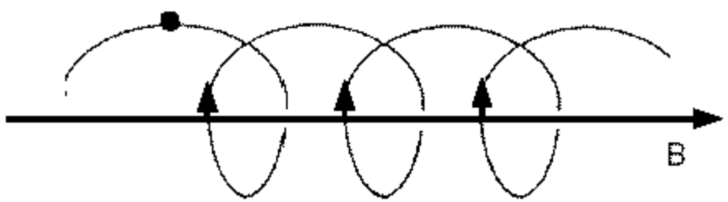
Gyration radius: $r_{ge} = \dfrac{m_e \mathrm{V}_\perp}{e \cdot B}$ $\qquad r_{gi} = \dfrac{m_i \mathrm{V}_\perp}{Z \cdot e \cdot B}$

Single particle motion is diamagnetic

Electron



Ion



Fluid description of Plasmas

Possible description of plasmas:

- single particle description (but order of 10^{23} particles)
- kinetic equation (statistical description)

Number of particles in phase space $d^3r\, d^3v$: $f_\alpha(\vec{r},\vec{v},t)\, d^3r d^3v$

Conservation of number of particles : $\frac{d}{dt}\int f_\alpha(\vec{r},\vec{v},t)\, d^3r d^3v = 0$

Without sources and sinks one finds (flow in phase space):

$$\boxed{\frac{d}{dt} f_\alpha(\vec{r},\vec{v},t) = 0}$$

Derivatives in fluid dynamics (Lagrangian derivative)

Total rate of change a specific „flow parcel“ experiences

$$\frac{df_\alpha(\vec{r}(t))}{dt} = \frac{\partial f_\alpha}{\partial t} + \frac{\partial f_\alpha}{\partial x} v_x + \frac{\partial f_\alpha}{\partial y} v_y + \frac{\partial f_\alpha}{\partial z} v_z = \frac{\partial f_\alpha}{\partial t} + (\vec{v} \cdot \nabla) f_\alpha$$

variations in fixed
coordinate system

variations due to
motion

One particle distribution function

$$\boxed{\frac{d}{dt} f_\alpha(\vec{r},\vec{v},t) = 0} \Rightarrow \frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + (\nabla_r f_\alpha)\frac{d\vec{r}}{dt} + (\nabla_v f_\alpha)\frac{d\vec{v}}{dt} = 0$$

Splitting of interaction into mean field and collisions:

$$\frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\nabla_r f_\alpha + \frac{\vec{F}}{m}\cdot\nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{Sto\beta}$$

Force due to mean field (produced by plasma particles or external sources):

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

Fluid description of Plasmas

Start with kinetic equation:

$$\frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla_r f_\alpha + \frac{q_\alpha}{m}\left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{Sto\beta}$$

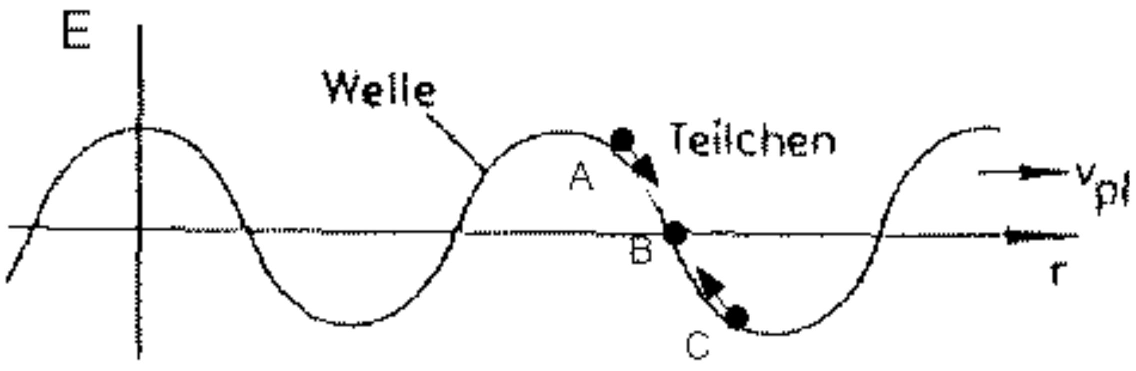
do not consider the distribution function but moments:

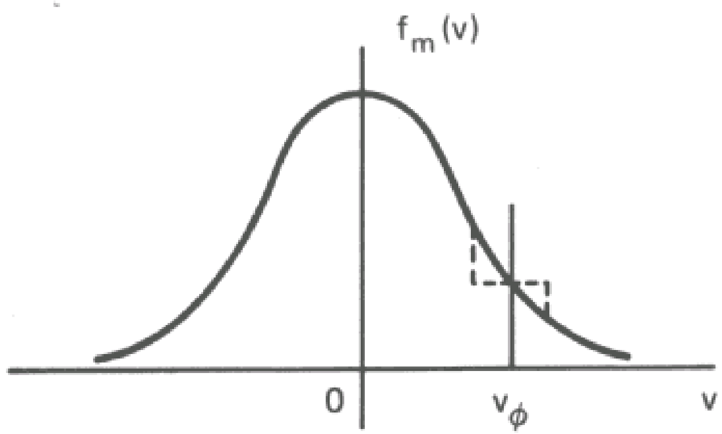
- moments of the distribution function: $\int (\vec{\mathrm{v}})^k \cdot f_\alpha(\vec{\mathrm{v}}, \vec{r}, t) d\vec{\mathrm{v}}$

- neglect kinetic effects, i.e. the different response of particles with different velocities to external fields (e.g. Landau damping)

Landau Damping: wave damping even without collisions

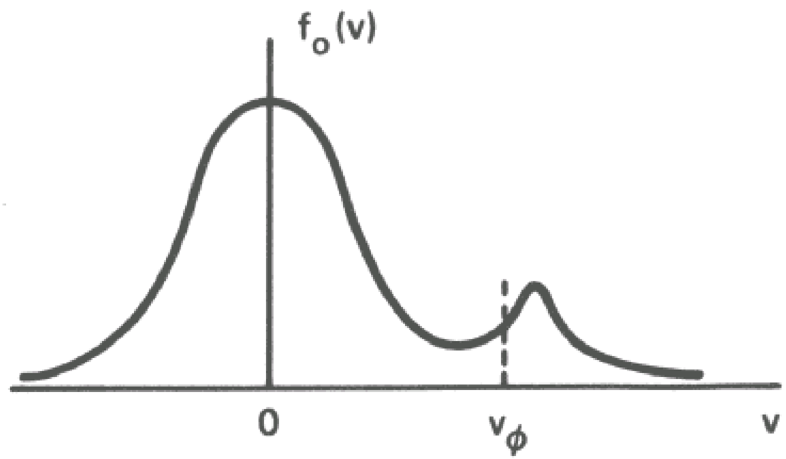
Simple picture for non-linear Landau damping





Landau damping itself also works without collisions, but collisions needed to restore original distribution function

Damping rate can become negative -> Instabilities



Moments of the distribution function

density (k=0):

$$n_\alpha(\vec{r},t) \equiv \int (\vec{\mathrm{v}})^0 \cdot f_\alpha(\vec{\mathrm{v}},\vec{r},t)d\vec{\mathrm{v}}$$

centre-of-mass velocity (k=1):

$$\vec{\mathrm{u}}_\alpha(\vec{r},t) \equiv \int (\vec{\mathrm{v}})^1 \cdot f_\alpha(\vec{\mathrm{v}},\vec{r},t)d\vec{\mathrm{v}} / n_\alpha(\vec{r},t)$$

temperature (k=2): $\frac{3}{2}k_B T = \frac{m}{2}\left\langle (v-u_a)^2 \right\rangle$

$$kT_\alpha(\vec{r},t) \equiv \frac{m_\alpha}{3}\int (\vec{\mathrm{v}}-\vec{u}_\alpha)^2 \cdot f_\alpha(\vec{\mathrm{v}},\vec{r},t)d\vec{\mathrm{v}} / n_\alpha(\vec{r},t)$$

Moments of the distribution function

local quantities i.e. fluid description is only possible if the mean free path in smaller than the scale length of the processes under investigation!

$$\lambda_c << L_H$$

problematic for:

- collision-free plasmas
- small-scale processes

time scale of the processes under investigation has to be much longer than the collision time :

$$\tau_{ii} << \tau_H$$

Magnetohydrodynamics (MHD)

single fluid description:

assumption: fluids and fields fluctuate on the same time and length scales (ions length and time scales)

$\Rightarrow$ all effects that are connected to the electron dynamics are neglected

$$\frac{\omega}{k} \sim \frac{L_H}{\tau_H} \sim u_i << c$$

non-relativistic description

$T_e = T_i$ even higher collision rate necessary than for two fluids

energy exchange between ions and electrons must happen on the time scale under consideration:

$$\tau_{ii} << \boxed{\sqrt{\frac{m_e}{m_i}}} \; \tau_H \qquad \left(\tau_{ei} = \sqrt{\frac{m_i}{m_e}} \left(\frac{T_e}{T_i}\right)^{3/2} \tau_{ii} << \tau_H \right)$$

MHD-equations

continuity equation $$\frac{\partial \rho}{\partial t} + \nabla(\rho \cdot \overline{\mathrm{v}}) = 0$$

force equation $$\rho\left(\frac{\partial \overline{\mathrm{v}}}{\partial t} + (\overline{\mathrm{v}} \cdot \nabla)\overline{\mathrm{v}}\right) = -\nabla p + \bar{j} \times \overline{B}$$

