2. Plasma frequency, collisions (discussion and correction: 19.11.2013)

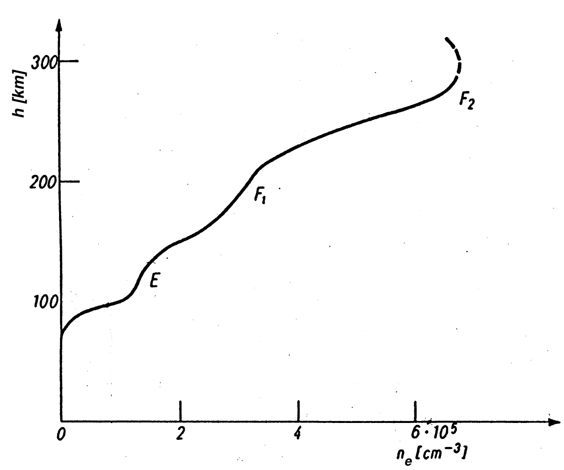
A) What is the plasma frequency, cut-off? What assumptions are made?
B) Show that the plasma frequency is

$$\omega_p{}^2 = \omega_{pe}^2 + \omega_{pi}^2 = \frac{n_e\, e^2}{\epsilon_0\, m_e} + \frac{n_i\, Z^2 e^2}{\epsilon_0\, m_i},$$

if also the motion of the background ions is included. Here, Z is the charge number of the ions. Use Poisson's equations and the equation of motions and continuity for both ions and electrons. As ansatz use small perturbations in the form of plane waves, i.e.

$$n_e = n_{e0} + n_{e1} e^{i(kr - \omega t)}.$$

(also for n_i, v_i, v_e and E).
C) In the ionosphere air molecules are ionised by photons. Due to the decreasing density and different absorption processes there are several layers at different heights (D, E, F_1, F_2 layer). The ionisation grade varies due to different solar activity (day/night) up to 2 orders of magnitude. The follwing graph shows the electron density for some intermediate activity:



Calculate the critical frequency that separates reflected and transmitted electromagnetic waves! What are the implications for long-distance radio waves?

D*) Describe the role of neutrals and Coulomb collisions in the Ionosphere!

Solution (19.11.2013):

A) see lecture

B)

Die zu linearisierenden Gleichungen sind:

$$m_{e,i} n_{e,i} \frac{\partial \vec{v}_{e,i}}{\partial t} = q_{e,i} n_{e,i} \vec{E}$$

$$\frac{\partial n_{e,i}}{\partial t} + \vec{\nabla} \cdot (n_{e,i} \vec{v}_{e,i}) = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} (q_i n_i + q_e n_e)$$

mit $q_e = -e, \quad q_i = Ze.$
Mit dem in den Hinweisen gegebenen Ansatz lautet das linearisierte homogene Gleichungssystem:

$$\begin{aligned} i m_e \omega v_{e1} + q_e E_1 &= 0 \\ i m_i \omega v_{i1} + q_i E_1 &= 0 \\ -\omega n_{e1} + k n_{e0} v_{e1} &= 0 \\ -\omega n_{i1} + k \frac{n_{e0}}{Z} v_{i1} &= 0 \\ \epsilon_0 i k E_1 - (q_i n_{i1} + q_e n_{e1}) &= 0 \end{aligned}$$

Eine von Null verschiedene Lösung für $v_{e1,i1}, n_{e1,i1}, E_1$ existiert nur wenn die Determinante verschwindet, also

$$0 = -\omega^2 m_i m_e \epsilon_0 + q_i^2 m_e \frac{n_{e0}}{Z} + m_i q_e^2 n_{e0}$$

und

$$\omega^2 = \omega_p^2 = \frac{e^2 n_{e0}}{m_e \epsilon_0} + \frac{Z e^2 n_{e0}}{m_i \epsilon_0} = \frac{e^2 n_e}{m_e \epsilon_0} + \frac{Z^2 e^2 n_i}{m_i \epsilon_0} = \omega_{pe}^2 + \omega_{pi}^2$$

Für ein Wasserstoffplasma gilt:

$$\omega_p^2 = \omega_{pe}^2 (1 + \frac{m_e}{m_i})$$

$$\Rightarrow \frac{m_e}{m_i} = \frac{(\omega_p + \omega_{pe})(\omega_p - \omega_{pe})}{\omega_{pe}^2} \approx 2 \frac{\omega_p - \omega_{pe}}{\omega_p} = 5.4 \cdot 10^{-4}$$

Der relative Fehler den man bei der Verwendung von ω_{pe} statt ω_p macht, beträgt also nur 0.00027.

C) The maximal electron density is $6.9 \cdot 10^{11} m^{-3}$. The plasma frequency is:

$$\omega_p = \sqrt{\left(\frac{e^2 n_e}{\epsilon_0 m_e}\right)}; \quad f_p = \omega_p/(2\pi) = 7.45\text{MHz}$$

Consequently, VHF (=UKW) waves can penetrate the ionosphere, HF (=KW) and low frequency waves are reflected. For that reason the low frequency radio channels can be recieved after one (or more) reflection in a very large distance from the sender. Note that the damping of waves is larger for low frequency waves. Therefore, there is a frequency window that varies from day to night and is influenced by the solar activity.

D) see given literature; in short: below 200km collisions with neutrals dominate (high neutral density), above 200 km Coulomb collisions are more important (higher $n_{i,e}$, less neutrals).