Discrete Flow Mapping

In collaboration with:
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<u>Aim</u>: Predicting the wave intensity distributions for the vibro-acoustic response of mechanical structures at mid-to-high frequencies

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→ general tool for ray-tracing algorithms on meshes

Outline of the talk

- **From waves to rays – ray-tracing with linear operators**
- **Numerical implementation of ray tracing algorithms: Dynamical Energy Analysis and Discrete Flow Mapping**
- **Examples: Applications in the ship industry (*Germanischer Lloyd*), car industry (*Land Rover Ltd*) and aviation industry (*EADS & Airbus*).**

I. Background and Method

Linear wave dynamics:

Point source

$$\left(-\frac{\partial^2}{\partial t^2} - H\right)\hat{G}(\mathbf{r},\mathbf{r}_0;t) = \delta(\mathbf{r}_0-\mathbf{r})\delta(t);$$

$$(\omega^2 - H)\quad G(\mathbf{r},\mathbf{r}_0;\omega) = \delta(\mathbf{r}_0-\mathbf{r})$$

with

$$\begin{aligned} H &= -c^2\Delta && \text{Helmholtz Eqn.}\\ &= -\frac{1}{\kappa(r)}\nabla\frac{1}{\rho(r)}\nabla && \text{Acoustic Wave Eqn.}\\ &= \frac{D}{\rho h}\Delta^2 && \text{Biharmonic Eqn.}\\ &= -\mu\Delta - (\lambda+\mu)\nabla\nabla && \text{Navier-Cauchy Eqn.} \end{aligned}$$

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Numerical Methods:

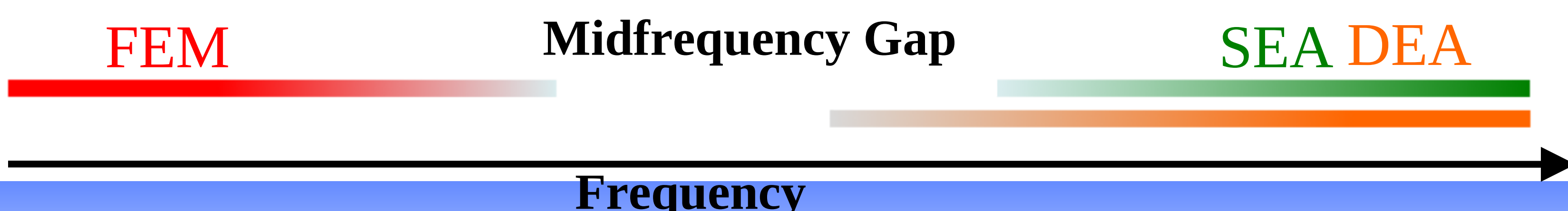
<u>Low Frequencies:</u> (wave length ~ object size; L ~ 1m … f < 0.5-1.0 kHz)

- **Finite Element Method**
- **Boundary Element Method**
- **Spectral Methods**

<u>High Frequencies:</u> (wave length << object size; L ~ 1m … f > 1-5 kHz)

- **Statistical Energy Analysis (SEA)**
- **Ray Tracing**
- **Dynamical Energy Analysis (DEA)**

Not based on meshes! (applies to SEA and Ray Tracing)



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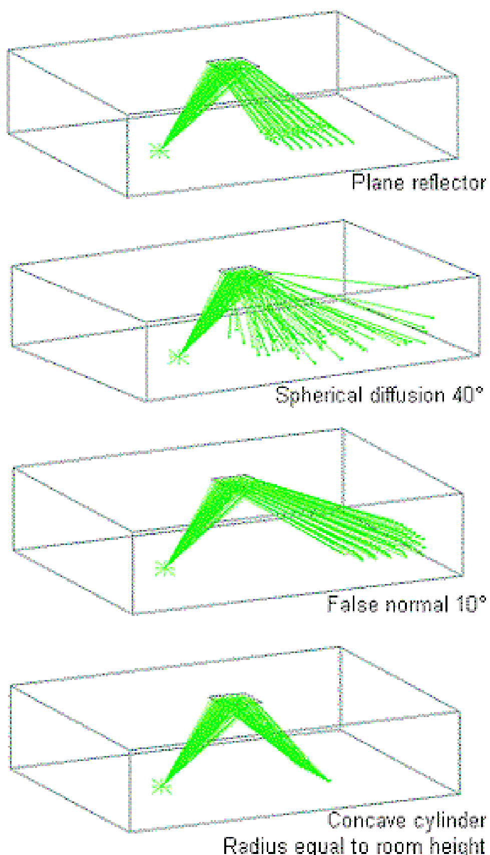
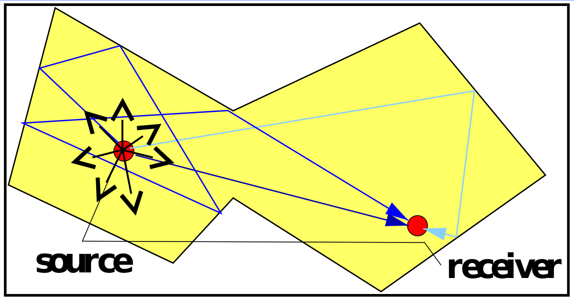
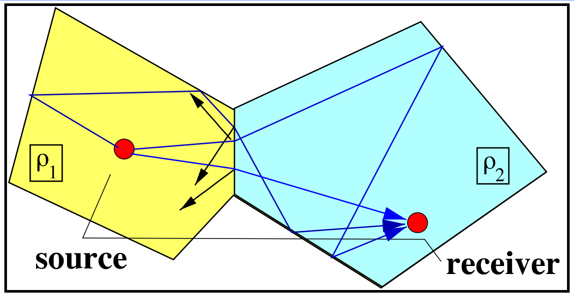
Ray Tracing:







Applications:

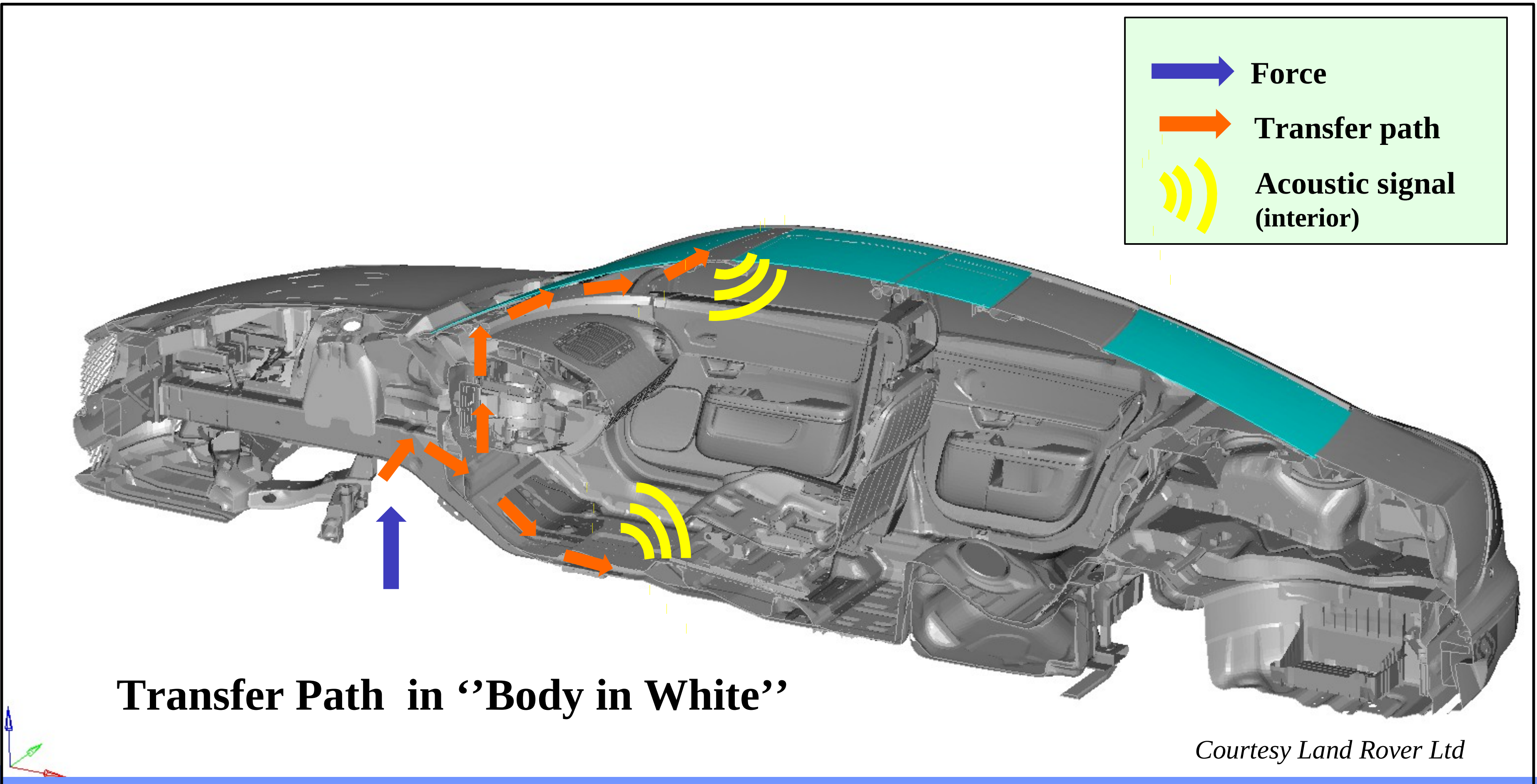
Acoustic, geometric optic, computer graphics, …

Airplanes/trains (interior), vehicles (exterior), room acoustics …

Contains full information about geometry – inefficient for multiple reflections, complex structures, … → **many different paths**

http://www.akustikon.se/eng/software_e.html

Transfer Path Analysis



Transfer Path in ''Body in White''

Courtesy Land Rover Ltd

Linear Wave Equation:

$$\left(-\frac{\partial^2}{\partial t^2} - H(r,\nabla)\right)\Psi(r) = 0$$

$$\Psi = A\exp(i\omega S)$$

Hamilton – Jacobi Eqn:

$$S_t^2 - H(r,\nabla S) = 0$$

Transport Eqn for *A(r)* driven by phase *S*

Hamilton Equations:

(Characteristics of HJ; non-linear ODE)

$$\dot{r} = \quad \nabla_p H(r,p)$$

$$\dot{p} = -\nabla_r H(r,p) \qquad p \equiv \nabla S$$

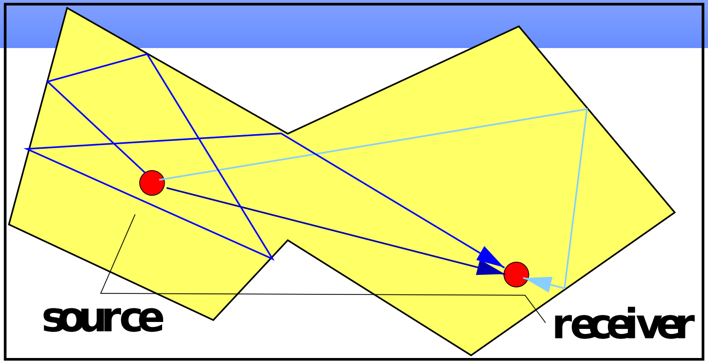
Liouville Equation:

(linear)

$$\rho_t + \dot{X}\,\nabla\rho = 0 \quad X = (r,p)$$

$$|A(r)|^2 = \int \mathrm{dp}\,\rho(r,p)$$

Short wave length asymptotics for Green function



$k = \omega / c >> 1/L$

(L = typical lengthscale)

$$G(r, r_0, \omega) = C \sum_{j: r_0 \to r} A_j e^{i \int dr' k_j(r', \omega) - i\nu_j \frac{\pi}{2}}$$

with

j : sum over rays from $r_0 \to r$

$$C = \frac{\pi}{\omega} \frac{1}{(2\pi \mathrm{i})^{(d+1)/2}}$$

$$A_j = A_j^{(g)} A_j^{(d)}$$

geometrical ($A_j^{(g)}$), damping ($A_j^{(d)}$)

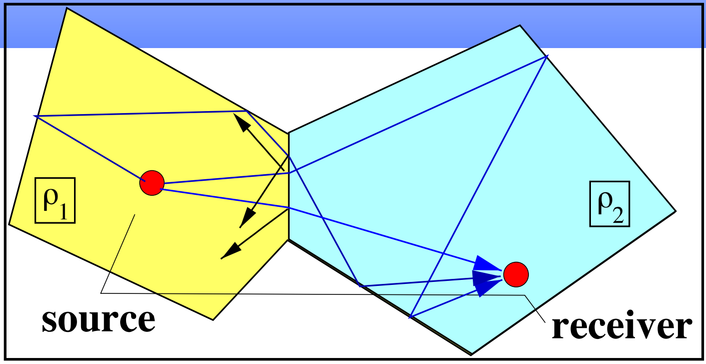
$k_j(r, \omega)$: local wavelength

ν_j : Maslov index

Short wave length asymptotics for Green function

$k = \omega/c >> 1/L$

(L = typical lengthscale)



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with

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$$A_j = A_j^{(g)} A_j^{(d)} A_j^{(c)}$$

geometrical, damping, mode conversion

$k_j(r,\omega)$: local wavelength

ν_j : Maslov index

Energy density in the structure emanating from source at r_0

$$\epsilon_{r_0}(r,\omega) \propto \omega^2 |G(r,r_0,\omega)|^2$$

$$\approx \sum_{j,j':r_0\to r} A_j A_{j'}\, e^{i(S_j - S_{j'} - (\nu_j - \nu_{j'})\frac{\pi}{2})}$$

$$= \rho(r,r_0,\omega) + \text{off-diagonal terms}$$

with

$$\rho(r,r_0,\omega) = \sum_{j:r_0\to r} |A_j|^2$$

Diagonal Approximation

~ RAY TRACING

Stationary density:

$$X = (r,p)$$

$$\rho(r,r_0;\omega) = \int_0^\infty \mathrm{d}\tau \int \mathrm{dp} \int \mathrm{dX'}\; \underbrace{w(X',\tau)\delta\left(X - \varphi^\tau(X')\right)}_{\mathcal{L}(X,X')}\, \rho_0(X';\omega)$$

$\mathcal{L}(X,X')$: Frobenius-Perron operator

with initial density

$$\rho_0(X';\omega) = \delta(r' - r_0)\delta(\omega^2 - H(X')),$$

$X(\tau) = \varphi^\tau(X')$ — phase space flow: propagating for time τ; solution of – in general – nonlinear ODE

$X = (r,p)$ — phase space coordinates; *k*: wave vector, momentum

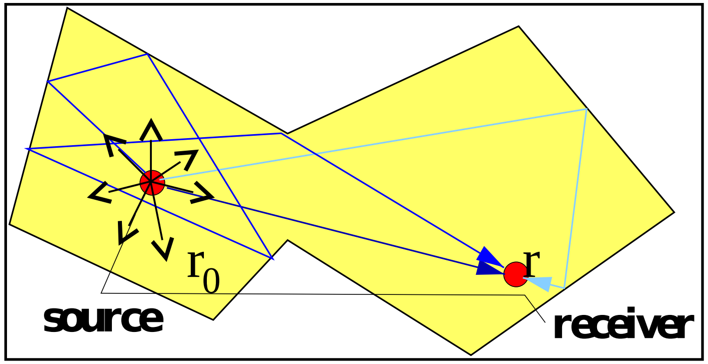
$w(X,\tau)$ — weight function - multiplicative

Stationary density:

$$X = (r, p)$$

$$\rho(r, r_0; \omega) = \int_0^{\infty} \mathrm{d}\tau \int \mathrm{dp} \int \mathrm{dX}' \; \underbrace{w(X', \tau)\delta\left(X - \varphi^{\tau}(X')\right)}_{\mathcal{L}(X,X')} \rho_0(X'; \omega)$$

