A High Order A-Stable Wave Propagation Method

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The goal of this work is the development of a low storage implicit Maxwell solver for application to problems in plasma physics. The key issue motivating this work is the vast scale separation in temporal scales in these problems. The two common approaches are sub-cycling and the introduction of implicit methods. The problem with sub-cycling is the time and loss of accuracy do to hundreds of EM solves per single time step of the plasma species whereas the issue with implicit solvers is the time associated with inversion of the matrix.

In recent work, we developed a new A-stable approach to wave propagation problems based on the Method of Lines Transpose (MOL^T) formulation combined with alternating direction implicit (ADI) schemes. Because our method is based on an integral solution of the ADI splitting of the MOL^T formulation, we are able to easily embed non-Cartesian boundaries and include point sources with high accuracy. Further, we developed an efficient $O(N)$ convolution algorithm for rapid evaluation of the kernels. We have demonstrated the utility of this method by applying it to a range of problems with complex geometry, including cavities with cusps.

However, one of the well-known drawbacks of ADI methods is the high degree of dispersion that they introduce. Furthermore, the Dahlquist barrier is a well-known theorem which prevents the construction of a scheme of order higher than 2, which remains unconditionally stable. We note that this barrier can be removed if the scheme is not a linear multistep scheme.

In this work, we present several important modifications to our recently developed wave solver. We obtain a family of wave solvers which are unconditionally stable, accurate of order $2P$, and require $O(NP)$ operations per time step, where N is the number of spatial points. We obtain these schemes by including higher derivatives of the solution, rather than increasing the number of time levels, thus removing the Dahlquist barrier. The novel aspect of our approach is that the higher derivatives are constructed using successive applications of the convolution operator, which does not formally change the size of the stencil. One way to view this method is to consider it through the lens of defect correction, as with each application of the convolution kernel, we are eliminating the next highest error term.

We develop these schemes in one spatial dimension, and then extend the results to higher dimensions, by reformulating the ADI scheme to include recursive convolution. Thus, we retain a fast, unconditionally stable scheme, which does not suffer from the large dispersion errors characteristic to the ADI method. We demonstrate the utility of the method by applying it to a host of wave propagation problems. In our future work we plan to combined these methods with mesh based and particle based Vlasov solvers.