Ohm's law $$\vec{E} + \vec{u} \times \vec{B} = \eta \vec{j}$$

in addition

Maxwell's equations $$-\frac{\partial \overline{B}}{\partial t} = \nabla \times \overline{E}, \quad \mu_0 j = \nabla \times \overline{B}, \quad \nabla \cdot \overline{B} = 0$$

adiabatic gas law: $$\frac{d(p\rho^{-\gamma})}{dt} = 0$$

Magnetic confinement

$$\boxed{\nabla p = \vec{j} \times \vec{B}}$$

Pressure gradient is balanced by Lorentz force
(currents perpendicular to magn. field)

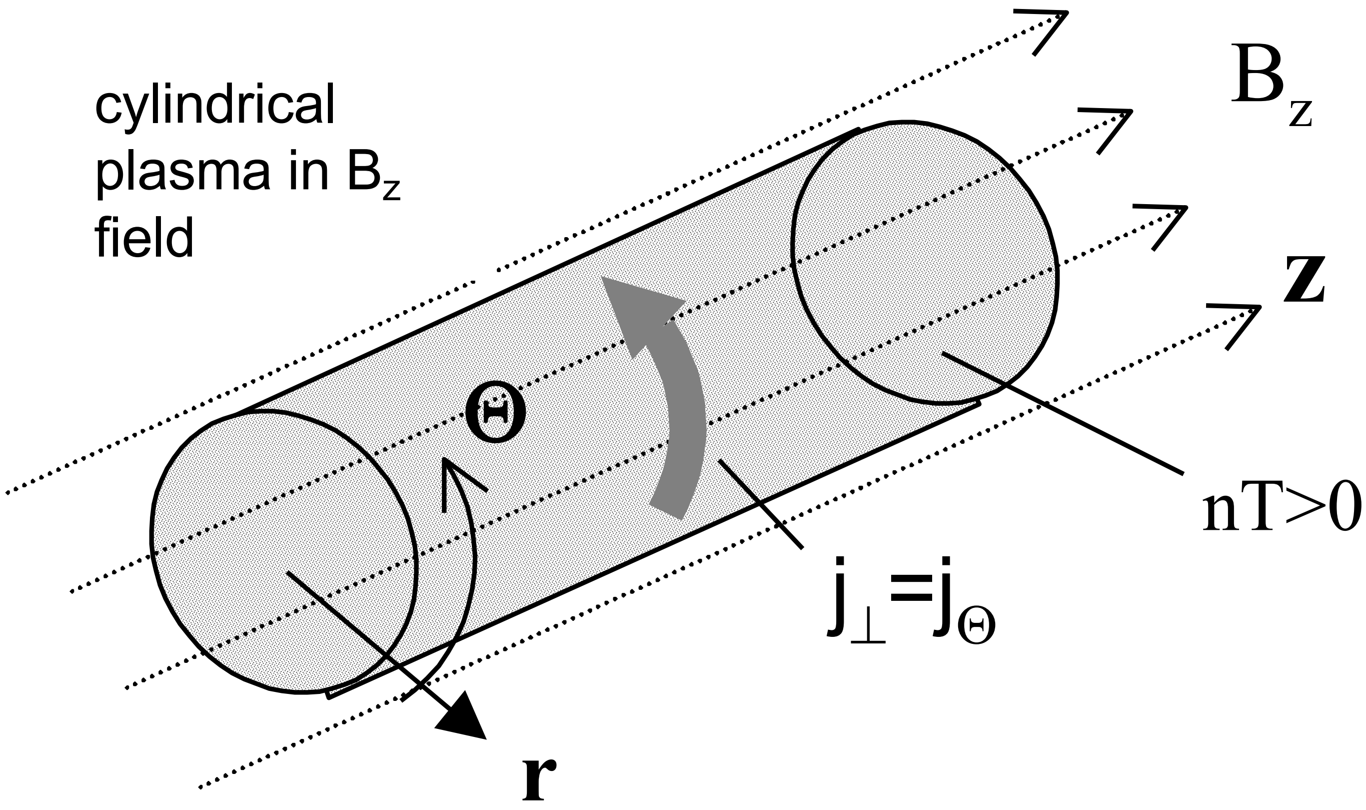
$$\vec{B} \cdot \nabla p = 0$$

Pressure along magnetic field lines is constant

$$\vec{j} \cdot \nabla p = 0$$

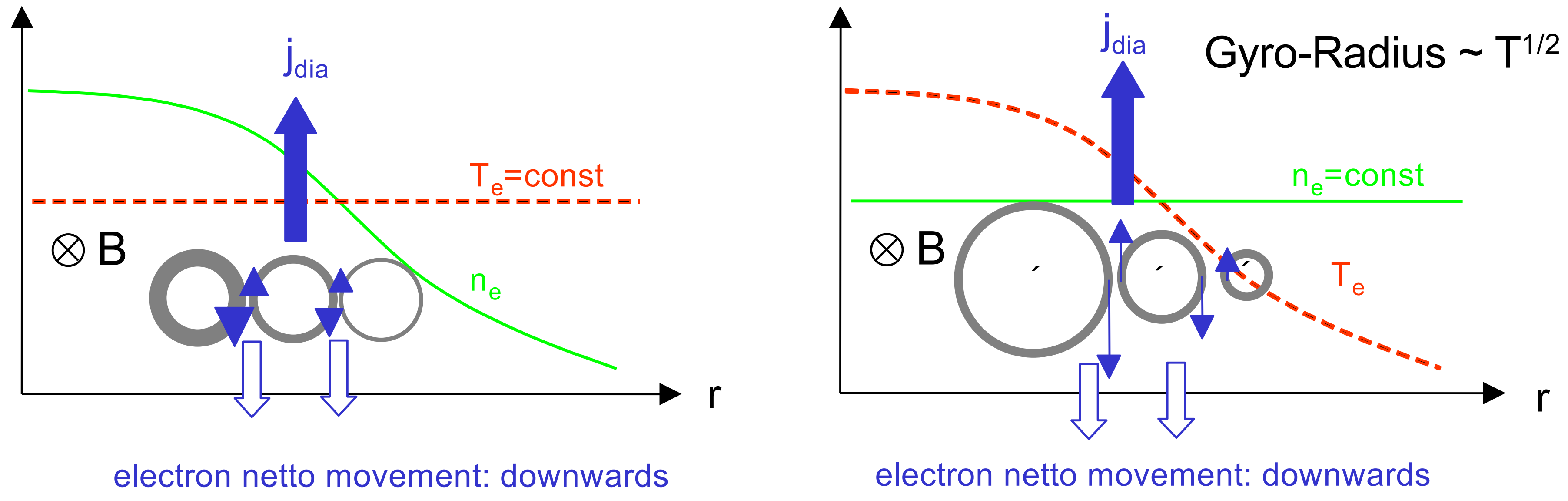
Current lines lie surfaces of constant pressure

θ-Pinch

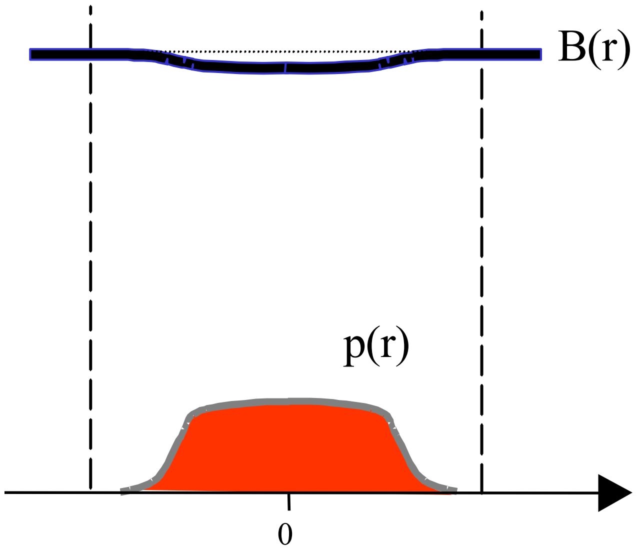


Diamagnetic current reduces external magnetic field

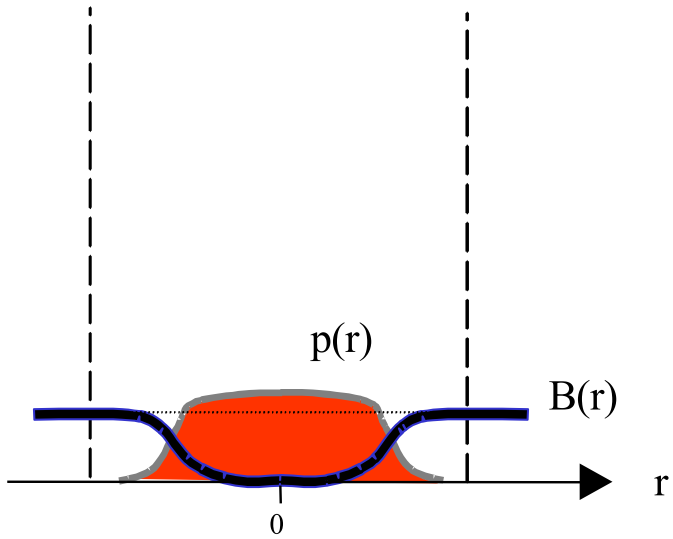
Diamagnetic currents



Pressure gradient generates current perpendicular to B field if ß is small: almost no change of external magnetic field