$\mathcal{L}(X, X')$: Frobenius-Perron operator



$\rho(r, r_0, \omega)$: Density of rays starting uniformly in r_0 with $H(r_0, p) = \omega^2$ and reaching r including absorption

Classical ray tracing = following individual trajectories (room acoustics, computer graphics)

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Ray Tracing in terms of integral equations:

Ray density ρ (x, p) **→ linear integral equation:**
i.e. Rendering Equation (Computer Graphics), Radiosity Equation (Acoustics)

$$\rho(X,t) = \int \mathrm{dX}' \, \mathcal{L}(X,X',t) \, \rho(X',0) \, ; \qquad X = (r,p)$$

with linear integral kernel:
(Frobenius-Perron operator)

$$\mathcal{L}(X,X',t) = \delta\left(X - \varphi^t(X')\right)$$

Ray dynamics:

$$\varphi^t(X') = X(t); \; X(0) = X'$$

DEA: Finite Volume Method for solving the integral equation!

Numerical solution methods solving for $\rho(X,\tau)$ or $\rho(X,\omega)$

Integral Equation $$\rho(X,\tau) = \int \mathrm{dX'}\, \mathcal{L}(X,X',\tau)\, \rho(X',0)$$

Admits non-smooth solutions: Ergodic theory

Liouville Equation – differential form:

Smooth solutions – extensions: viscous LE in limit D→ 0 (Fokker-Planck Equation)

$$\rho_t + \dot{X}\,\nabla\rho - D\Delta\rho = 0$$

Characteristics – ray-tracing:

Solutions constant along characteristics = classical trajectories;
Boundary value problem, small sampling problem

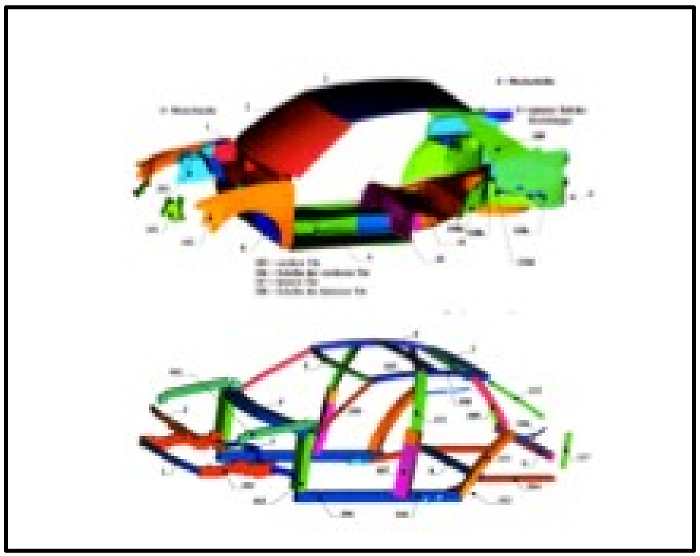
Markov approximation – Ulam method, Statistical Energy Analysis

Transition probabilities between cells in *phase space*

Statistical Energy Analysis:

- divide system into substructures
- determine average transmission/ reflection coefficients

Thermodynamic approach



Ennes Sarradj: www.tu-cottbus.de/fakultaet3/fileadmin/uploads/aeroakustik/files/sarradj_sea_daga2004.pdf

$$P_{ij} \quad = \quad \omega \bar{d}_i \eta_{ij} \left(\frac{E_i}{\bar{d}_i} - \frac{E_j}{\bar{d}_j} \right) ,$$

P_{ij} : Power flowing from subsystem i to j

$\bar{d}_i$: mean density of eigenmodes in i

η_{ij} : Coupling loss factors

E_i : wave energy stored in i

Expert tool - general conditions: irregular structure, low absorption, many reflections, well separated substructures, …

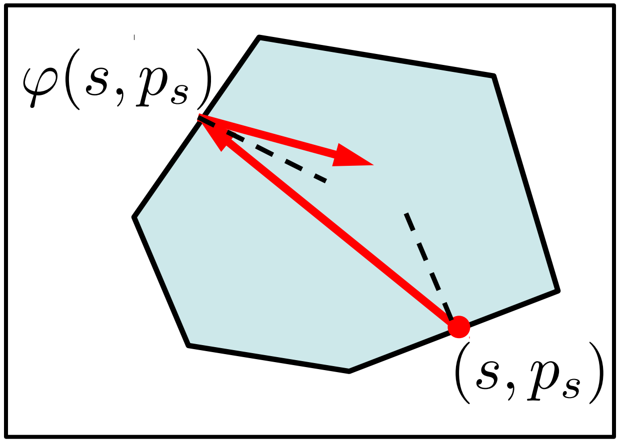
Energy ~ |Amplitude|^2

SEA: classical flow method for phase space densities

Dynamical Energy Analysis = Finite Volume Method:

For single cavity – fixed frequency

Boundary value problem – determine the ray density ρ_B on the boundary:



