Kinetic instabilities driven by runaway electrons

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Outline

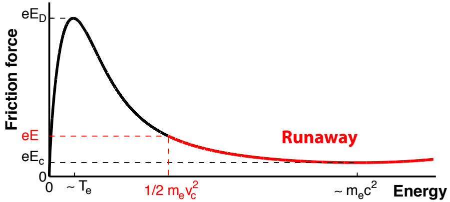
- Kinetic instabilities driven by runaway electrons
  - Whistler waves
  - Extraordinary electron waves
- Synchrotron radiation diagnostics
- Magnetic perturbations

Runaways in plasmas

- Runaway acceleration of some electrons if $E > E_c$

$$E_c = \frac{n_e e^3 \ln \Lambda}{4\pi\epsilon_0^2 m_e c^2}$$

- Hot tail generation in case of rapid cooling (dominates over Dreicer in ITER if $\tau_{th} \sim 1\ \mathrm{ms}$).
- Secondary generation due to close Coulomb collisions $\rightarrow$ Exponential growth of runaways.

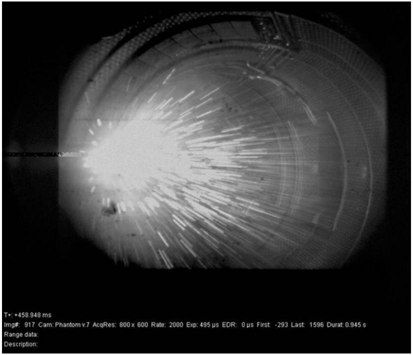


$$E_c(\mathrm{V/m}) \simeq 0.1 n_e (10^{20}\ \mathrm{m}^{-3})$$

$$\frac{E_c}{E_D} = \frac{T_e}{m_e c^2} \ll 1$$

Runaway avalanches

- In tokamak disruptions:
  - the plasma cools quickly,
  - the resistivity $\eta \propto T^{-3/2}$ rises, and
  - a high electric field is induced to maintain the plasma current.
- The pre-disruption current is partly replaced by a current of runaway electrons.

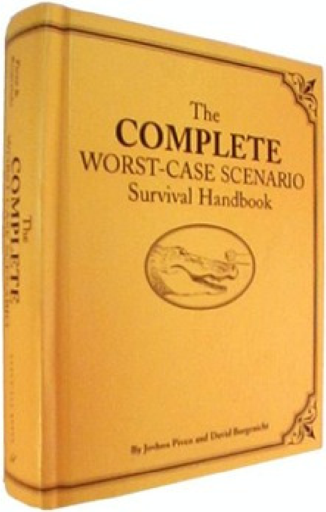


Carbon dust particles produced when runaways hit a plasma-facing component in Tore Supra.

Damaging potential is huge

- “Several kg of molten material can be produced (and moved around by gravity and $\mathbf{j} \times \mathbf{B}$ forces) by a single runaway event.”

  [Progress in ITER Physics Basis, NF 47 S180 (2007)]
- Mitigation:
  - gas injection
  - magnetic perturbations
- Kinetic instabilities
  - increased transport
  - diagnostics



Avalanche is primary mechanism in ITER

- Growth rate of runaway current
$$\gamma_{RA} = \frac{1}{j_{RA}} \frac{dj_{RA}}{dt} \simeq \frac{E-1}{c_z \tau}$$
where $c_z = \sqrt{3(Z+5)/\pi} \ln \Lambda$ and $E = E_{\|}/E_c$.
- Total number of e-folds during an avalanche
$$\gamma_{RA} t \simeq \frac{\int (E-1)dt}{c_Z \tau} \simeq \frac{2 I_p}{c_Z I_A}$$
where $I_A = 0.017\ \mathrm{MA}$.
- Avalanche multiplication in ITER $\sim e^{50}$ ($\ln \Lambda = 15$, $Z = 1$, $I_p = 15\ \mathrm{MA}$).
- Plasma current is a key parameter and defines the runaway formation.

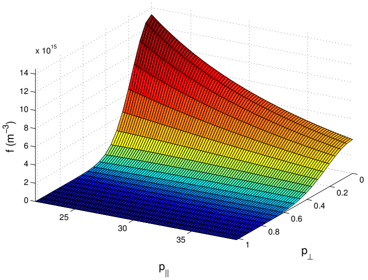
Runaway distribution

The distribution of relativistic secondary runaways is

$$f(p_\perp, p_\|, t) = \frac{\alpha n_r(t)}{2\pi c_Z p_\|} e^{-\frac{p_\|}{c_Z} - \frac{\alpha p_\perp^2}{2p_\|}}$$

where $\alpha = (E-1)/(1+Z)$,
$dn_r/dt = (E-1)n_r/c_Z\tau$.

