GRAY:
a quasi-optical beam tracing code for Electron Cyclotron absorption and current drive

Daniela Farina

Istituto di Fisica del Plasma
Consiglio Nazionale delle Ricerche
EURATOM-ENEA-CNR Association, Milano, Italy

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Quasi-optical ray equations (1)

*E. Mazzucato, Phys. Fluids, **1**, 1855 (1989)*

- solution of the wave equation of the form:

$$\mathbf{E}(\mathbf{x},t) = \mathbf{e}(\mathbf{x})E_0(\mathbf{x})\exp\left[-ik_0 S(\mathbf{x}) + i\omega t\right]$$

- complex eikonal function:
  - real part describes beam propagation
  - imaginary part describes beam shape

$$\mathbf{E} \sim \exp[k_0 S_I]\exp[-ik_0 S_R + i\omega t]$$

$$S = S_R(\mathbf{x}) + iS_I(\mathbf{x})$$

$$\bar{\mathbf{k}} = \mathbf{k} + i\mathbf{k}' = k_0(\nabla S_R + i\nabla S_I)$$

$$k_0 = \omega/c$$

- three scalelengths: λ wavelength, w beam width, L system dim.

$$\lambda/w \sim w/L \sim \delta \ll 1$$

- asymptotic analysis of the wave equation in the small parameter δ

Quasi-optical ray equations (2)

Complex eikonal function satisfies:

$$D(\mathbf{x}, \bar{\mathbf{k}}, \omega) = D(\mathbf{x}, \mathbf{k}, \omega) + i\mathbf{k}' \cdot \frac{\partial D}{\partial \mathbf{k}} - \frac{1}{2}\mathbf{k}'\mathbf{k}' : \frac{\partial^2 D}{\partial \mathbf{k} \partial \mathbf{k}} = 0$$

D expanded up to order δ^2

cold EC dispersion relation assumed in the following:

$$D(\mathbf{x}, \mathbf{k}, \omega) = N^2 - N_s^2(\mathbf{x}, N_{||}, \omega) = 0$$

N_s : local cold refractive index (Appleton-Hartree expression)

real and imaginary part of the QO dispersion relation:

$$D_R(\mathbf{x}, \mathbf{k}, \omega) = N^2 - N_s^2(\mathbf{x}, N_{||}, \omega) - |\nabla S_I|^2 + \frac{1}{2}\nabla S_I \nabla S_I : \frac{\partial^2 N_s^2}{\partial \mathbf{N} \partial \mathbf{N}} = 0$$

$$D_I(\mathbf{x}, \mathbf{k}, \omega)] = \nabla S_I \cdot \frac{\partial D}{\partial \mathbf{k}} = 0$$

additional terms with respect to geometric optics (GO) approximation:
$\Rightarrow$ *diffraction effects*

Quasi-optical ray equations (3)

QO ray equations *at dominant order in δ*

$$\frac{d\mathbf{x}}{ds} = \left.\frac{\partial D_R/\partial \mathbf{k}}{|\partial D_R/\partial \mathbf{k}|}\right|_{D_R=0}$$

$$\frac{d\mathbf{k}}{ds} = -\left.\frac{\partial D_R/\partial \mathbf{x}}{|\partial D_R/\partial \mathbf{k}|}\right|_{D_R=0}$$

formally equal to GO ray eqs. with D_R depending also on ∇S_I
⇒ rays are coupled together

$$\frac{\partial D_R}{\partial \mathbf{k}} \cdot \nabla S_I = 0$$

partial differential eq. coupled to ray eqs. :

S_I conserved along the ray trajectories

QO dispersion relation

$$D_R(\mathbf{x}, \mathbf{k}, \omega) = N^2 - N_s^2(\mathbf{x}, N_{\|}, \omega) - |\nabla S_I|^2 + \frac{1}{2}(\mathbf{b} \cdot \nabla S_I)^2 \frac{\partial^2 N_s^2}{\partial N_{\|}^2}$$

Solution QO ray equations (1)

Integration scheme

- the Gaussian beam is described in terms of N_T coupled rays with initial conditions on a given surface
- the QO ray eqs for the N_T rays are simultaneously advanced by an integration step by means of a standard integration scheme
- the ray pattern is then mapped onto a new surface
- the derivatives of the imaginary part of the eikonal function are computed by means of a difference scheme based on adjacent ray points on the mapped surface (S_I conserved along the QO rays)
- the QO ray equations are advanced by a further step and the scheme is iterated

Solution QO ray equations (2)