Large modification of external field for “high-ß”-case (ß=1 if B=0)



Magnetic confinement in θ-pinch

Ions and electrons contribute to diamagnetic current:

$$(\bar{j}_e + \bar{j}_i) \times \overline{B} = \nabla p_e + \nabla p_i$$

$$\mu_0 j = \nabla \times \overline{B} \qquad \nabla p = \left(\frac{1}{\mu_0} \cdot \nabla \times \overline{B} \right) \times \overline{B}$$

Pressure gradient is balanced by $\left(\nabla \times \vec{B}\right) \times \vec{B} = -\frac{\nabla B^2}{2} + \vec{B} \cdot \nabla \vec{B}$

B –field pressure

Field line tension

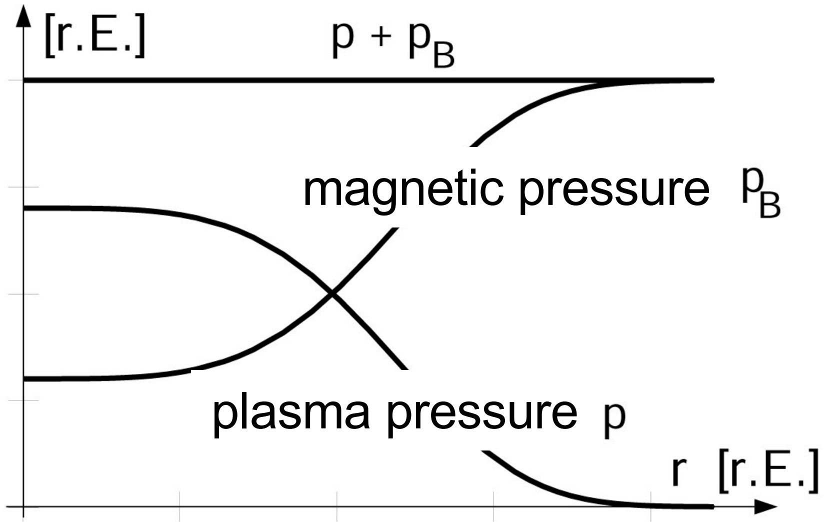
$$\nabla\left(p + \frac{B^2}{2\mu_0} \right) = \frac{\left(\overline{B}\nabla\right)\overline{B}}{\mu_0}$$

Magnetic confinement in θ-pinch

In θ-pinch no field line tension (MF constant along MF-lines):

$$\nabla\left(p+\frac{B^2}{2\mu_0}\right)=\cancel{\frac{(\overline{B}\nabla)\overline{B}}{\mu_0}}$$

Plasma pressure + mf- pressure = const: $$p+\frac{B^2}{2\mu_0}=const=\frac{B_0^2}{2\mu_0}$$



Magnetic confinement in θ-pinch

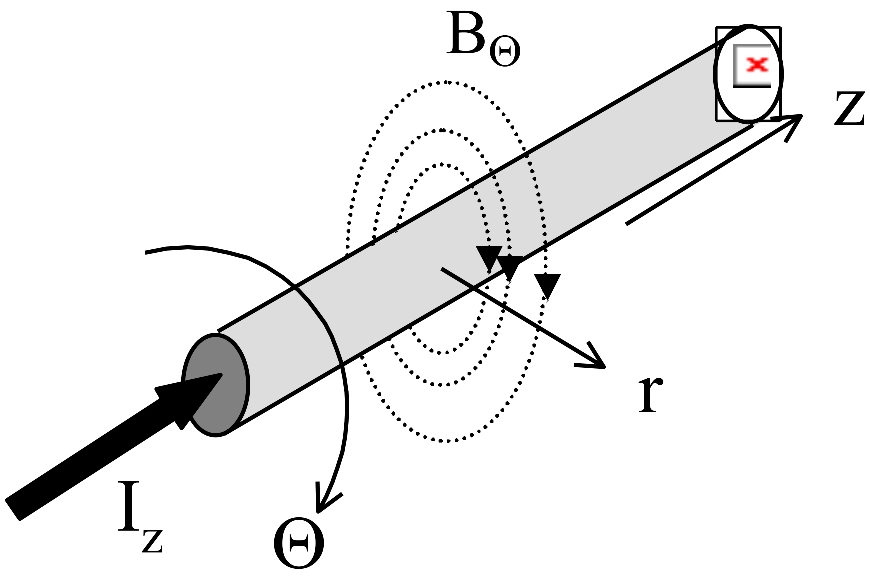
In θ-pinch no field line tension (MF constant along MF-lines):

$$\nabla\left(p+\frac{B^2}{2\mu_0}\right)=\cancel{\frac{\left(\overline{B}\nabla\right)\overline{B}}{\mu_0}}$$

Plasma pressure + mf- pressure = const: $$p+\frac{B^2}{2\mu_0}=const=\frac{B_0^2}{2\mu_0}$$

Normalised plasma pressure $$\beta \equiv \frac{p}{B_0^2/2\mu_0} = 1-\frac{B_i^2}{B_a^2}$$

Z-Pinch



$$\nabla p = \vec{j} \times \vec{B} = \cancel{j_\Theta \cdot B_z} \quad - \quad j_z \cdot B_\Theta$$

Z-Pinch-equilibrium

$$\boxed{\nabla p = \vec{j} \times \vec{B} = -j_z \cdot B_\Theta}$$

Ampere's law:

$$\mu_0 j_z = \nabla \times \vec{B} = \frac{1}{r}\frac{d}{dr}\left(rB_\Theta\right)$$

$$B_\Theta = \frac{\mu_0}{2\pi r} \cdot \int_0^r 2\pi r' \cdot dr' \cdot j_z(r') = \frac{\mu_0}{2\pi r} \cdot I_z(r)$$

$$j_z = \frac{1}{2\pi r}\frac{dI_z}{dr}$$

$$\nabla p = -\frac{\mu_0}{(2\pi r)^2} \cdot \frac{dI_z}{dr} \cdot I_z = -\frac{\mu_0}{2 \cdot (2\pi r)^2} \cdot \frac{d(I_z^2)}{dr}$$

Z-Pinch Equlibrium – total pressure

$$\nabla p = -\frac{\mu_0}{(2\pi r)^2} \cdot \frac{dI_z}{dr} \cdot I_z = -\frac{\mu_0}{2 \cdot (2\pi r)^2} \cdot \frac{d(I_z^2)}{dr}$$

Integration by parts

$$\int_0^a (2\pi r')^2 \nabla p(r') dr' = -\frac{\mu_0}{2} I_z^2 \qquad -4\pi \int_0^a 2\pi r' \, p(r') dr' = -\frac{\mu_0}{2} I_z^2$$

assume p=const. $\pi a^2 p = \frac{\mu_0}{8\pi} I_0^2$

Confinement for Z-Pinch: $V = \pi a^2, NkT = pV$

(per unit length)

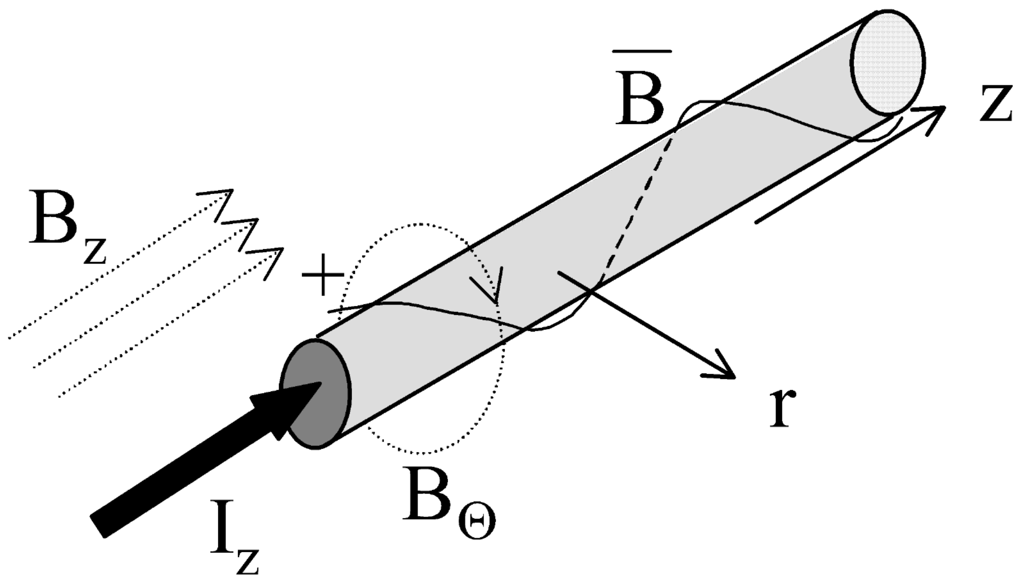
Bennet Condition: $\boxed{NkT = \mu_0 \cdot \frac{I_0^2}{8\pi}}$

Total current determines confined plasma pressure

Screw-Pinch

Θ-pinch has bad confinement properties (losses along magnetic field lines)
Z-pinch is very unstable (detailed analysis later)