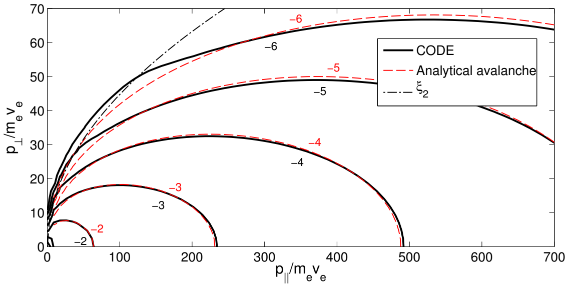
[Rosenbluth & Putvinski, NF 37 (1997)]



[Fülöp *et al*, PP 13, 0625 (2006)]

COllisional Dynamics of Electrons (CODE)

- CODE is a 2D continuum code for computing the distribution function of electrons, including both primary and secondary runaways.
- Successfully benchmarked to analytical results in relevant limits.



[Landreman, Stahl & Fülöp, submitted to CPC, http://arxiv.org/abs/1305.3518]

Kinetic instabilities

- High-frequency electromagnetic waves can be destabilized by the secondary runaway electron beam through resonant interaction at the anomalous Doppler resonance $\omega - k_{||} v_{||} = -\omega_{ce}/\gamma$.
- The main result of the instability is rapid pitch-angle scattering of the runaway electrons.
- The instability threshold depends on the magnetic field and the temperature.

Non-exhaustive list of previous work

- Resonant interaction of runaways with lower-hybrid waves
  [Parail & Pogutse, NF, 18, 303 (1978); Rev PP, 11, 1 (1986);
  Liu & Mok, PRL, 38, 162, (1977)]
- Instability threshold of the upper-hybrid wave is $Z_{\rm eff}\sqrt{n_{20}}/(T_{eV})^{3/2} < 0.01$
  [Rosenbluth *et al* IAEA Montreal 1996]
- Whistler wave instability (WWI) driven by non-relativistic electrons
  [Kennel, PFl 9, 2190 (1966)]
- WWI driven by primary (Dreicer) runaways
  [Elfimov & Galvao, PPCF 45, L63 (2003)]
- WWI driven by relativistic bi-Maxwellian distribution with temperature anisotropy.
  [R Ciurea-Borcia *et al*, PP 7, 359 (2000)]
- WWI driven by a *relativistic secondary* runaway electron beam.
  [Fülöp *et al*, PP 13, 0625 (2006);
  Pokol *et al*, PPCF 50, 045003 (2008)]

WWI analysis

- Without runaways, the dispersion relation of the fast wave $(\epsilon_{11} - k_\parallel^2 c^2/\omega^2)(\epsilon_{22} - k^2 c^2/\omega^2) + \epsilon_{12}^2 = 0$ simplifies to the whistler branch $\omega = k k_\parallel v_A^2/\omega_{ci}$, if $\omega_{ci} \ll \omega \ll \omega_{ce}$, $k_\parallel^2 c^2/\omega_{pi}^2 \gg 1$ and $(k^2 + k_\parallel^2) v_A^2 \ll \omega_{ci}\omega_{ce}$.
- The instability growth rate for a small perturbation due to runaways $\omega = \omega_0 + \delta\omega$, where $\gamma_i = \mathrm{Im}\delta\omega$, is given by

$$\frac{\gamma_i}{\omega_0} = -\frac{k^2 v_A^2}{2\omega_{pi}^2} \mathrm{Im}\chi_{11}^r$$

Runaway contribution

The runaway contribution to the susceptibility is

$$\chi_{11}^{r} = \frac{\omega_{pr}^2 \omega_{ce}^2}{k_\perp^2 c^2 \omega^2} \int d^3 p \sum_{n=-\infty}^{\infty} \frac{n^2 J_n^2(z) \hat{\Pi} f_r}{\gamma(\omega - k_\parallel c p_\parallel/\gamma - n\Omega)},$$

with

$$\hat{\Pi} = \frac{\omega - k_\parallel v_\parallel}{p_\perp} \frac{\partial}{\partial p_\perp} + \frac{k_\parallel v_\perp}{p_\perp} \frac{\partial}{\partial p_\parallel}$$

where $\Omega = eB/m_e = \omega_{ce}/\gamma$, $z = k_\perp c p_\perp / \omega_{ce}$, $f_r = f/n_r$ is the normalized runaway distribution function.

Most unstable wave

- The growth rate can be simplified to

$$\gamma_i(\omega_0, k, k_\parallel) = \frac{\pi}{4c_Z}\frac{\omega_{pr}^2}{\omega_{pi}^2}\frac{k^2 v_A^2}{\omega_0}\exp\left[\frac{-\omega_{ce}}{(k_\parallel c - \omega_0)c_Z}\right]$$

- The growth rate of the fastest growing wave can be obtained by using $\partial\gamma_i/\partial\boldsymbol{k} = 0$ and is

$$\gamma_i^{\max} = 1.3 \cdot 10^{-9} n_r/B_T$$

- The most unstable wave has $k_0 = \omega_{pi}/2v_A$, $k_{\parallel 0}c = 2\omega_{ce}/c_Z$ and $\omega_0 = \omega_{ce}/c_Z$.