Initial conditions

Eikonal function for an astigmatic Gaussian beam propagating in vacuum in the $\tilde{z}$ direction:

$$S_R = \tilde{z} + \frac{\tilde{x}^2}{2R_{cx}} + \frac{\tilde{y}^2}{2R_{cy}}$$

$$S_I = -\frac{1}{k_0}\left(\frac{\tilde{x}^2}{w_x^2} + \frac{\tilde{y}^2}{w_y^2}\right)$$

R_{ci} , w_i : curvature radius and beam width

initial ray positions on the contourlines of the S_I function in the $(\tilde{x}, \tilde{y})$ plane:

$$\tilde{x}_0 = w_{x0}\tilde{\rho}\cos\theta,\ \tilde{y}_0 = w_{x0}\tilde{\rho}\sin\theta$$

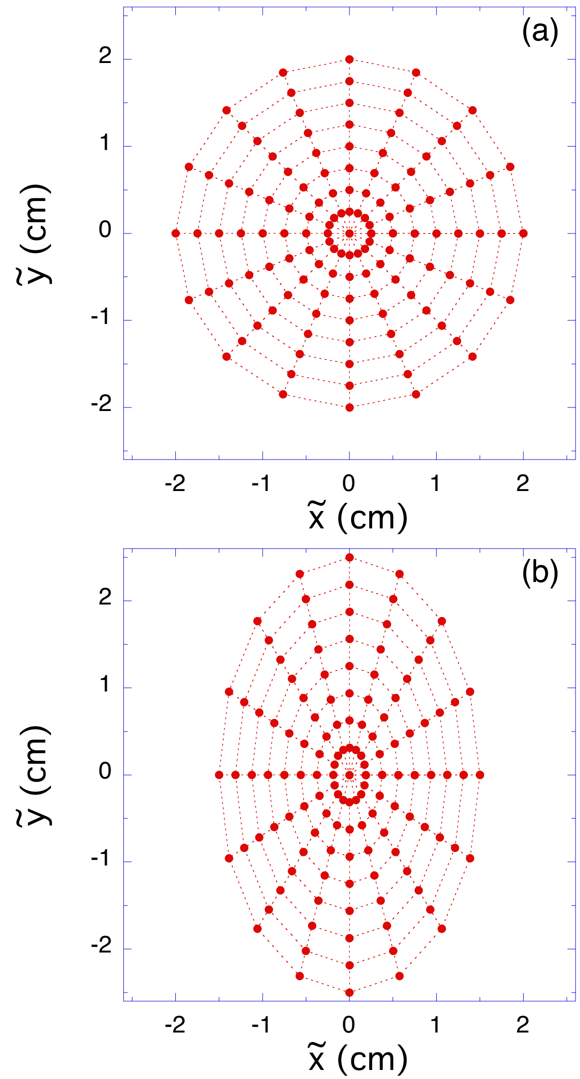
initial ray conditions:

$$\tilde{\rho}_j = j\tilde{\rho}_{mx}/N_r \quad j = 0, \ldots, N_r$$

$$\theta_k = 2k\pi/N_a \quad k = 1, \ldots, N_a$$

$$\mathbf{N}_0 = \nabla S_R$$

$N_T = N_r x\, N_a + 1$



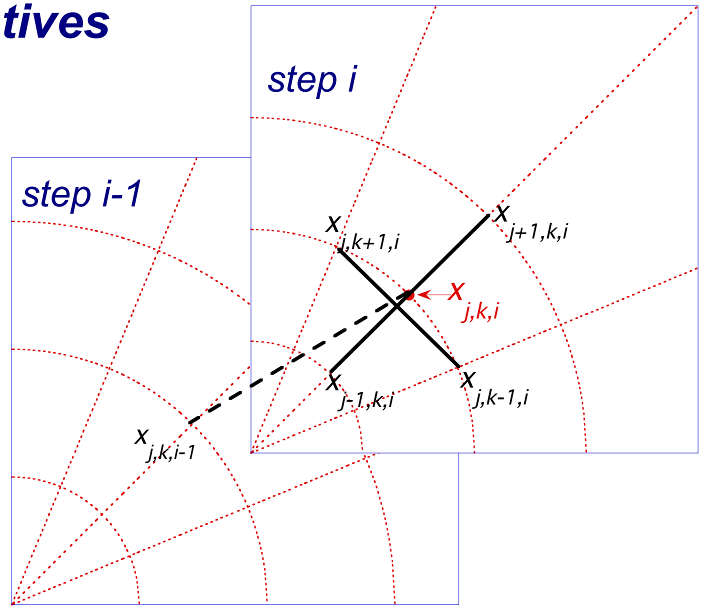
circular gaussian beam

astigmatic gaussian beam

$N_T = 129$

Solution QO ray equations (3)

algorithm for S_I derivatives



$\mathbf{x}_{j,k,i}$ ⟶ coordinate on the *(j,k)* ray at *i-th* integration step

computation of S_I derivatives at $x_{j,k,i}$ involves adjacent points:

$x_{j+1,k,i}$ $x_{j-1,k,i}$ $x_{j,k+1,i}$ $x_{j,k-1,i}$ $x_{j,k,i-1}$

derivatives of S_I computed in terms of u derivatives with : $k_0 S_I = -u^2/N_r^2$

f(x) generic function of position ⟶ (at lowest order) $\Delta f(\mathbf{x}) = \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \frac{\partial f}{\partial x_3}\Delta x_3$

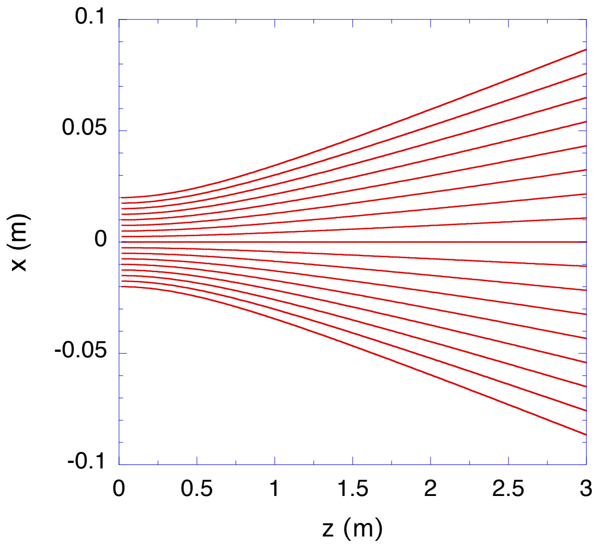
derivatives $\partial f/\partial x_s$ obtained from linear system of three difference eqs. ⇧ evaluated for:

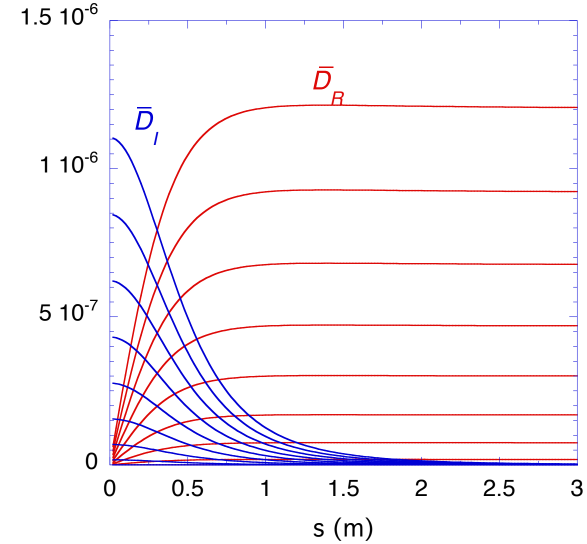
$$\Delta\mathbf{x}_a = \mathbf{x}_{j,k,i} - \mathbf{x}_{j-1,k,i}, \qquad \Delta f_a = f_{j,k,i} - f_{j-1,k,i},$$
$$\Delta\mathbf{x}_b = \mathbf{x}_{j,k+1,i} - \mathbf{x}_{j,k-1,i}, \qquad \Delta f_b = f_{j,k+1,i} - f_{j,k-1,i},$$
$$\Delta\mathbf{x}_c = \mathbf{x}_{j,k,i} - \mathbf{x}_{j,k,i-1}, \qquad \Delta f_c = f_{j,k,i} - f_{j,k,i-1},$$

the algorithm is applied putting $f \rightarrow u$ and then $f \rightarrow \partial u/\partial x_s$

Beam trajectories in vacuum (1)