Current and B-field in z- and Θ- direction



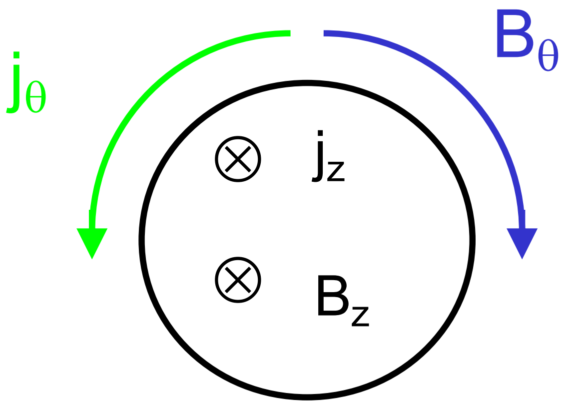
Pitch of field lines:

$$\frac{r d\theta}{dz} = \frac{B_\theta(r)}{B_z(r)}$$

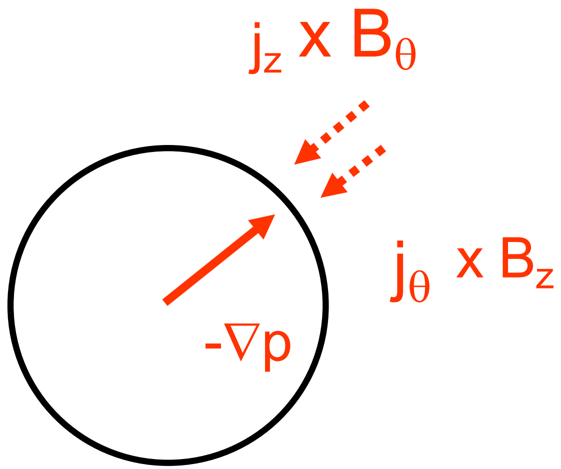
$$\boxed{\nabla p = \vec{j} \times \vec{B} = j_\Theta \cdot B_z - j_z \cdot B_\Theta}$$

Screw-Pinch is (mostly) diamagnetic

B_z is weakened
(as in θ-pinch)

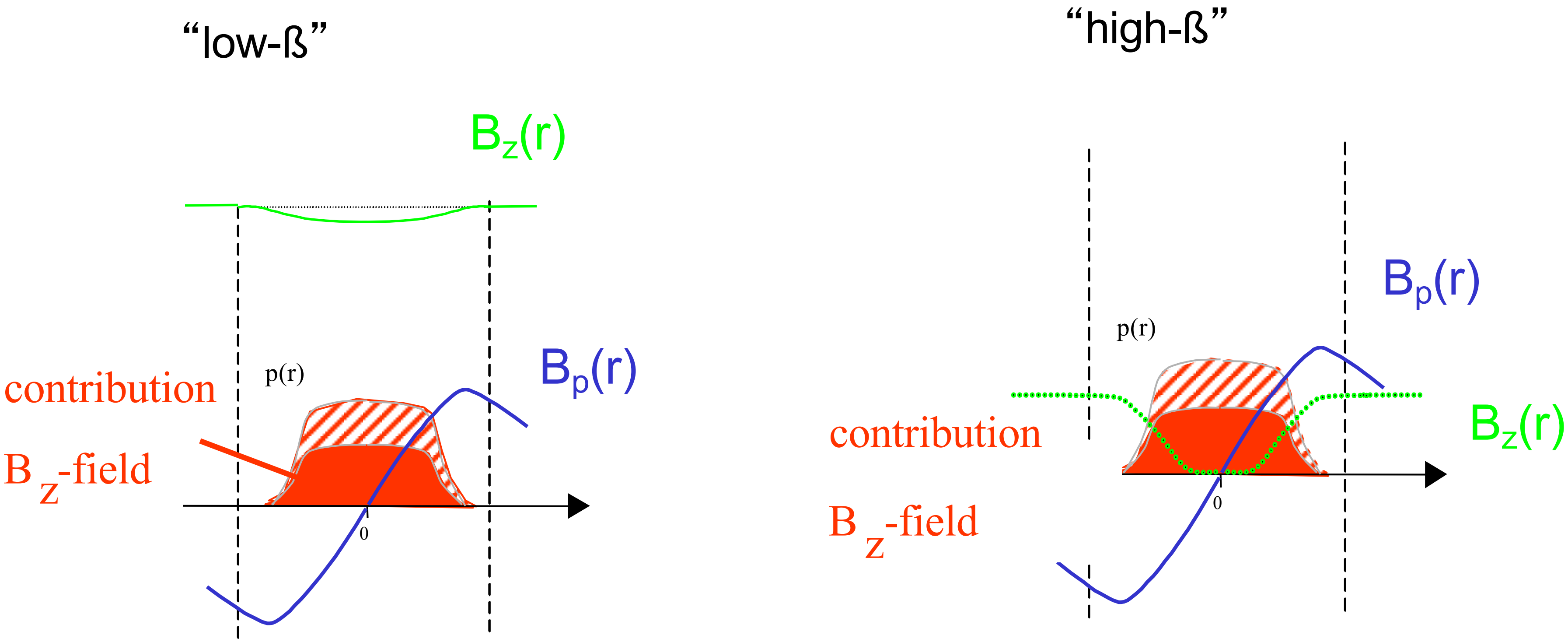


B_z and B_θ contribute to the confinement



Confinement is better than in z-pinch, pressure and current profiles can be chosen

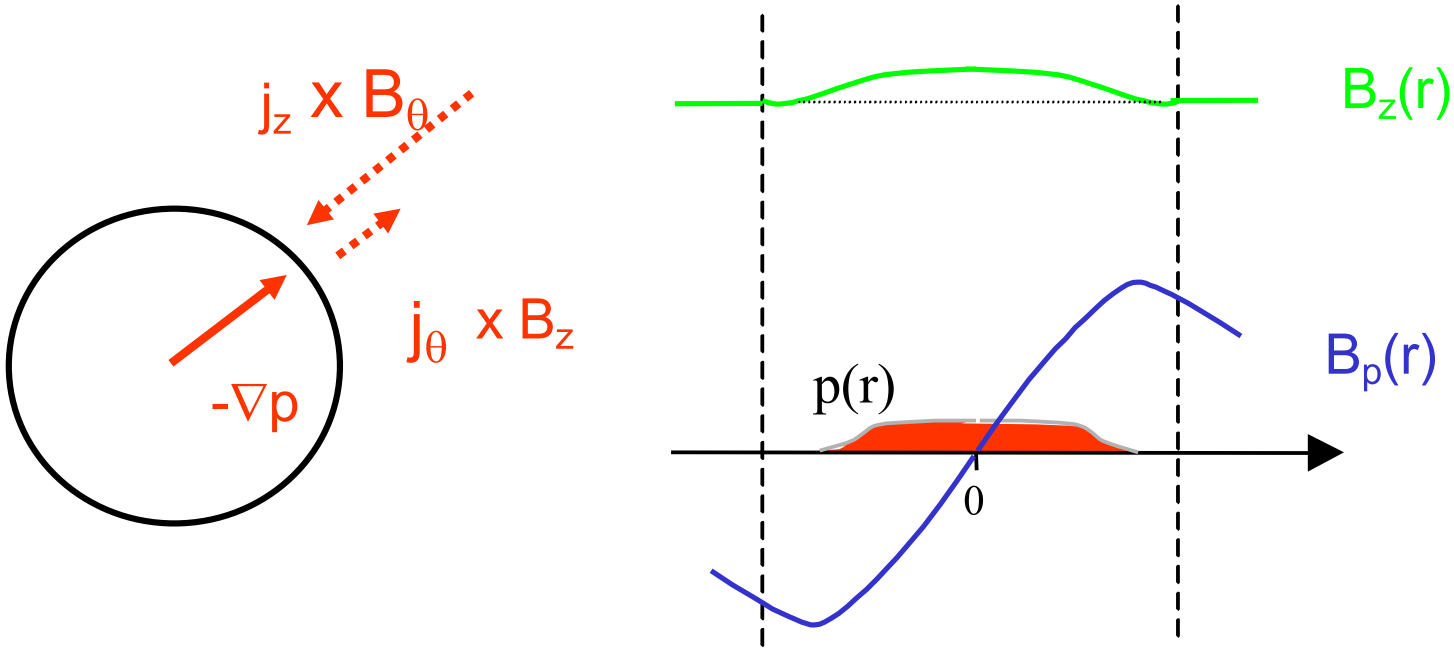
Screw-Pinch: high and low ß



Only the part of B_z that is generated by the current contributes to the confinement
(homogeneous MF influences only stability)