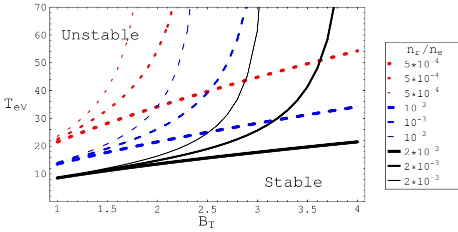
[Fülöp *et al*, PP 13, 0625 (2006)]

Instability threshold

- In the cold post-disruption plasmas collisional and convective damping dominates.
- The threshold of the instability is

$$\frac{n_r}{n_e} > \frac{Z^2 B_T}{20\, T_{eV}^{3/2}} + \frac{B_T^3}{90 c_Z n_{19}^2 L_r}$$

- Thickest $L_r = \infty$
  Middle-thickness $L_r = 0.4\ \mathrm{m}$
  Thinnest $L_r = 0.2\ \mathrm{m}$;
- Density $n_e = 5 \cdot 10^{19}\ \mathrm{m}^{-3}$



[Fülöp et al, PP 16 022502 (2009)]

Quasi-linear analysis of WWI

- Quasi-linear analysis is needed to determine how the instability affects the runaway electrons.
- Breizman Rev of PP Vol. 15 (1990), considers the effect of high-frequency instabilities on runaway electron beams, but uses simpler models for the distribution function.

Quasi-linear equations

$$\frac{\partial f}{\partial t} = \frac{\pi e^2}{m_{e0}^2 c^2} \int d^3k \hat{\Pi} \frac{|E_k|^2}{4} \frac{p_\perp^2}{\omega^2} \delta(\omega + \Omega - k_\| p_\| c/\gamma) \hat{\Pi} f, \qquad (1)$$

where $W_k(t) = \epsilon_0 |E_k(t)|^2/2$ grows as $dW_k/dt = 2\gamma_k(t) W_k$, with

$$\gamma_k(t) = -\frac{k^2 v_A^2 \omega_0}{2\omega_{pi}^2} \mathrm{Im}\chi^r(t) - \gamma_d - \gamma_v, \text{where}$$

$\gamma_d = 1.5\tau_{ei}^{-1}$, $\gamma_v = (\partial\omega/\partial k_\perp)/4L_r$, L_r is the radius of the runaway beam.

Diffusive solution

If we assume $k_{\|} v_{\perp} \partial f / \partial p_{\|} \ll \Omega \partial f / \partial p_{\perp}$, the solution is

$$f(p_{\perp}, p_{\|}, t) = \frac{n_r \alpha}{2\pi c_z \phi(p_{\|}, t)} \exp\left(-\frac{p_{\|}}{c_Z} - \frac{\alpha p_{\perp}^2}{2\phi(p_{\|}, t)} \right)$$

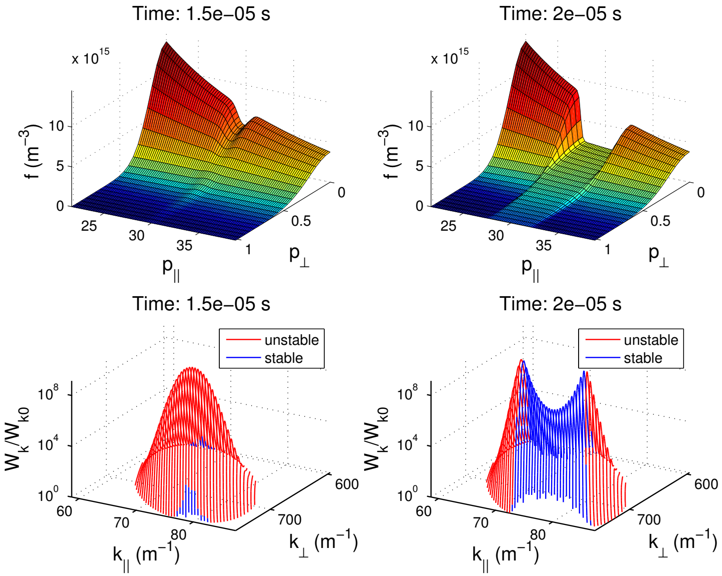
where $\phi(p_{\|}, t) = 2\alpha \hat{\tau}(p_{\|}, t) + p_{\|}$,

$$\hat{\tau}(p_{\|}, t) = \frac{\pi e^2 \omega_{ce}^2}{2\epsilon_0 m_{e0}^2 c^2 p_{\|}^2} \int_0^t dt' \int d^3k \frac{W_k}{\omega^2} \delta(\omega + \omega_{ce}/p_{\|} - k_{\|} c)$$