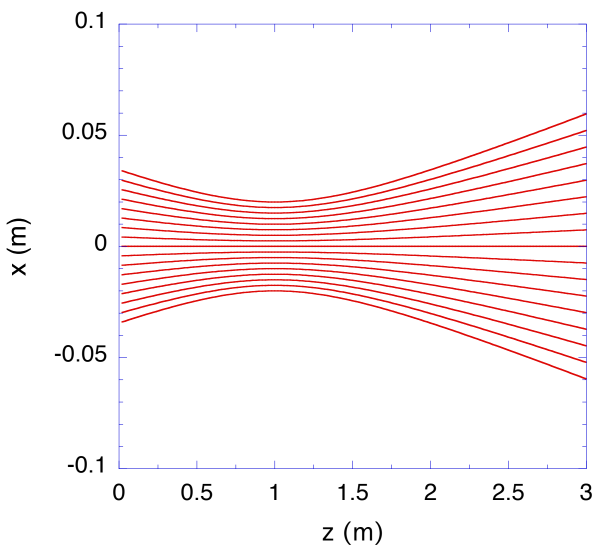
divergent beam

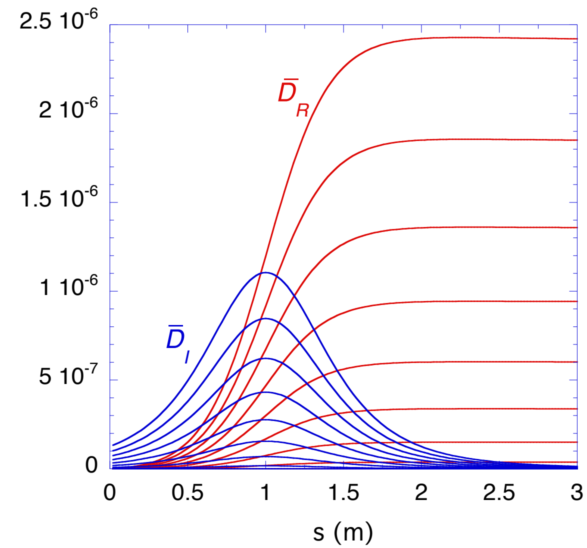




f=170 GHz
λ=1.76 mm
$w_1=w_2$=2 cm
ds=1 mm

convergent beam





numerical accuracy estimated from conservation of real and imag. QO dispersion relation

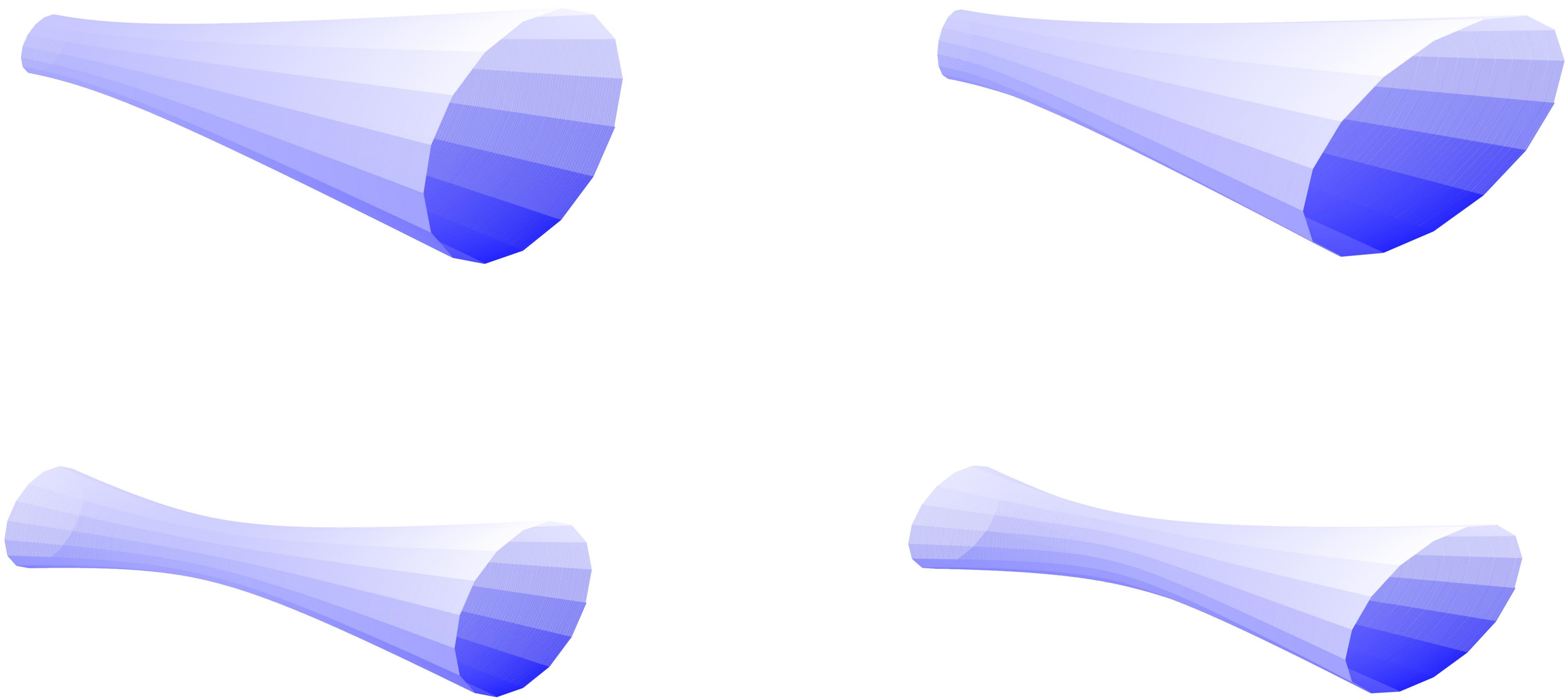
QO ray trajectories *numerical accuracy*