Boundary map: $\varphi : (s,p_s) \to \varphi(s,p_s)$

Boundary operator: $\mathcal{L} : \rho_B(s,p_s) \to [\mathcal{L}\rho_b](s,p_s)$

Stationary solution with multiple reflections:

$$\rho_B = \sum_{n=0}^{\infty} \mathcal{L}^n \rho_0 = (1-\mathcal{L})^{-1}\rho_0$$

source

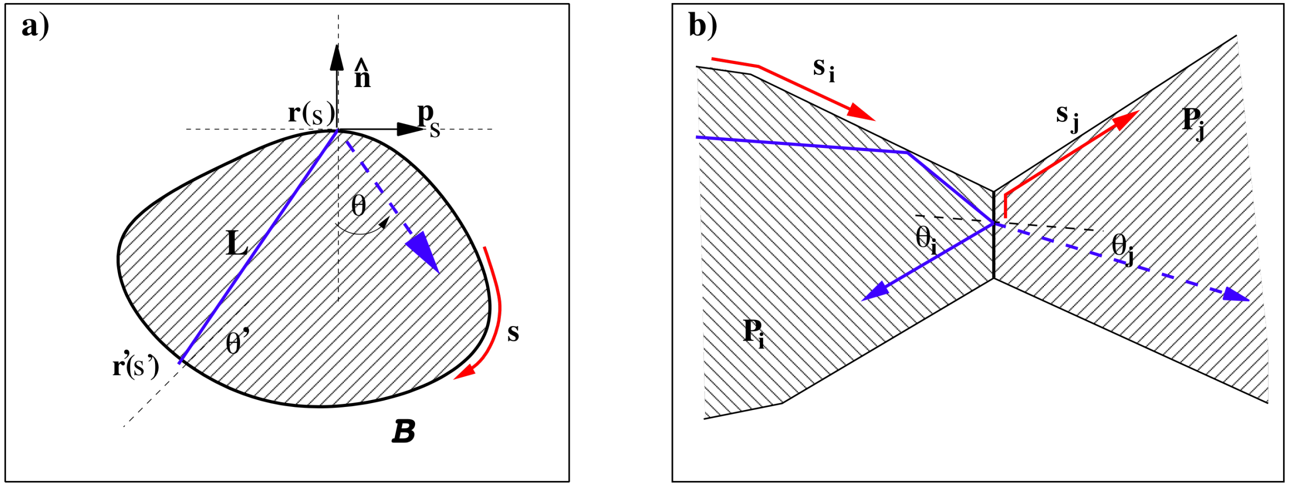
Summation over multiple reflections

Solve as boundary integral problem – many sub-domains:

Steady state solution:

Boundary map: $\varphi : (s, p_s) \to \varphi(s, p_s)$

Boundary operator: $\mathcal{L} : \rho_B(s, p_s) \to [\mathcal{L}\rho_b](s, p_s)$



$$\rho_B^{(n+1)} = \mathcal{T}\, \rho_B^{(n)}$$

Numerical Implementation:

- Fourier basis – periodic BC, problems at corners

- Chebyshev polynomials – separate expansion along edges **x 10**

- Collocation method in spatial, Legendre in momentum coordinates **x 10**

- Discrete Flow Mapping – ‘’Ulam’’ type method on grid with semi–analytic solution

x 100

efficiency gain

Discrete Flow Mapping

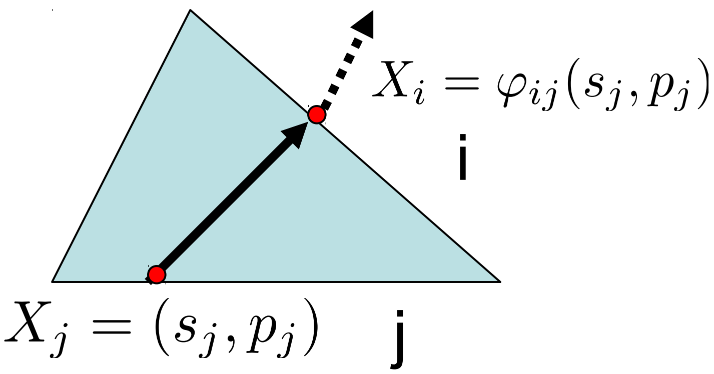
<u>In typical engineering problems:</u>

- Elasticity – different modes (shear, pressure, bending)
- Abrupt changes in material properties; corners, edges …; → mode mixing
- Curved shells → ray dynamics along geodesics (if $\lambda < R$)

Geometry data given as mesh/FEMgrids → use mesh to generate DEA matrix

Discrete Flow Mapping:

Define boundary map φ_{ij}
for each mesh region *j* with neighbour *i*:



Hamiltonian:

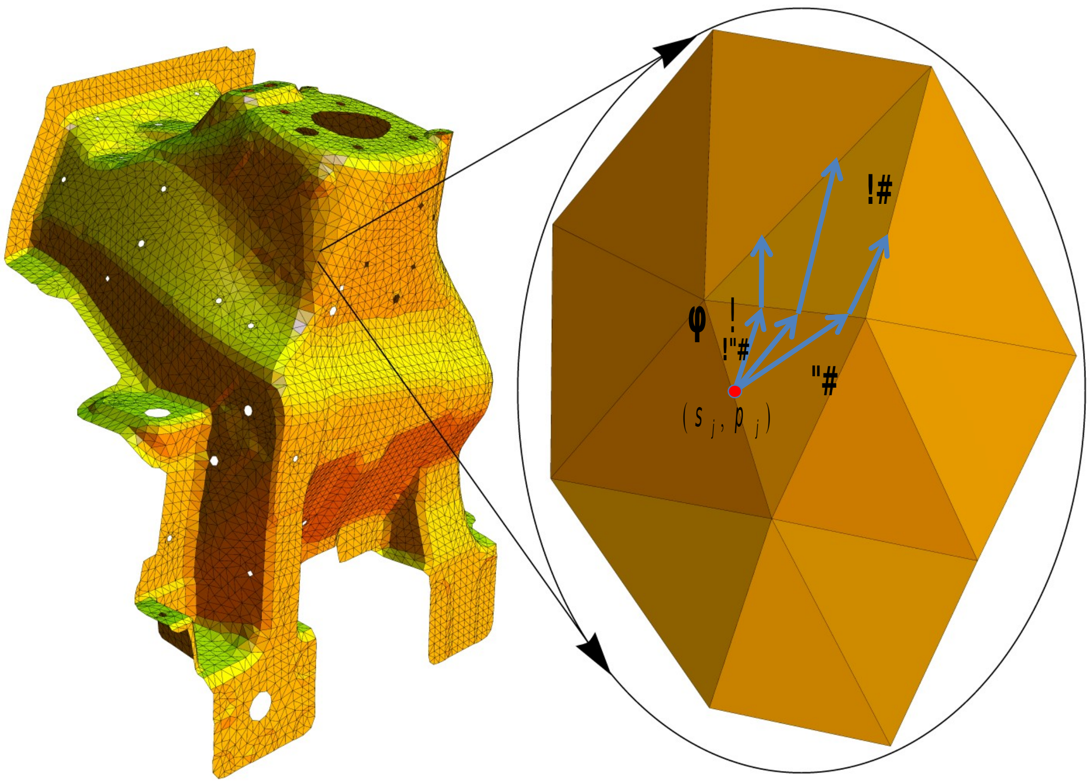
$$H = f(x)\,p^2 = \omega^2$$

$f(x_j) = f_j$; piecewise constant

Linear Operator
between edge j and i

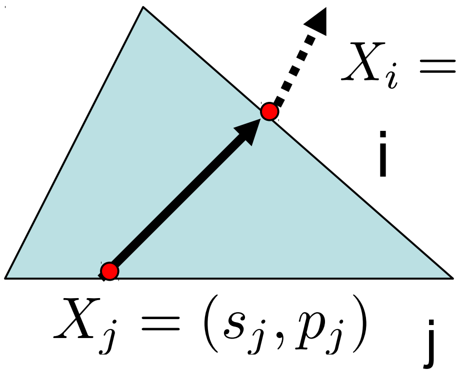
$$\mathcal{L}_{ij} : \rho_B^{(i)} \to \mathcal{L}[\rho_B^{(i)}] = \rho_B^{(j)}$$

Discrete Flow Mapping:



Range Rover - shocktower

Map φ_{ij} defined on boundary



Linear Operator between edge j and i

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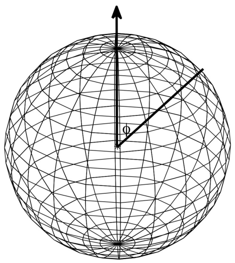
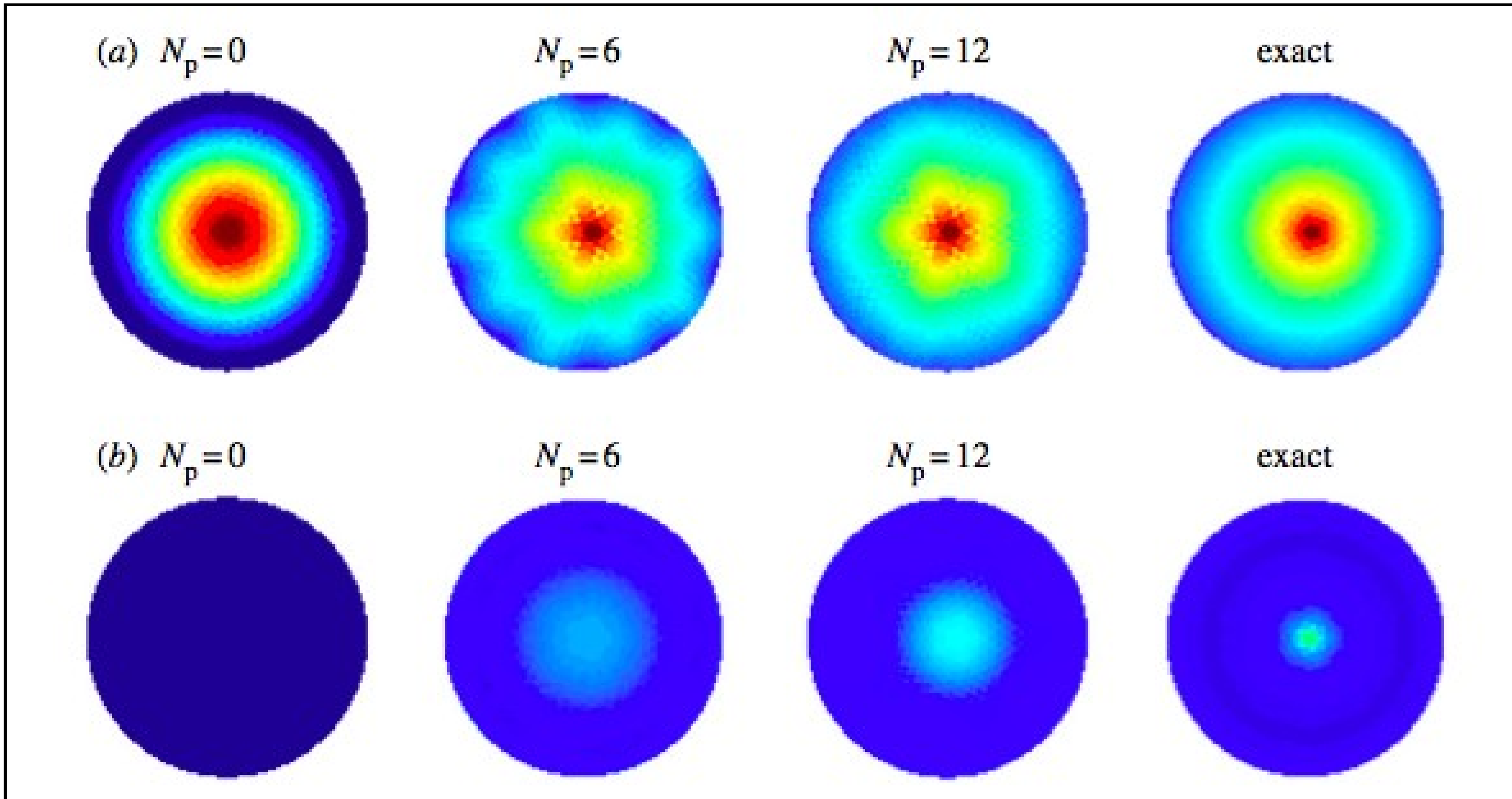