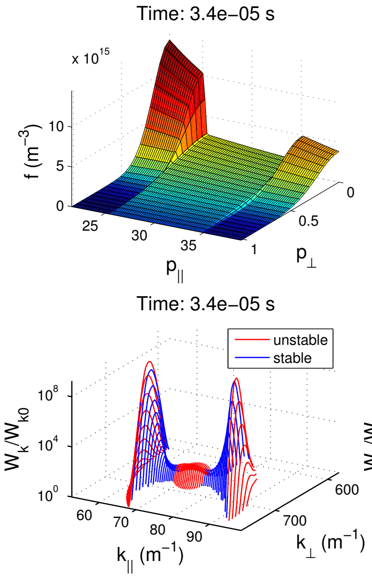
The growth rate is

$$\gamma_k(k, k_{\|}, t) = \frac{\pi}{4c_Z} \frac{\omega_{pr}^2}{\omega_{pi}^2} \frac{k^2 v_A^2}{\omega_0} \frac{I_1(\zeta)}{\zeta} e^{\left[-\zeta - \frac{\omega_{ce}}{(k_{\|} c - \omega_0) c_Z}\right]} - \gamma_d - \gamma_v,$$

where $\zeta = k_{\perp}^2 c^2 \phi(\omega_{ce}/(k_{\|} c - \omega_0), t)/\omega_{ce}^2 \alpha$.



[Pokol *et al*, PPCF 50, 045003 (2008)]



[Pokol *et al*, PPCF 50, 045003 (2008)]

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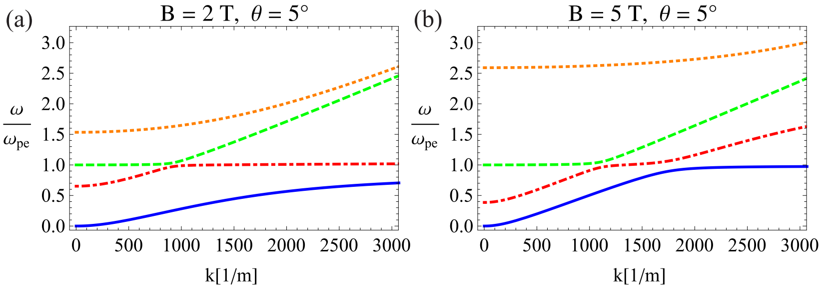
General dispersion relation

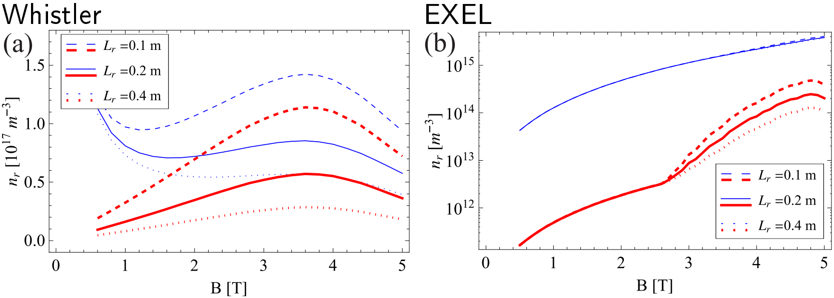
$$\left[\left(\epsilon_{11}-\frac{k_{\parallel}^2c^2}{\omega^2}\right)\left(\epsilon_{22}-\frac{k^2c^2}{\omega^2}\right)+\epsilon_{12}^2\right]\left(\epsilon_{33}-\frac{k_{\perp}^2c^2}{\omega^2}\right)-\frac{k_{\parallel}^2k_{\perp}^2c^4}{\omega^4}\left(\epsilon_{22}-\frac{k^2c^2}{\omega^2}\right)=0,$$



The solid blue line corresponds to the whistler wave and the red dashed line is the extraordinary electron (EXEL) wave. The branches with higher frequencies are not destabilized by the suprathermal electrons.

[Kómár et al, JPCS **401** 012012 (2012)]

EXEL wave has lower stability threshold



The electron temperatures are $T_e = 20$ eV (blue thin lines) and $T_e = 1$ keV (red thick lines).

[Kómár et al, JPCS **401** 012012 (2012)]

- Quasi-linear analysis of EXEL-runaway interaction is ongoing.

Outline

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Synchrotron radiation

- Synchrotron spectrum depends on the velocity-space distribution of the radiating particles.
- Most previous studies used either appoximate electron distributions or a single particle with a certain energy and pitch-angle.
- Average emitted synchrotron power per runaway

$$P(\lambda) = \frac{2\pi}{n_r}\int \mathcal{P}(p,\chi,\lambda) f_r(p,\chi) p^2 dp d\chi$$

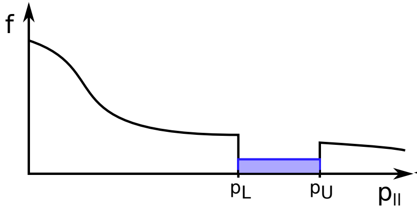
with

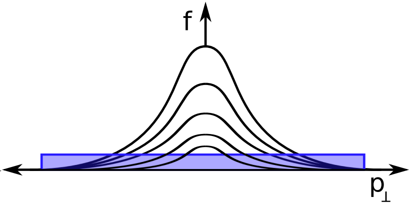
$$\mathcal{P}_{\mathrm{cyl}}(\lambda) = \frac{1}{\sqrt{3}}\frac{ce^2}{\epsilon_0 \lambda^3 \gamma^2}\int_{\lambda_c/\lambda}^{\infty} K_{5/3}(l)\mathrm{d}l$$