Beam trajectories in vacuum (2)

surface at e^{-2} power level

divergent beams

convergent beams



circular gaussian beams

astigmatic gaussian beams

Beam trajectories in plasmas

QO ray equations are integrated in the Cartesian reference system (x,y,z)

The equilibrium and the plasma profiles are given either numerically or analytically

density, temperature, ... : function of poloidal flux function ψ

Magnetic field: $\mathbf{B} = I(\psi)\nabla\phi + \nabla\phi \times \nabla\psi$ (ϕ toroidal angle)

NUMERICAL EQUILIBRIUM: ψ and $I(\psi)$ from **EQDSK** file

(spline interpolation of ψ , $I(\psi)$, $n(\psi)$, ..., is performed in case of numerical data)

the flux surfaces are characterized by a flux label ρ, *e.g.,*

$$\rho_\psi = |\psi - \psi_{axis}|/|\psi_{edge} - \psi_{axis}|, \quad \rho_p = \sqrt{\rho_\psi}, \quad \rho_t = \sqrt{\Phi_t(\psi)/\Phi_t(\psi_{edge})}$$

ECRH and ECCD calculations require the following quantities to be computed on a generic magnetic surface for the given equilibrium:
B_m, B_M, $\langle B \rangle$, $\langle 1/R^2 \rangle$, $A(\psi)$, $V(\psi)$