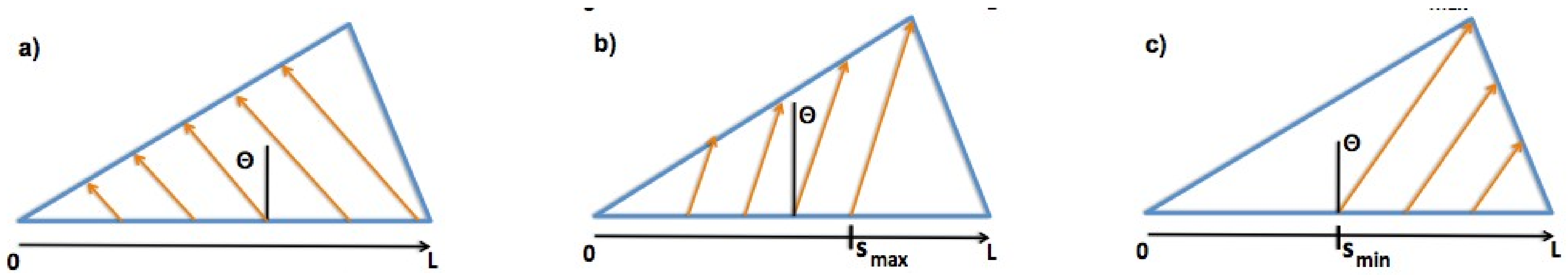


(a) $N_p = 0$
$N_p = 6$
$N_p = 12$
exact
(b) $N_p = 0$
$N_p = 6$
$N_p = 12$
exact

Constructing the operator:

$$T_{ij} = \int\int dX dX' \Psi_i(X) K(X, X') \Psi_j(X');$$
$$K(X, X') = w(X)\delta(X - \phi(X'))$$

Basis function $\Psi_i(s, p)$: piecewise constant in *s* along each edge;
Legendre Polynomial up to order *n* in *p*



Reduction to single integral for each mesh domain!

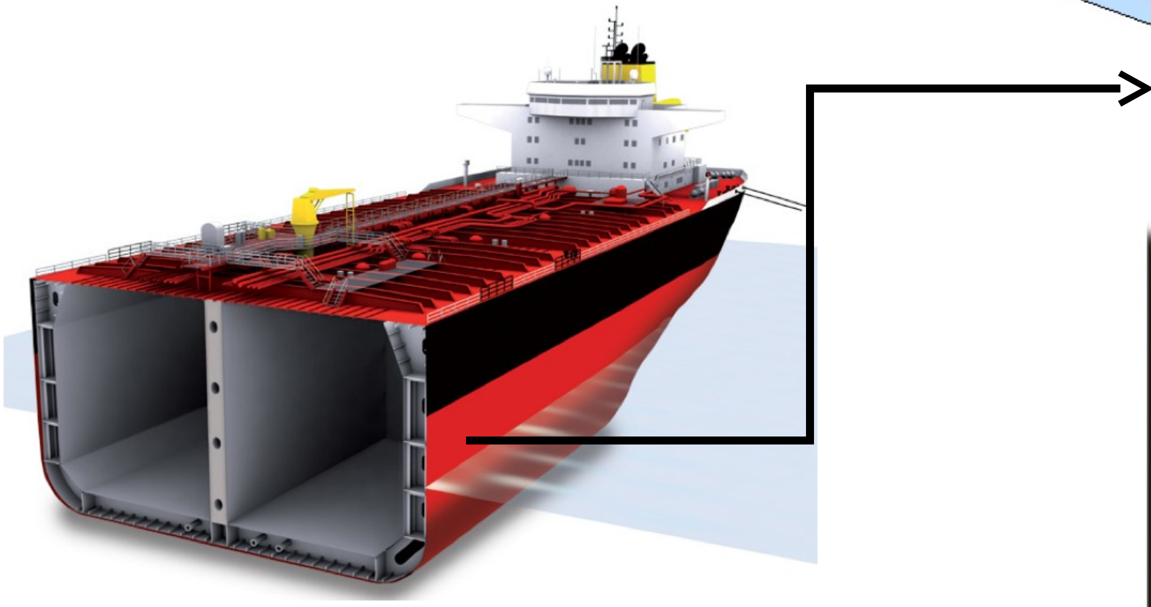
DFM: Num. method for solving the stationary integral equation

- Propagation by linear integral operator on each mesh cell → *Finite Volume Method*
- Operator: represent in basis functions or collocation method;
- Reflection/transmission at interfaces → scattering matrices;
- Linear system of equation – standard solver.