$$p + \frac{B^2}{2\mu_0} = const = \frac{B_0^2}{2\mu_0}$$

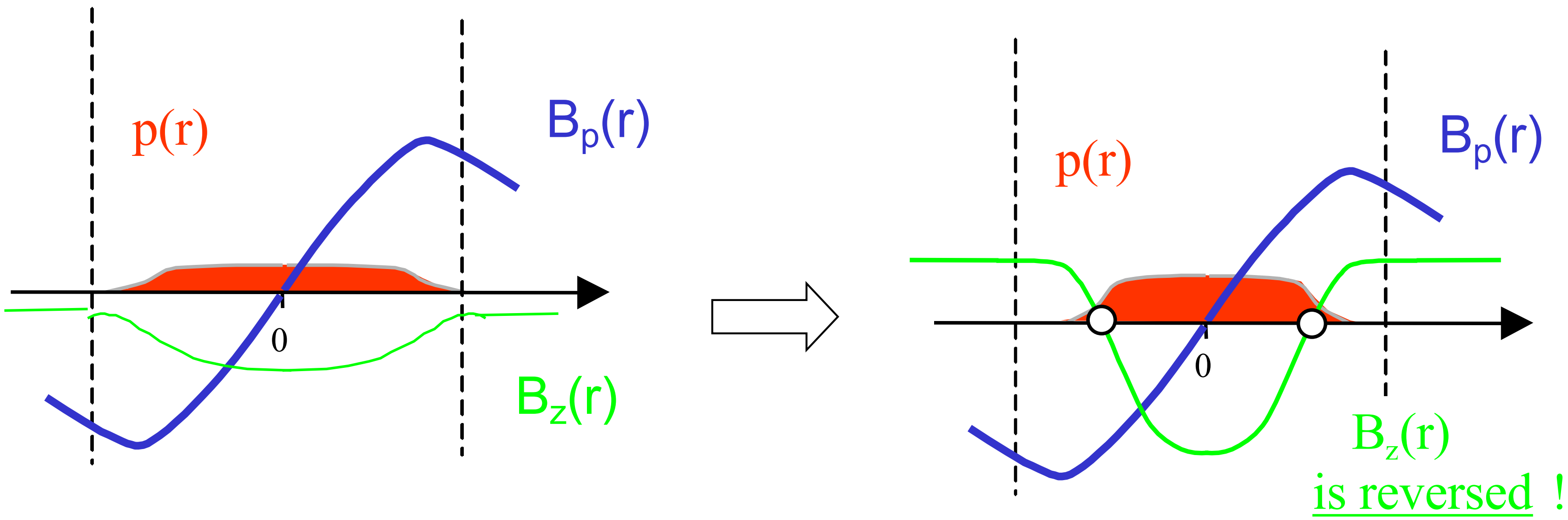
Usually, the plasma is diamagnetic, but at large currents it can be paramagnetic:



Confinement in that case worse than in Z-pinch

Reversed Field Pinch

Not stationary, sustained by turbulent plasma flows (dynamo effect) only



“Reversed-Field-Pinch“

- start phase -

“Reversed-Field-Pinch“

- final phase -

Equilibria with B_z and B_p-field

$$\boxed{\nabla p = \vec{j} \times \vec{B} = j_\Theta \cdot B_z - j_z \cdot B_\Theta}$$

θ-Pinch:

$$\nabla\left(p + \frac{B_z{}^2}{2\mu_0}\right) = 0$$

Z-Pinch:

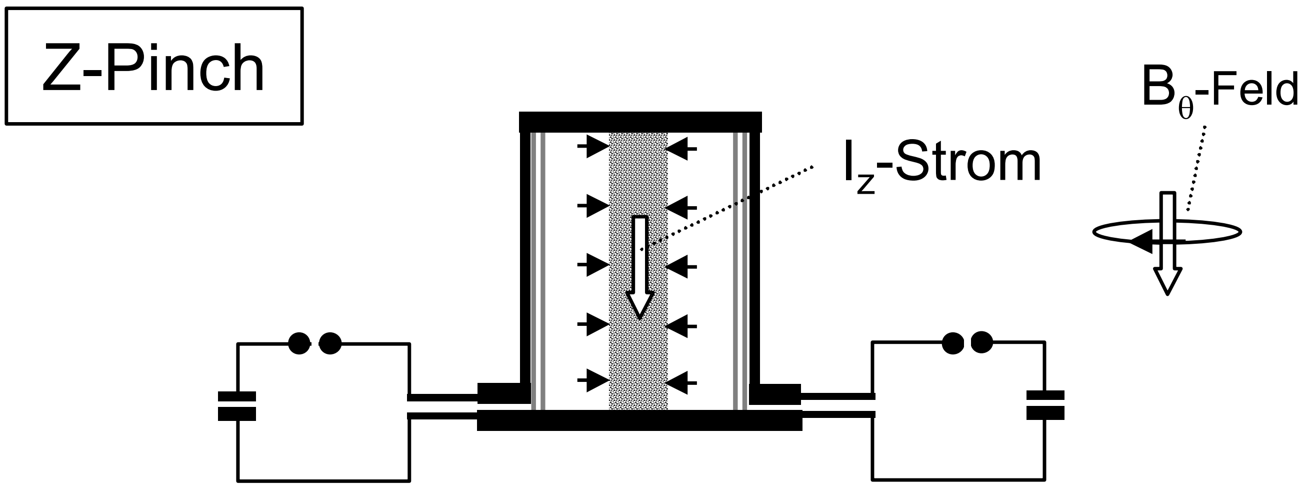
$$\nabla p = -\frac{\mu_0}{2\cdot(2\pi r)^2}\cdot\frac{d(I_z^2)}{dr}$$

B_z and B_p-field:

$$\boxed{\nabla\left(p + \frac{B_z{}^2}{2\mu_0}\right) = -\frac{\mu_0}{2\cdot(2\pi r)^2}\cdot\frac{d(I_z^2)}{dr}}$$

Z-pinch fusion experiments

- Start of plasma current by a capacitor bank
- Fast increase of current, but only at the edge (skin effect)
- Large magnetic field at the edge – plasma compression by $j_z x B_\theta$



Z-pinch fusion experiments

B_θ on axis =0, i.e. β=1

Fusion experiments in the 1960…1970:

- high β, but only very short pulses
- no methods for plasma heating available, thus adiabatic compression used