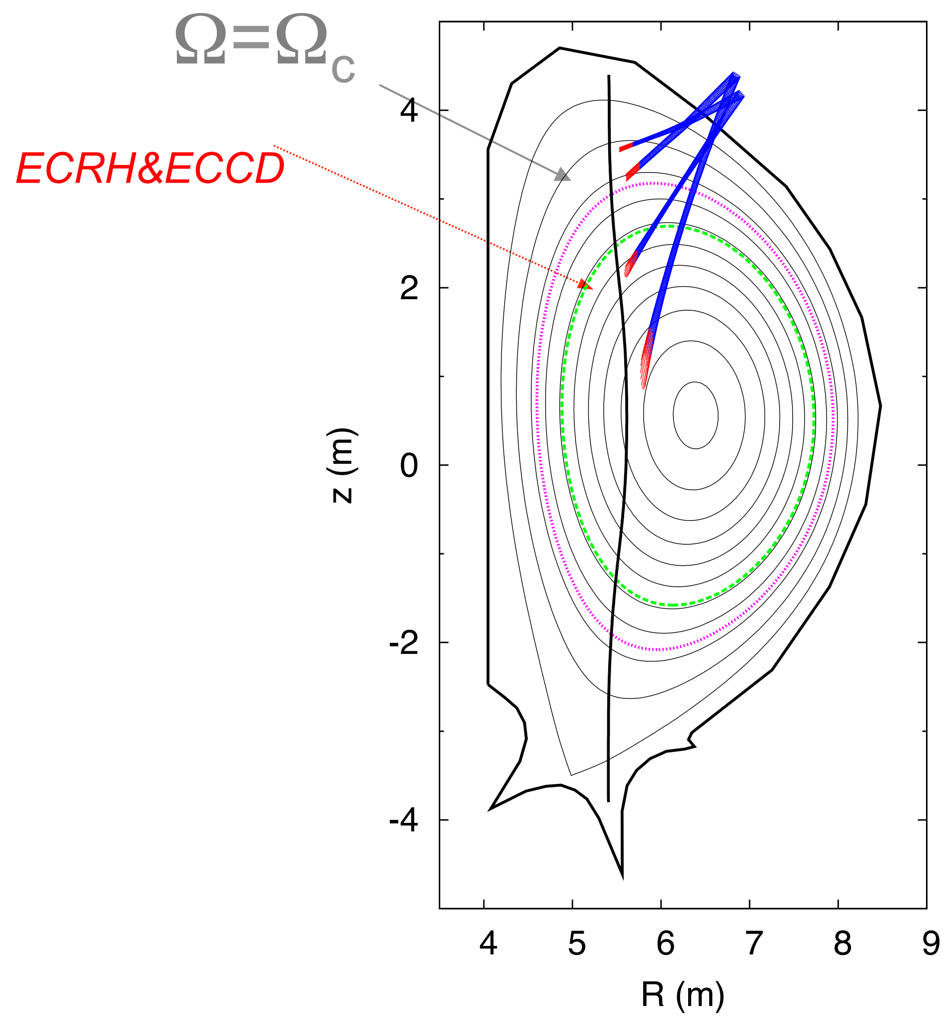
Beam trajectories in ITER

ITER Scenario 2, 15 MA, 5.3 T, $n_{e0} \approx 10^{19}$ m^{-3}, $T_{e0} \approx 25$ keV

f=170 GHz

convergent beams

$w_1 = w_2 = 1.6$ cm

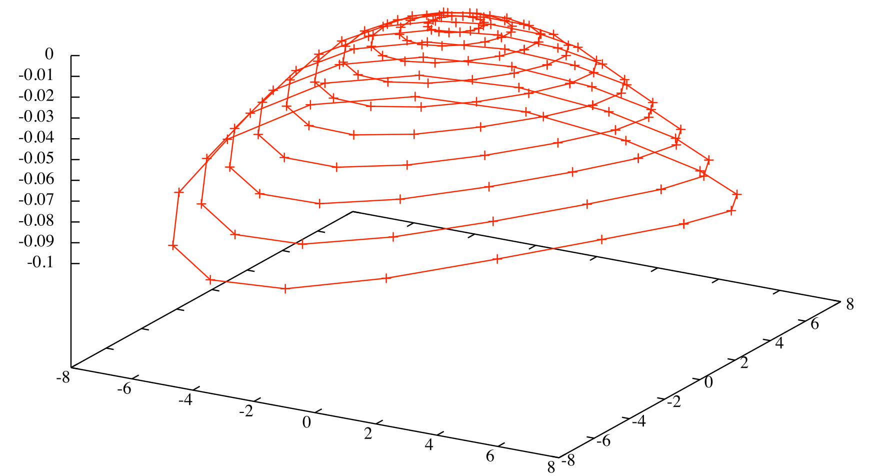


$N_{//}$ spectrum: heuristic approach

- Integration of QO equations at constant Re[S] -> advancing the phase front (initial ray conditions on a phase front)

- local reference system with z along k vector *(not v): surface ~paraboloid*

- Via suitable fit of this surface: curvature radii & beam widths



beam width & ray tracing codes

- Effects due to spectrum width are taken into account in present codes only partially *(not self consistently in (**x**,**k**) space)*
- In case of gaussian beams, two contributions to spectrum width can be identified, due to
  a) phase front curvature
  b) beam width
- Multi-ray codes *(almost)* take into account contribution a) in the spectrum
- In addition, multi-ray codes take into account the spread due to the finite illumination region
- the spectral width due to b) is practically neglected
- this last effect can be important close the focal region

Resonance condition

plane wave ⇒ "infinite" interaction time : δ function

$$\delta(\gamma - N_{\|}u_{\|} - n\Omega/\omega)$$

wave beam ⇒ "finite" interaction time : function Δ *(exponential function)*

$$\Delta = \sqrt{\frac{2}{\pi}}\frac{1}{\left|\Delta N_{//}u_{//}\right|}e^{-2\left(\gamma - N_{//}u_{//} - n\Omega/\omega\right)^2 / \left(\Delta N_{//}^2 u_{//}^2\right)}$$

⇒

$$\Delta = \sqrt{\frac{2}{\pi}}\frac{1}{\left|\Delta N_{//}u_{//}\right|}e^{-2\left(N_{//} - N_{//res}\right)^2 / \Delta N_{//}^2}$$

$$N_{\|res} = \frac{\gamma - n\Omega/\omega}{u_{\|}}$$

Westerhof et al, RELAX (1992), Demeio, Engelman, PPCF 1986, Farina, Pozzoli (1990?)