Modeling of Electron Cyclotron Current Drive applied for the suppression of magnetic islands in tokamaks

E. Westerhof, B. Ayten, and the ASDEX Upgrade team
Motivation

- Magnetic islands in tokamaks must be controlled to prevent reduction of performance and disruptions
- Design of effective control strategies requires a reliable model
- ECCD inside islands is main control strategy

Picture © http://www.vacet.org/gallery/fusion.html
Modeling ECCD in magnetic islands

1. Field topology, temperature, and density distributions in presence of magnetic island
   Equilibrium + perturbation model

2. High frequency wave propagation and absorption
   Ray-tracing code (TORAYFOM)

3. Kinetic plasma response and current drive
   Quasi-linear Fokker–Planck code (RELAX)

4. Evolution of magnetic island
   Generalized Rutherford Equation (GRE)
Field topology and plasma profiles

Example: ASDEX Upgrade #26827

Plasma equilibrium and profiles from IDA Integrated Data Analysis
Adding the magnetic island

Single helicity \( m=3, n=2 \) perturbation to poloidal flux

\[
\tilde{\psi} = \psi(r_{c,s}) \frac{r_c^2}{r_{c,s}^2} \frac{(1 - \frac{r_c}{a})^2}{(1 - \frac{r_{c,s}}{a})^2} \cos(m\theta_s + n\phi + \xi_0)
\]

Flux surfaces = contours of constant helical flux

Temperature (density) profile

\( r_c > r_{c,s} \):
\[
T(r_c, \theta, \phi) = T_0 \left( \sqrt{\frac{S(\psi(r_c, \theta, \phi), \sigma)}{\pi}} \right)
\]

island:
\[
T_{isl} = T_0 \left( \sqrt{\frac{S(\psi_{sep}, 1)}{\pi}} \right)
\]

\( r_c < r_{c,s} \):
\[
T(r_c, \theta, \phi) = T_0 \left( \sqrt{\frac{S(\psi(r_c, \theta, \phi), \sigma)}{\pi}} \right) - \Delta T,
\]

\[
\Delta T = T_0 \left( \sqrt{\frac{S(\psi_{sep}, -1)}{\pi}} \right) - T_0 \left( \sqrt{\frac{S(\psi_{sep}, 1)}{\pi}} \right)
\]
Wave propagation and absorption

TORAYFOM solves ray traces using cold plasma Appleton-Hartree dispersion relation.

Absorption uses (weakly) relativistic lowest order FLR dielectric tensor.

Focused Gaussian beam modeled with 9(toroidal) x 41(poloidal) rays:
- FWHM (toroidal) 3° (focused)
- FWHM (poloidal) 1.7 cm

Note the ray propagation tangential to flux surfaces.
Power deposition profiles (1 MW injected)

Equilibrium topology

- $P$ (flux surface) projected on normalized LFS minor radius

Island topology (4 cm wide)

- Ten fold increase in $P$ inside island -> nonlinear kinetic effects
Kinetic plasma response

Bounce-averaged, quasi-linear Fokker–Planck equation

\[
\frac{\partial f_e}{\partial t} = \left\langle \sum_s C(f_e, f_s) \right\rangle_{\phi_B} - \langle \Gamma_{ql} \rangle_{\phi_B}
\]

Balance between collisions and quasi-linear diffusion

Bounce average along single poloidal turn of orbit

\[
\langle Q \rangle_{\phi_B} = \frac{1}{\tau_B} \int Q \frac{ds}{v \cos \theta}
\]

not significantly affected by presence of island, but we do need to take account of the flux surface topology
Nonlinear effects when

\[ H \equiv \frac{p_{ECCD} \text{ [MW/m}^3\text{]}}{(n_e \text{ [10}^{19}\text{ m}^{-3}\text{]}^2 \gtrsim 0.5} \]

Ref. Harvey et al. PRL 62 (1989) 426

Results of RELAX code
Equilibrium:
Tangential propagation allows single ray / single surface calculation

Nonlinear effects reduce ECCD efficiency (absolute value)
Reduced nonlinear efficiency explained

Local efficiency decreases along ray as resonance is approached

Quasi-linear flattening reduces absorption and shifts power deposition towards resonance
Nonlinear effects inside island

FP equation solved on 40 surfaces:
$$10 \text{ for } r_c < r_{c,s}, \text{ 20 in island, and } 10 \text{ for } r_c > r_{c,s}$$

Island phase fixed for O-point power deposition

Nonlinear effects are function of averaged $H$:
$$< H > = \frac{\int H p_{ECCD} dV}{\int p_{ECCD} dV}$$
Time dependence: rotating 8 cm island

- Averaged over all phases nonlinear effect is weak
- Nonlinear effects result in a minor fluctuation of the total driven current in the case of a rotating island
- 23 kHz (experiment) > collision frequency > 3 kHz
Strong fluctuations in current density inside island

- O-point
- Middle of island
- Just outside separatrix

23 kHz

3 kHz
Magnetic island evolution

The generalized Rutherford equation

- obtained by averaging the relevant helicity of Ohm’s law over the entire island region

\[ 0.82 \frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta'_0(w) - r_s \sum_i \Delta'(\delta j_i) \]

island width evolution any current perturbation classical stability

- Contribution from ECCD

\[ r_s \Delta'_{CD} = \frac{16 \mu_0 L_q r_s}{B_p \pi w^2} \int_{-\infty}^{\infty} dx \int d\xi j_{CD} \cos(m\xi) \]
We define the nonlinear shape function $\gamma_{CD}$

$$\gamma_{CD} \equiv \frac{1}{\eta_{CD, \text{lin}} P} \int_{-\infty}^{+\infty} dx \int d\xi J_{CD} \cos(m\xi)$$

**locked mode**

**23 kHz / 3 kHz rotating mode**
Ray tracing and Fokker–Planck calculation of ECCD applied for suppression of magnetic islands

- Island topology results in ~ten fold increase of maximum deposited power density
- Nonlinear effects reduce ECCD efficiency
- Especially J(O-point) is reduced in (slowly) rotating/locked islands
- The stabilizing effect of ECCD is reduced (although negligibly so for experimental condition)
Closing the circle

Self-consistent modeling of magnetic island suppression

Flux surface topology

Island width evolution from GRE

EC wave propagation TORAYFOM ray tracing

ECCD efficiency RELAX Fokker-Planck
Open issues

This still leaves important aspects untreated, for example concerning transport:

- Transport inside magnetic islands
- Transport of energetic electrons
- Influence of RF heating on turbulence
- ...

Modeling Kinetic Aspects of Global MHD Modes

Workshop: 2-6 December 2013, Leiden, the Netherlands

Scientific Organizers
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- Arthur Peeters, U Bayreuth
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