Gaussian Beam Approximations of High Frequency Waves

Olof Runborg

We consider high frequency wave propagation problems, where the wave length is significantly smaller than the overall size of the computational domain. This is a multiscale problem for which direct discretization methods are computationally very expensive. Instead numerical methods based on various asymptotic approximations, such as geometrical optics, can be used. In this talk we discuss Gaussian beams, which are asymptotically valid high frequency solutions to hyperbolic partial differential equations, concentrated on a single curve through the physical domain. They can also be extended to some dispersive wave equations, such as the Schrödinger equation. Superpositions of Gaussian beams provide a powerful tool to generate more general high frequency solutions that are not necessarily concentrated on a single curve. In contrast to the standard geometrical optics, the Gaussian beam approximation does not break down at caustics. We will discuss numerical methods based on Gaussian beam superpositions as well as error estimates in terms of the small wavelength.