Cubic finite–element methods to simulate wave propagation and absorption in fusion plasmas

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Abstract

Numerical simulations of radio-frequency heating amount to solve partial differential equations for the propagation and absorption of electromagnetic waves in fusion plasmas. The low-cost computational description of wave propagation and absorption with asymptotic methods, such as geometrical optics (ref. Dr Maj's tutorial), is not viable for waves in the ion cyclotron range of frequencies, characterized by wavelengths comparable with the characteristic lengths of plasma equilibrium profiles. Therefore the wave equations have to be directly solved in realistic geometry, at a rather high computational price.

The wave equations are derived from the Maxwell's equations under the ansatz $\exp(-i\omega\ t)$, with ω the frequency of the source. Upon using the spectral ansatz in two spatial directions, the starting wave equations are reduced to a set of ordinary differential equations in the radial coordinate. These equations describe the linear coupling of three wave branches, characterized by very different wavelengths and not always simultaneously propagative inside the plasma. These peculiarities make the problem mathematically stiff and for some numerical schemes the numerical noise can select the unphysical and exponentially growing solution at the expense of the physical one. On the other hand, finite–element methods are well suited for the wave propagation problems, since they look for the global solution of the boundary-value problem. Since the finite–element methods are applied to the weak formulation (Galerkin) of the wave equations, the use of *cubic* elements automatically guarantees also the continuity of the derivative of the solution (in this case the wave magnetic field).

In this talk the finite–element methods applied to the wave equations are presented with a few practical examples.