- More complicated expression valid in toroidal geometry derived in [Pankratov, Plasma Phys. Reports, 25 145 (1999)]

Wave-particle interactions modify the distribution

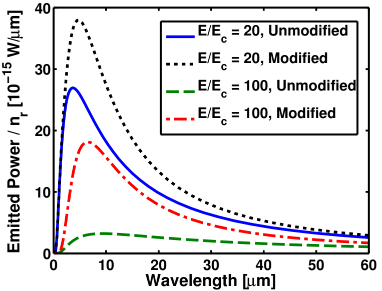
- Runaway distributions normally peaked around the parallel direction.
- Wave-particle interaction tends to drive the distribution towards isotropy.
- One can simulate the decrease in anisotropy by introducing a flat profile in part of the momentum space.





Wave-particle interaction

- Appreciable increase in the average emission of the runaways.
- The onset of a wave-particle resonance should be detectable.
- However, also other changes in plasma parameters could have similar effect on the synchrotron emission.



[Stahl et al, PP 20 093302 (2013)]

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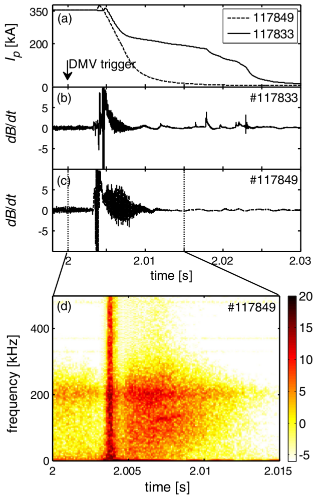
Losses due to magnetic perturbations in TEXTOR

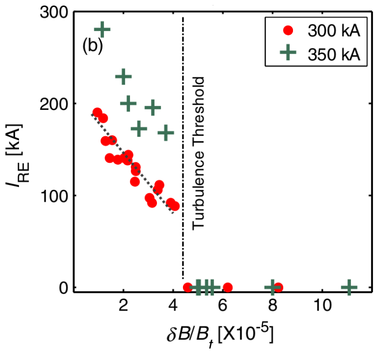
- Deliberately triggered disruptions by injection of large amounts of argon.
- Shots 117833 and 117849 are similar except for the toroidal magnetic field and the magnetic turbulence level.
- Frequencies form a wide distribution, most of the power is in the 60–260 kHz range.



[Zeng et al, PRL 110 235003 (2013)]

Runaway electron (RE) current in TEXTOR

- RE current as a function of the maximum magnetic turbulence during the current quench.
- If $\delta B/B$ exceeds a threshold, REs (which may be produced during the current quench) get quickly lost.
- The value of the critical fluctuation amplitude seems to depend only on the toroidal magnetic field and not on the plasma current.

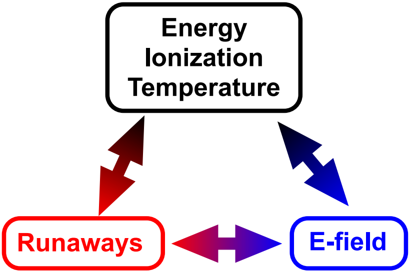


[Zeng et al, PRL 110 235003 (2013)]

Runaway modelling with GO

- 1D model for plasma cooling, runaway current and electric field evolution during impurity injection

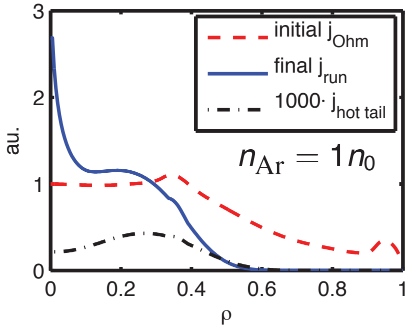
- Energy balance equations for all species, including
  - Ohmic heating
  - Line radiation and Bremsstrahlung
  - Rate equations for ionization
  - Collisional energy exchange



H Smith et al, PP 102502 (2006);
K Gál et al, PPCF 50 055006 (2008);
H Smith et al, PPCF 51 124008 (2009);
T Fehér et al, PPCF 53 035014, (2011);
G Papp et al, "The effect of ITER-like wall on runaway electron generation in JET", submitted to NF, http://arxiv.org/abs/1308.2616

Induction equation

- Electric field is induced to keep current constant
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right) = \mu_0 \frac{\partial}{\partial t}\left(\sigma_\parallel E + n_\mathrm{r} ec\right)$$
- Instead of modelling the velocity space dynamics for the electrons that are already inside the runaway region, we only consider their total density.

