On micro-macro numerical schemes for multiscale kinetic equations

Mohammed LEMOU
CNRS, IRMAR, University of Rennes 1
mohammed.lemou@univ.rennes1.fr

Abstract

The development of numerical methods to solve multi-scale kinetic equations has been the subject of many works in the past and is still a challenge in important international research programs, with applications in various fields: plasma physics, rarefied gas dynamics, aerospace engineering, semiconductors, radiative transfer, ... The general problem is to construct numerical schemes enjoying the so-called asymptotic preserving (AP) property: they are able to efficiently solve the kinetic model at various scales, while the numerical parameters remain independent of the stiffness character of these scales. In many systems of particles, one may have for instance several regimes in different regions of the physical space: microscopic regime (kinetic) or macroscopic regimes (fluid, diffusion,etc...). To avoid a domain decomposition method (according to these different regimes), which faces the delicate problem of handling the moving interfaces, we have developed an alternative approach which is based on a decomposition of the distribution function in the whole space. The original model is then decomposed into a system of two equations: an equation on a macro part (equilibrium or averaged part) whose evolution is coupled to an equation on the remaining micro (kinetic) part. Starting from suitable forms of this decomposition, AP numerical schemes are then constructed to solve the original kinetic model. This strategy is quite robust in the sense that it can be adapted to a large class of kinetic equations (Vlasov-BGK, Vlasov-Boltzmann, Vlasov-Landau, collisionless Vlasov, etc) and to various scales (kinetic/diffusion, kinetic/fluid, kinetic/high fields, two-scales asymptotics and guiding center approximations, highly oscillating Schrodinger equation, etc). The numerical schemes which are constructed from such formulations have the following AP property: they are consistent with the model at the kinetic regime and they degenerate into a scheme which is consistent with the macro model in the considered asymptotic limit, the numerical parameters being fixed. No inversion of collision operators (even linearized) is needed and any desired numerical discretization for the limiting model may be reached by such AP schemes. In this talk, we shall first present the main lines of this strategy for different asymptotics in the collisional regime: Fluid, diffusion and high field asymptotics. Then, we will show how to modify the approach in order to deal with the space boundary conditions and to provide good approximations of boundary layers as well. Finally we will discuss possible extensions to systems with high oscillations such as the Schrodinger equation.