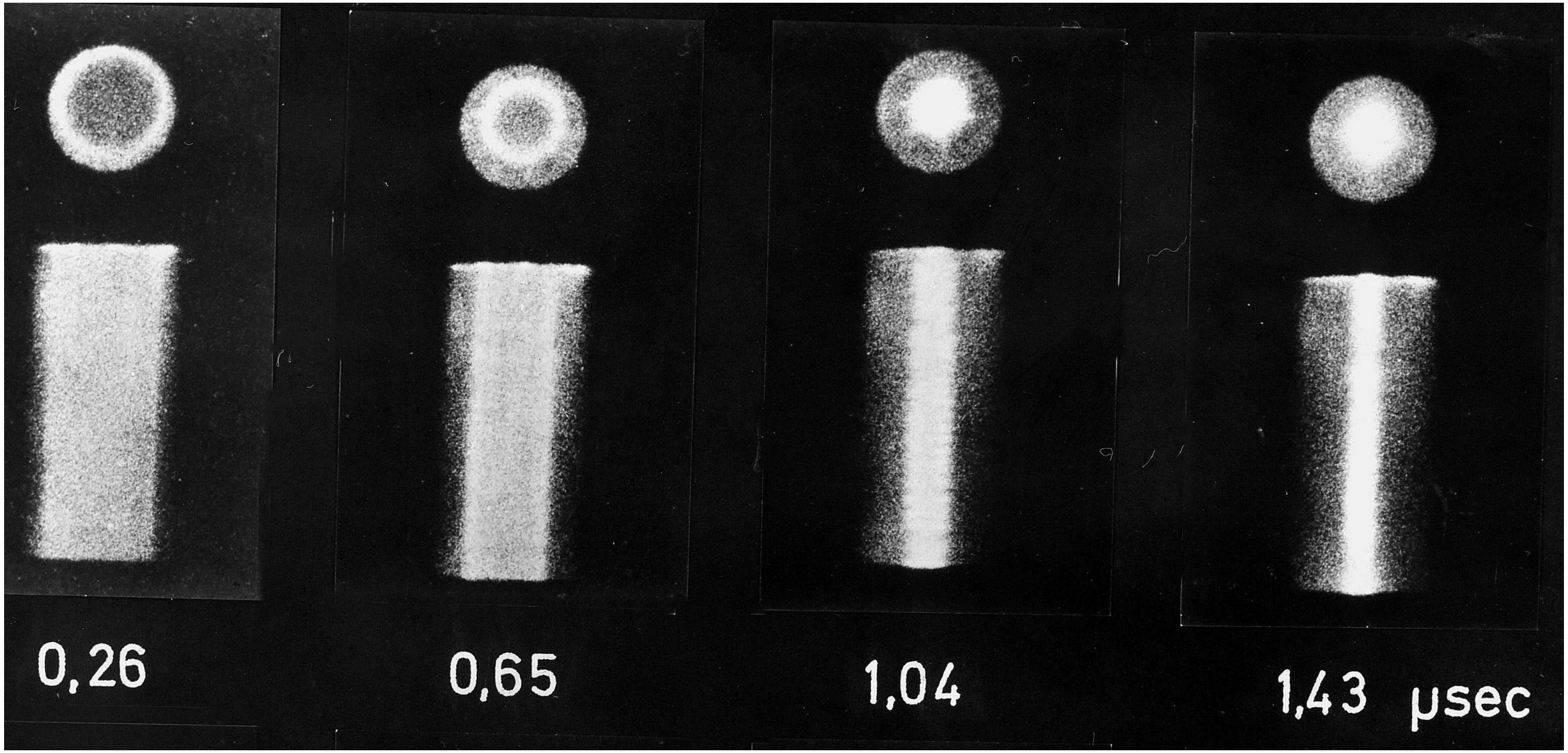
$$T \cdot V^{\gamma-1} = const$$

decrease in volume → increased density and temperature

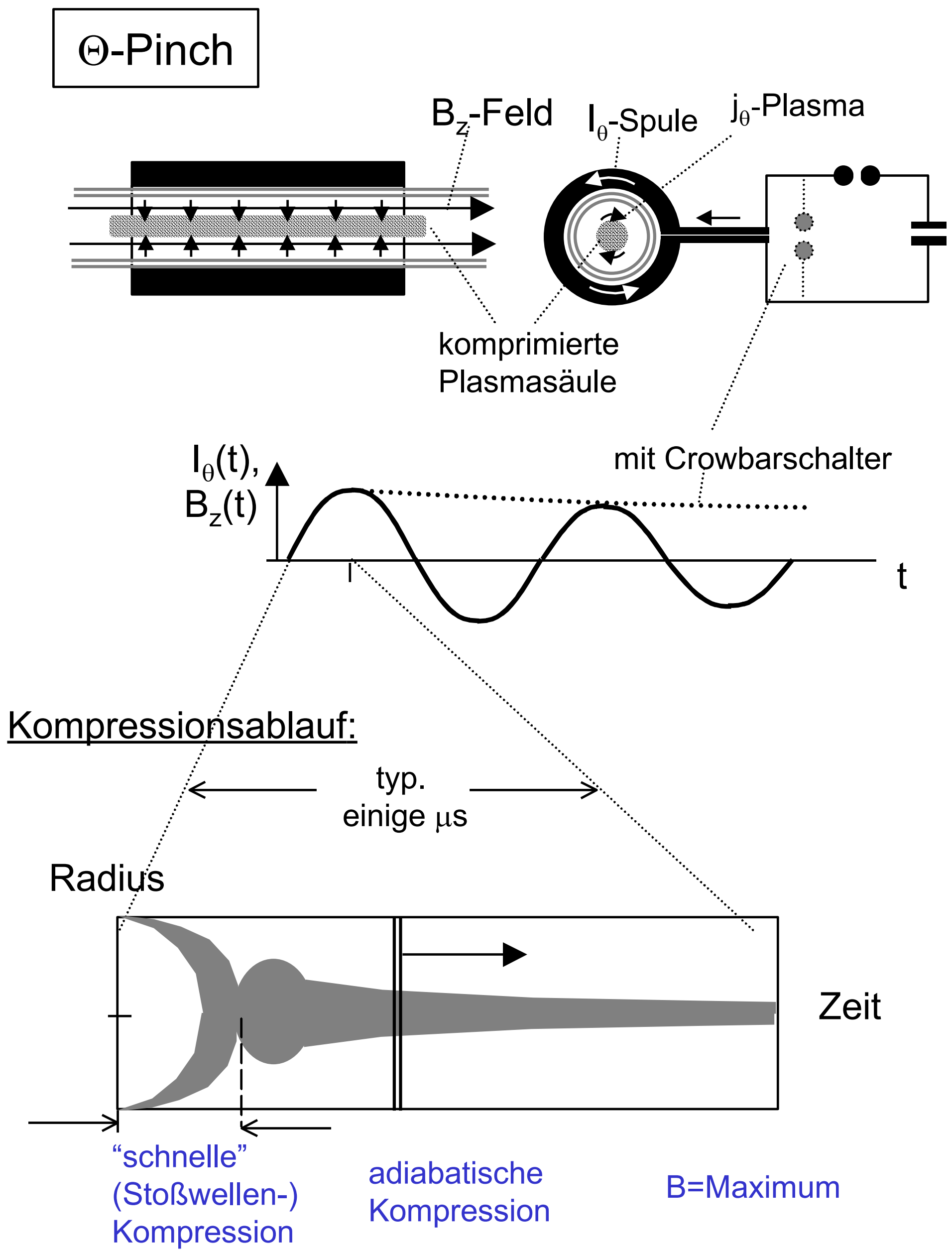
~100 Mio K, $10^{22} m^{-3}$, B~20 T

But very unstable!

Z-pinch fusion experiments



θ-pinch fusion experiments

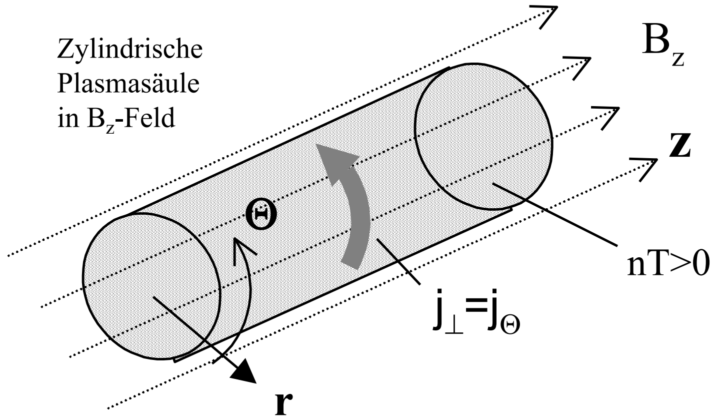


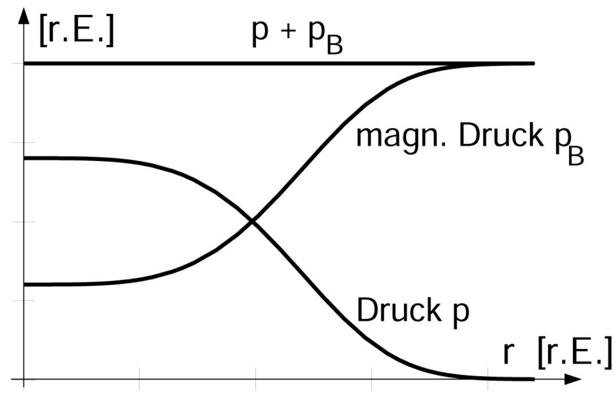
Current in shell induces plasma current in opposite direction and produces B field inside the plasma

- Fast compression: 200-300 ns
- afterwards further adiabatic compression due to increase of magnetic field

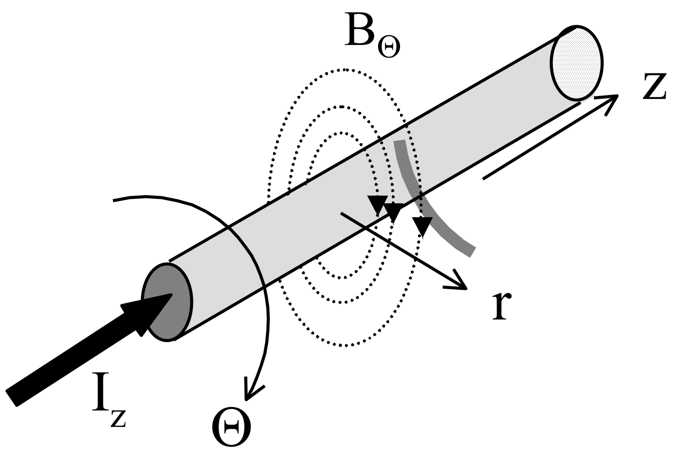
summary

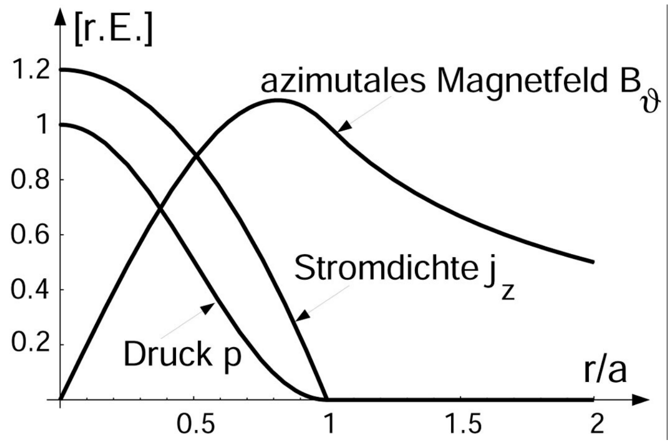
θ-pinch





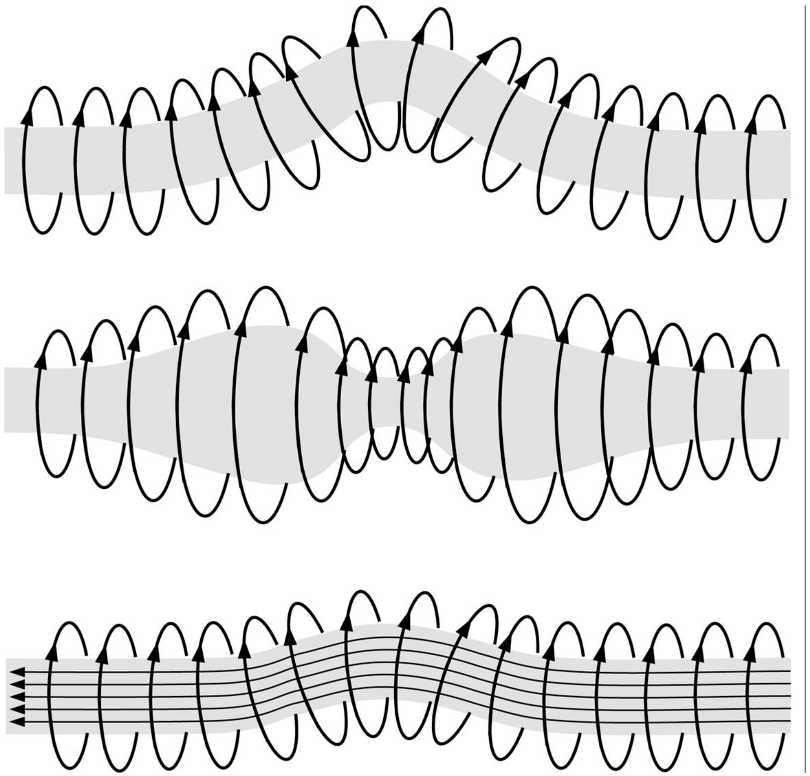
Z-pinch



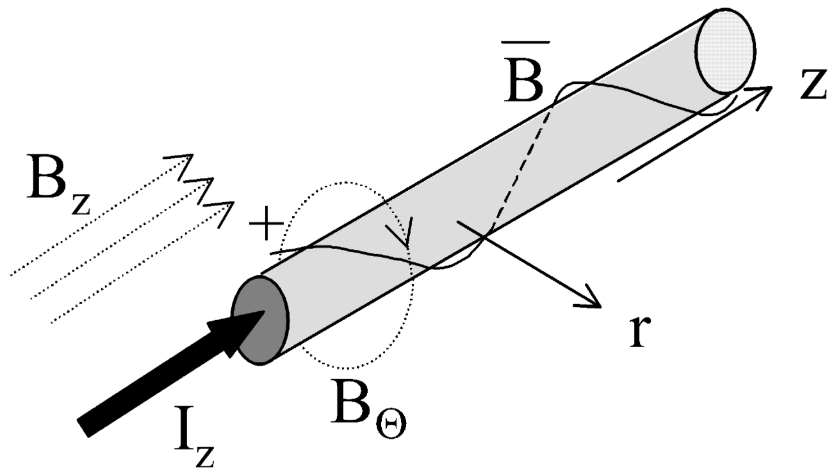


summary

Z- -pinches are unstable:



Screw-pinch has better stability properties!