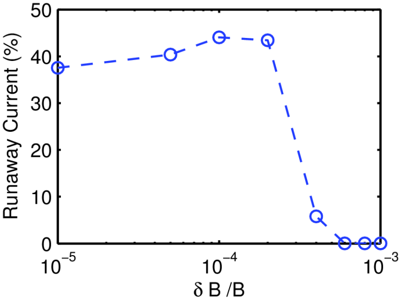


$$\frac{\partial n_\mathrm{r}}{\partial t} = \left(\frac{\partial n_\mathrm{r}}{\partial t}\right)^{\mathrm{Dreicer}} + \left(\frac{\partial n_\mathrm{r}}{\partial t}\right)^{\mathrm{hot\text{-}tail}} + \left(\frac{\partial n_\mathrm{r}}{\partial t}\right)^{\gamma} + \\ + \left(\frac{\partial n_\mathrm{r}}{\partial t}\right)^{\mathrm{avalanche}} + \frac{1}{r}\frac{\partial}{\partial r} r\, D_\mathrm{RR}\frac{\partial n_\mathrm{r}}{\partial r}.$$

[T Fehér et al, PPCF 53 035014, (2011)]

Losses due to magnetic perturbations in an ITER-like case

- Assuming that the radial diffusion coefficient is given by Rechester-Rosenbluth estimate $D_{RR} = \pi q v_{\|} R \left(\delta B/B\right)^2$, with $v_{\|} \simeq c$.
  - Maximum runaway current is shown.
  - Magnetic perturbations make the seed profile broader.
  - This allows the avalanche mechanism to create a higher runaway current for $\delta B/B = 10^{-4}$



[T Fehér et al, PPCF 53 035014, (2011)]

Magnetic perturbations in TEXTOR

- What drives them?
- Can they be driven by runaway ions?
- What energies can the ions reach?


ALL WE KNOW ABOUT THE ELECTRON RUNAWAYS
AND ABOUT THE ION RUNAWAYS

Ion runaway in fully ionized plasmas

- Equation of motion:

$$m\frac{dv}{dt} = eZE - \sum_j F_j$$

- The distortion of the electron distribution due to its drift in the imposed electric field must be taken into account

$$F_e \simeq F_{e0} + \frac{Z^2}{Z_i} eE,$$

where $Z_i = n_e^{-1} \sum_f n_f Z_f^2$ is the effective ion charge.

- The equation of motion:

$$m\frac{dv}{dt} = eZE^* - \sum_j F_{j0}$$

where $E^* = E(1 - Z/Z_i)$ is the effective electric field.

- If $Z/Z_i = 1$ the effective field is zero and the test ion will always slow down.

Ion runaway

- Ions accelerate when $eZE(1 - Z/Z_i) - \sum_j F_{j0} - F_x > 0$

$$\frac{E}{E_D} > \frac{\sum_j F_{j0} + F_x}{(Z|1 - Z/Z_i|)eE_D}$$

- solar flares

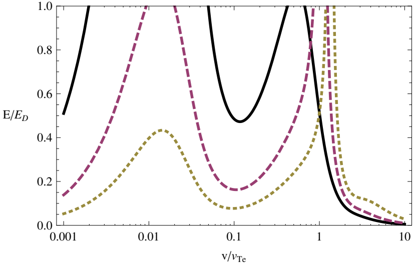
  [Holman, Astrophys J (1995)]

- magnetic reconnection events in tokamaks

  [Helander et al, PRL (2002)]

- lightning discharges

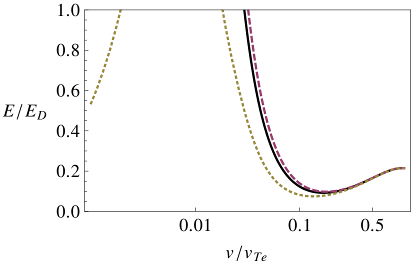
  [Fülöp and Landreman, Ion runaway in lightning discharges, PRL 111 115006 (2013)]



Normalized drag force E/E_D for $^{1}\mathrm{H}^{+1}$ (solid), $^{4}\mathrm{He}^{+2}$ (dashed) and $^{12}\mathrm{C}^{+6}$ (dotted). Plasma consisting of protons, electrons and 10% of helium ions is assumed. $Z_i \simeq 1.2$

Deuterium acceleration

- Deuterium ions from the tail of the thermal ion distribution can be accelerated.
- Resonant interaction between the deuterium and toroidal Alfvén eigenmodes may occur if $v_{\|} \simeq v_A$ or $v_{\|} \simeq v_A/3$.
- The instability growth rate depends on the ion distribution.



