

Variational Integrators for the Vlasov-Poisson System

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Variational integrators provide a systematic way to derive geometric numerical methods that preserve a discrete multisymplectic form (and therefore have good long time energy behaviour) as well as momenta associated to symmetries of the system by Noether's theorem. We first outline the fundamental theory of variational integrators for field theories including the derivation of discrete conservation laws by a discrete Noether theorem. We sketch how to derive variational integrators for systems that do not feature a natural variational formulation, like the equations of advection-diffusion type which are often found in fluid dynamics and plasma physics. We explain the detailed derivation of variational integrators and conservation laws for the advection equation as a prototypical example that shares many similarities with the Vlasov-Poisson system but is more instructive due to its reduced complexity. Finally, we present several variational integrators for the Vlasov-Poisson system which preserve the total particle number, momentum and energy as well as norms of the distribution function exactly (up to machine precision). These properties will be demonstrated by typical test cases from plasma and astro physics.