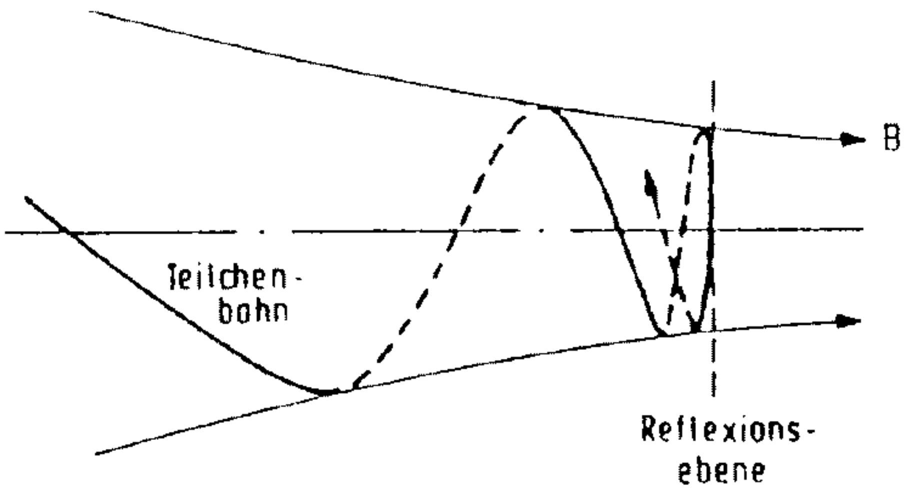


Avoiding end losses due to Magnetic Mirror?

Reflection of particles in regions of high magnetic field

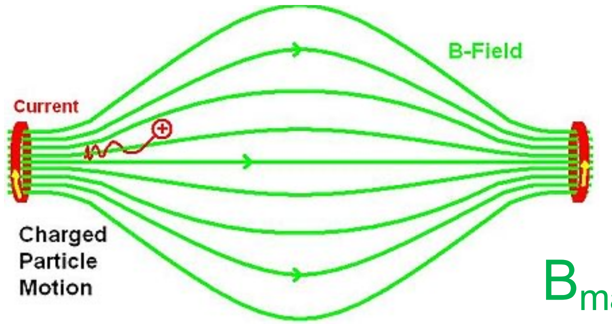
magnetic moment (adiabatic invariant): $\mu = \frac{mv_\perp^2}{2B}$

as energy conservation holds as well: reduced parallel energy when approaching regions with increasing B, down to $v_{||}=0$ (Reflection)



Magnetic Mirror

$$\mu = \frac{mv_\perp^2}{2B}$$



B_{max}

Mirror condition:

$$\frac{m}{2}\left(v_\perp^2(B_{min}) + v_\parallel^2(B_{min})\right) < \frac{m}{2} v_\perp^2(B_{max})$$

$$\frac{v_\perp^2(B_{max})}{B_{max}} = \frac{v_\perp^2(B_{min})}{B_{min}} \Rightarrow \quad \frac{m}{2} v_\perp^2(B_{max}) = \frac{B_{max}}{B_{min}} \frac{m}{2} v_\perp^2(B_{min})$$

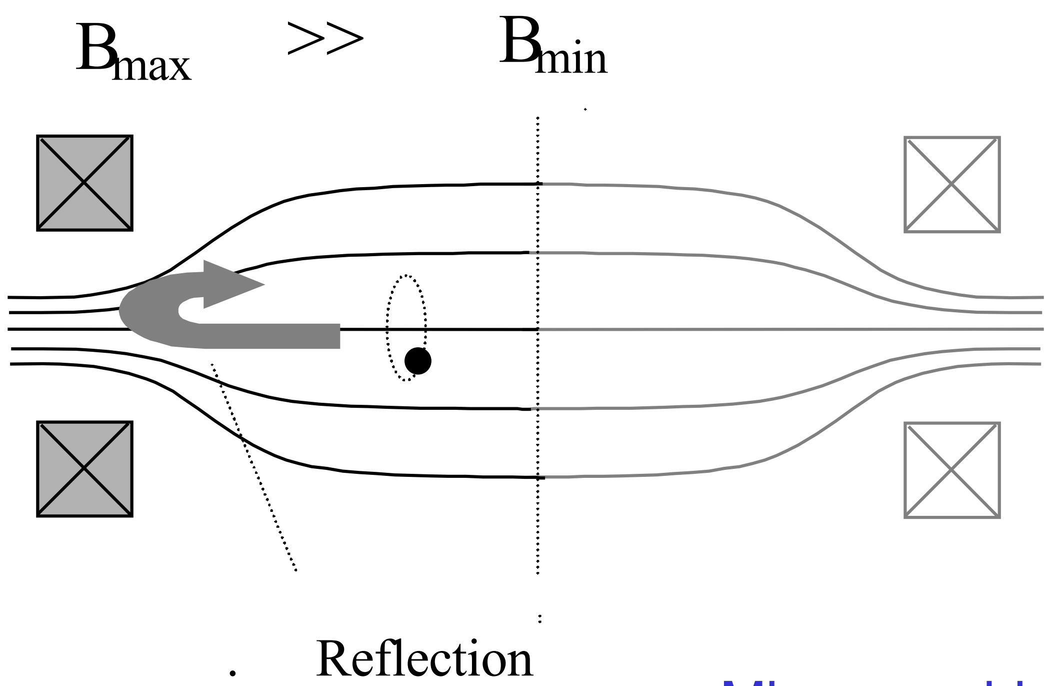
$$\frac{m}{2}\left(v_\perp^2(B_{min}) + v_\parallel^2(B_{min})\right) < \frac{B_{max}}{B_{min}} \frac{m}{2} v_\perp^2(B_{min})$$

$$1 + \frac{v_\parallel^2(B_{min})}{v_\perp^2(B_{min})} < \frac{B_{max}}{B_{min}}$$

Magnetic Mirror

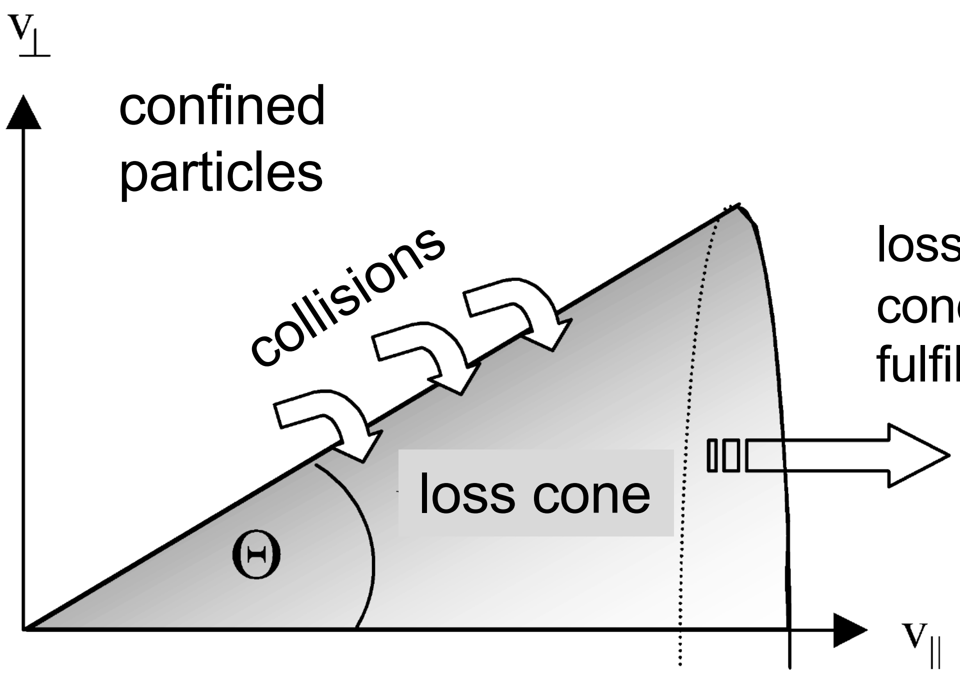
Mirror condition:

$$\frac{v_{\parallel}^2(B_{\min})}{v_{\perp}^2(B_{\min})} < \frac{B_{\max}}{B_{\min}} - 1$$



Mirror machine

Mirror machines for magnetic confinement of hot plasmas?