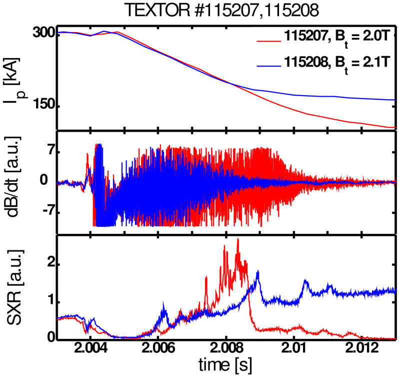
Normalized drag force E/E_D as function of normalized deuterium ion speed. Solid: deuterium plasma with argon puff, effective charge $Z_i = 3$. Purple dashed, $Z_i = 4$; yellow dotted: deuterium plasma with 2% fully ionized carbon.

Conclusions

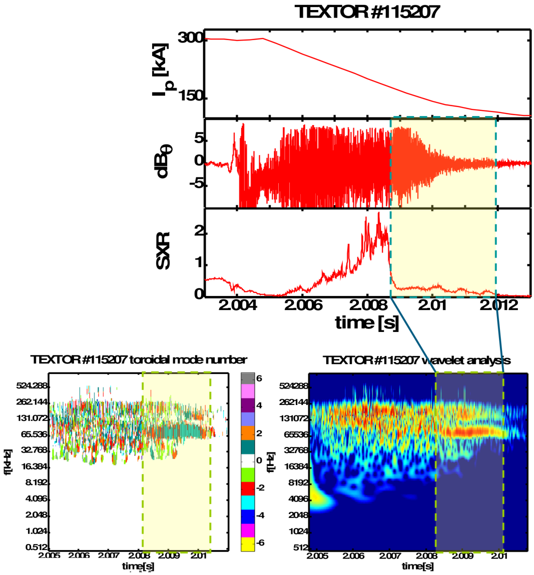
- Kinetic instabilities are likely to be driven by the velocity anisotropy of the runaway beam.
- Synchrotron radiation measurements can be used to detect signatures of kinetic instabilities.
- Runaway losses due to magnetic perturbations could counteract runaway generation.
- Such perturbations may be driven by runaway ions.
- An experimental simulation of ITER conditions is not possible. Good modelling capacity is crucial.

Spare slides

- 115207
  - $B = 2$ T
  - decrease in SXR signal
  - large magnetic fluctuations
  - no runaways
- 115208
  - $B = 2.1$ T
  - SXR signal increases
  - magnetic fluctuations disappear
  - runaways present



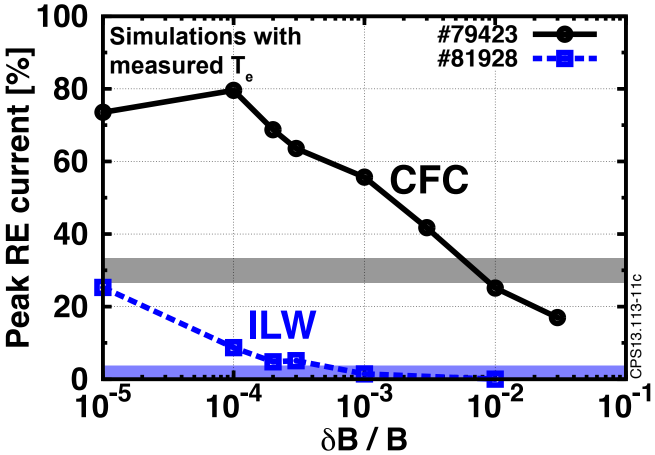
[Koslowski, EFDA project meeting 2012]



[Koslowski, EFDA project meeting 2012]

Effect of losses in JET-scenarios

- Comparison between the carbon and ITER-like wall in JET:



G Papp et al, "The effect of ITER-like wall on runaway electron generation in JET", http://arxiv.org/abs/1308.2616

Temperature evolution

- Energy balance equations for all species

$$\frac{3}{2}\frac{\partial(n_\mathrm{e}T_\mathrm{e})}{\partial t} = \frac{3n_\mathrm{e}}{2r}\frac{\partial}{\partial r}\left(\chi r\frac{\partial T_\mathrm{e}}{\partial r}\right) + P_\mathrm{OH} - P_\mathrm{rad} - P_\mathrm{ion} + P_\mathrm{c}^\mathrm{eD} + P_\mathrm{c}^\mathrm{eZ},$$

$$\frac{3}{2}\frac{\partial(n_\mathrm{D}T_\mathrm{D})}{\partial t} = \frac{3n_\mathrm{D}}{2r}\frac{\partial}{\partial r}\left(\chi r\frac{\partial T_\mathrm{D}}{\partial r}\right) + P_\mathrm{c}^\mathrm{De} + P_\mathrm{c}^\mathrm{DZ},$$

$$\frac{3}{2}\frac{\partial(n_\mathrm{Z}T_\mathrm{Z})}{\partial t} = \frac{3n_\mathrm{Z}}{2r}\frac{\partial}{\partial r}\left(\chi r\frac{\partial T_\mathrm{Z}}{\partial r}\right) + P_\mathrm{c}^\mathrm{Ze} + P_\mathrm{c}^\mathrm{ZD}.$$