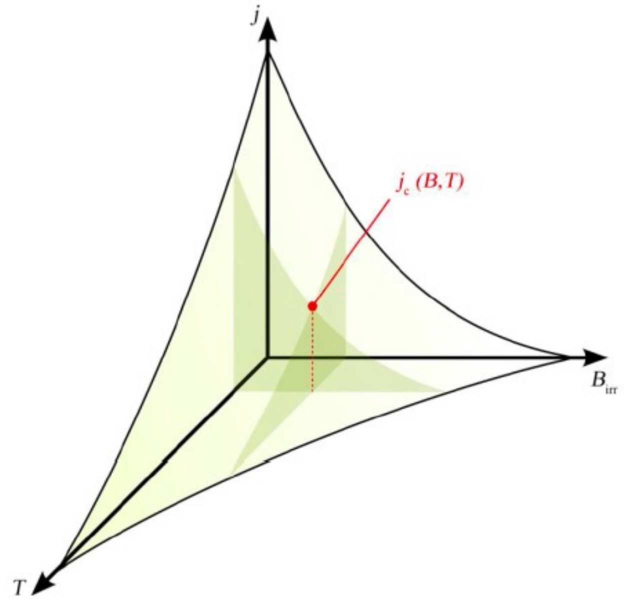
Particles with large parallel energy get lost!
(in thermal plasmas "loss cone" re-filled by collisions)

Mirror condition independent of mass and charge, but electrons have higher collision rate -> Electrons get lost faster

High temperature super conductors allow for higher magnetic fields and thus higher mirror ratios

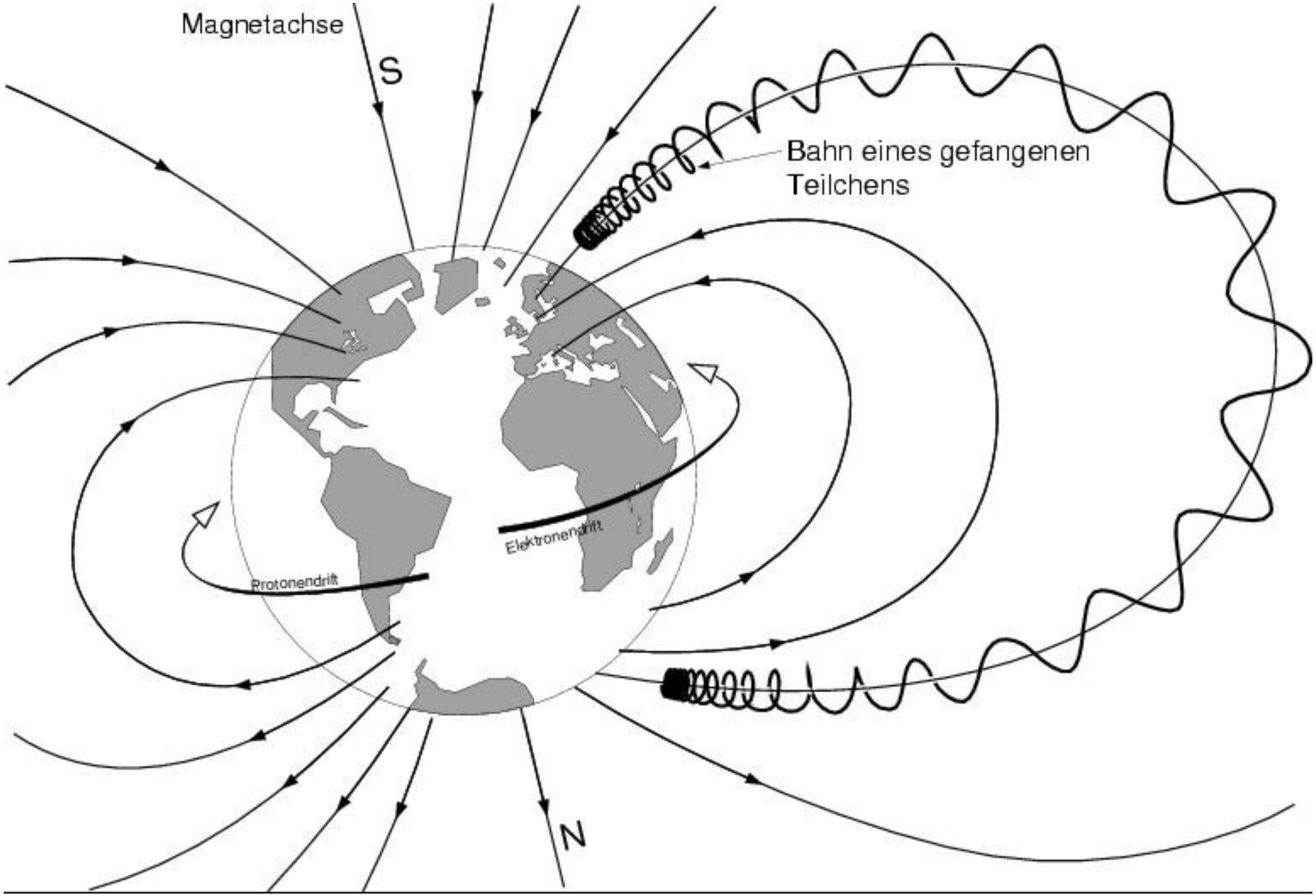
- Few HTS materials made it to commercial products
  - BSCCO (bismuth–strontium–calcium–copper oxide)
  - REBCO (rare–earth–barium–copper oxide)

- HTS: **High T_c and high critical magnetic field**
  - suitable for high field applications only at operating temperature <25 K
  - for high field applications helium or hydrogen cooling



Revival of the mirror concept, tandem mirrors

Another example for magnetic mirrors: Van-Allen belt




„Faszination Polarlicht“

New/alternative proposals towards fusion

- Many more than discussed here
- Selected those that are most open about their concept

Aim is often: smaller size

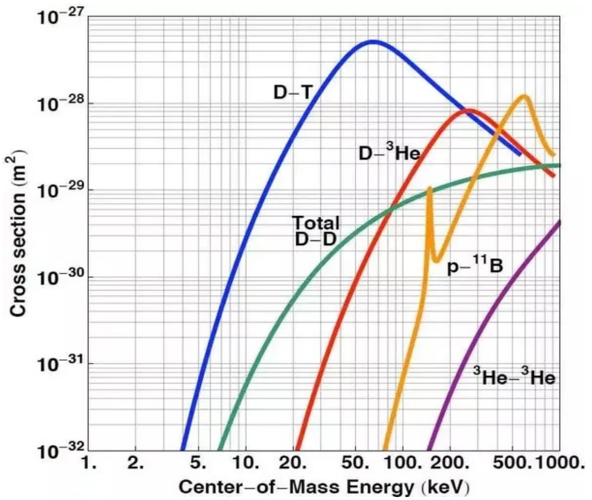
- But small size is not compatible with D-T fusion (neutron shielding, T breeding)
- Thus: many projects start with proposal to use neutron free reactions

Neutron-free fusion reactions

$^{2}D + ^{3}T \rightarrow ^{4}He\ (3.5MeV) + n\ (14.1MeV) + 17.6MeV$

$^{2}D + ^{3}He \rightarrow ^{4}He\ (3.7MeV) + p\ (14.6MeV) + 18.3MeV$ (but: D-D reaction and ^{3}He availability)

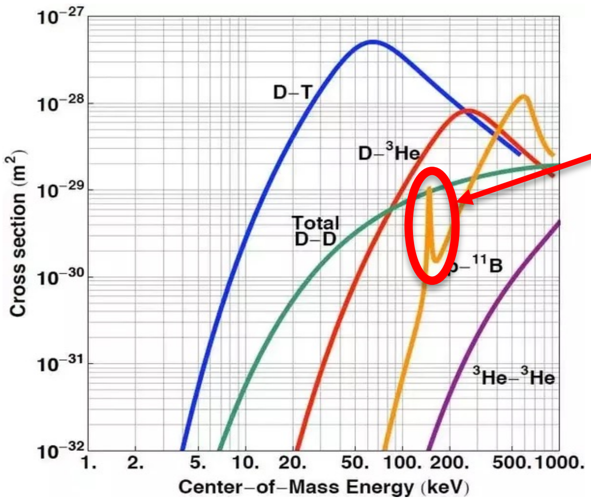
$p + ^{11}B \rightarrow 3\,^{4}He + 8.7MeV$



Parameter\Reaction	D-T	D-He^{3}	D-D	H-B^{11}
optimum composition for maximum fusion power at given pressure (Te=Ti)	1:1	3:2	1:1	3:1
maximum fusion power density at constant pressure (rel.units)	1,00	0,02	0,04	0,0013
maximum ratio $<\sigma v>/T^2$	1,00	0.022	0.013	0,008
burn temperature[keV] optimized for power density at given pressure	15,00	50,00	20,00	140,00
minimum required $nT\tau$ for ignition (rel.units)	1	11	16	100

Proton-Bor fusion: $p + {}^{11}B \rightarrow 3\,{}^{4}He + 8.7MeV$

- Lower power density at given pressure than other reactions (1000x lower than DT)
- → large volume or high pressure needed
- Fusion power output: marginal, only Bremsstrahlung losses about 90% of fusion power



Non-thermal reaction?

- Narrow velocity distribution of p needed
- Direct acceleration/heating of p
- Fusion alpha particle distribution is isotropic!