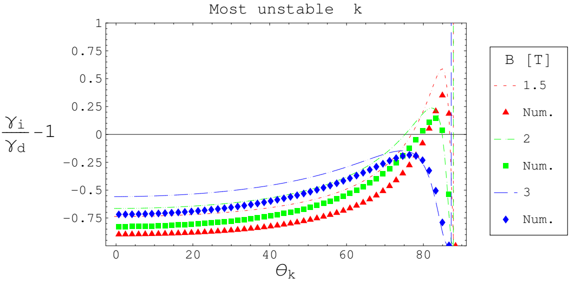
- Energy exchange in collisions: $P_\mathrm{c}^{kl} = \frac{3}{2}\frac{n_k}{\tau_{kl}}(T_l - T_k)$
- Radiation: $P_\mathrm{rad} = P_\mathrm{Br} + \sum_i P_{\mathrm{line},i}$, and $P_{\mathrm{line},i} = n_i n_\mathrm{e} L_i(n_\mathrm{e}, T_\mathrm{e})$.
- Impact ionization and radiative recombination determine n_i:

$$\frac{dn_i}{dt} = n_\mathrm{e}(I_{i-1}n_{i-1} - (I_i + R_i)n_i + R_{i+1}n_{i+1})$$

- Requires externally provided neutral impurity profile.

Numerical analysis

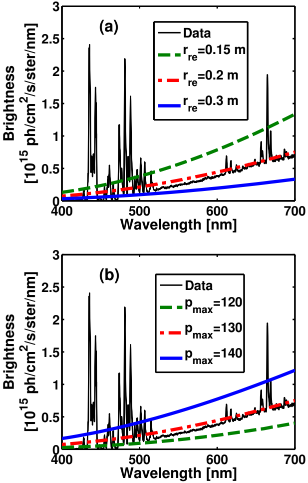
- Numerical solution of the full dispersion relation confirms the results of the perturbative analysis.



The figure shows analytical and numerical growth rates for deuterium plasma at different magnetic field strength values as a function of propagation angle θ_k ($n_e = 5 \cdot 10^{19}\ \mathrm{m}^{-3}$, $T = 10\ \mathrm{eV}$).

Comparison to DIII-D data

- Measured visible spectrum during the runaway plateau.
- Data is a superposition of synchrotron radiation from runaways and line radiation from the background plasma.
- Calculated spectra for various normalized momenta. Fit is best for $p_{max} = 130$, corresponding to 65 MeV.



[Stahl et al, PP 20 093302 (2013)]

Runaway generation

- $I_p = \sigma E_{\|} A + n_r ecA$ combined with $E_{\|} = -\frac{L}{2\pi R}\frac{dI_p}{dt}$
  $\Rightarrow \frac{d}{dt}(n + sE) = -E/\alpha$, with $\alpha \simeq 4 I_0$
- Runaway production= primary + secondary
  $dn/dt = F(E) + n(E-1)$

$$s\frac{dE}{dn} = -1 - \frac{E}{\alpha(F(E) + n(E-1))}$$

- Primary generation dominates initially: $s\dfrac{dE}{dn} = -1 - \dfrac{E}{\alpha F(E)}$
  and this gives a seed of runaways $n_* = \alpha s \int_E^{E_c} F(E) dE/E$
- This is amplified by the secondary mechanism $\dfrac{dn}{dE} = -\dfrac{\alpha s n}{1 + \alpha n}$

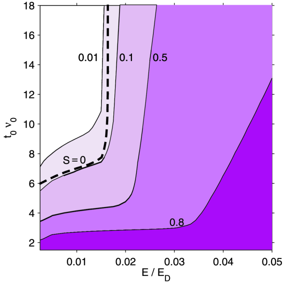
Runaway generation

- A criterion for large runaway generation before the induced toroidal electric field diffuses out of the plasma:

$$S = \alpha + \ln\left[\frac{\sqrt{2}\alpha \ln\Lambda}{\pi}\frac{m_e c^2}{T_e}\left(\frac{E_D}{E}\right)^{11/8} e^{-\frac{E_D}{4E}-\sqrt{\frac{2E_D}{E}}} + \frac{E_D}{E}\sqrt{\frac{m_e c^2}{T_e}\frac{\sqrt{2}u_c}{3\pi}e^{-u_c^2}}\right] > 0$$

where $\alpha \simeq 4I_0$, $u_c^3 = t_0\nu_0\left[2\ln\frac{E_{D0}}{2E_0} - \frac{4}{3}\ln\left(\frac{4}{3}t_0\nu_0\right) - \frac{5}{3}\right]$.

- Numerical simulations confirm the validity of this criterion. Figure shows GO-simulations for a JET-like case ($I_0 = 1.9\ \mathrm{MA}$, $j_0 = 1\ \mathrm{MA/m^2}$, $T_0 = 3\ \mathrm{keV}$ and $T_f = 10\ \mathrm{eV}$.)



[Fülöp et al, PP 16 022502 (2009)]