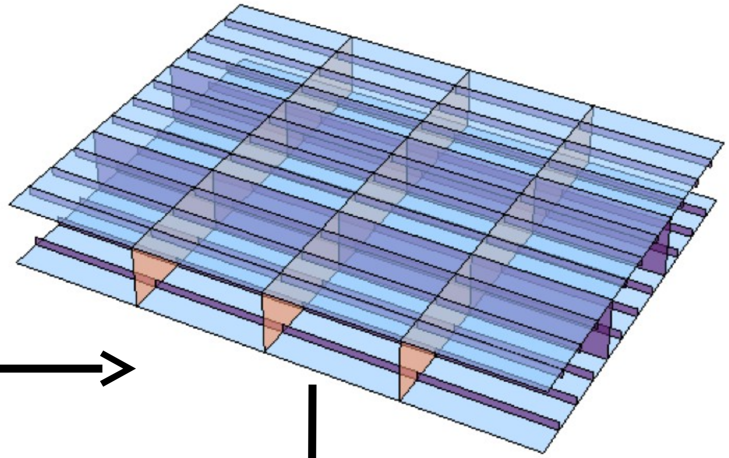
DEA interpolates between SEA and Ray Tracing

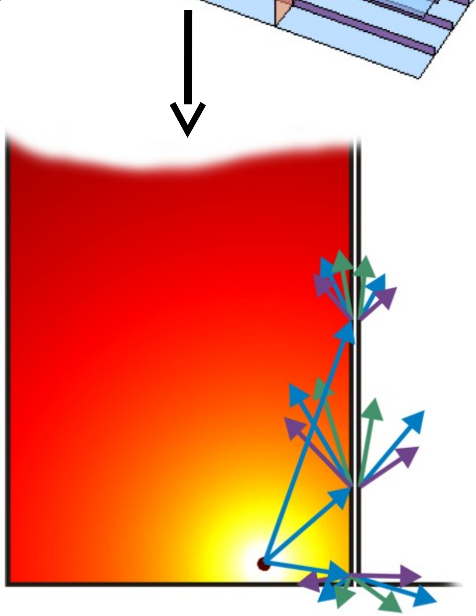
II. Applications

Vibro-acoustics for large structures: Tank ship



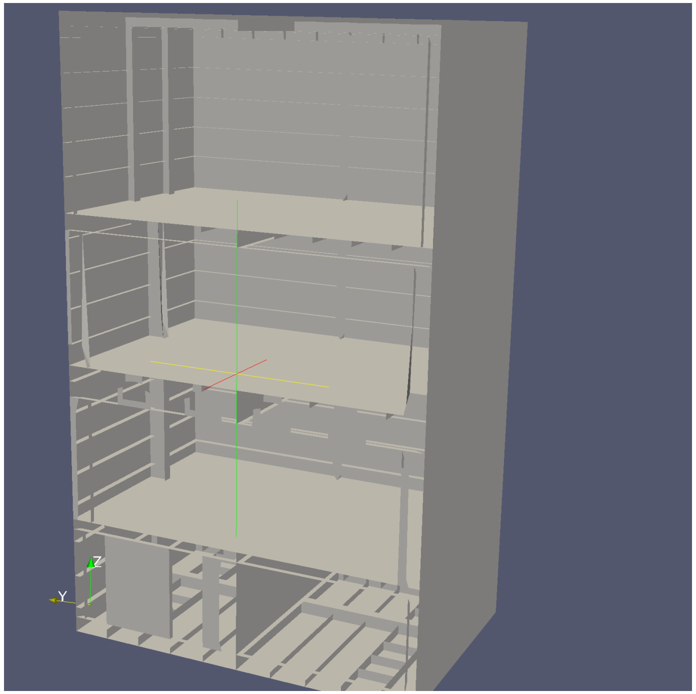
Model from *Germanischer Lloyd*, Hamburg

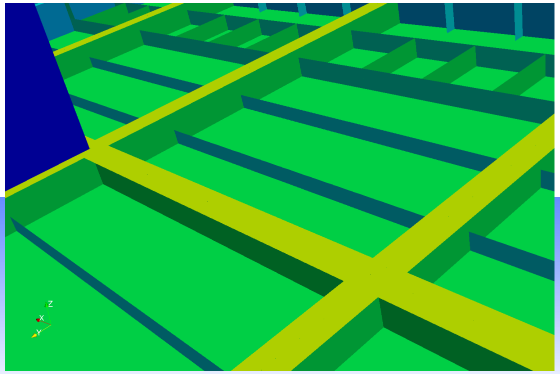




Mode conversion at edges

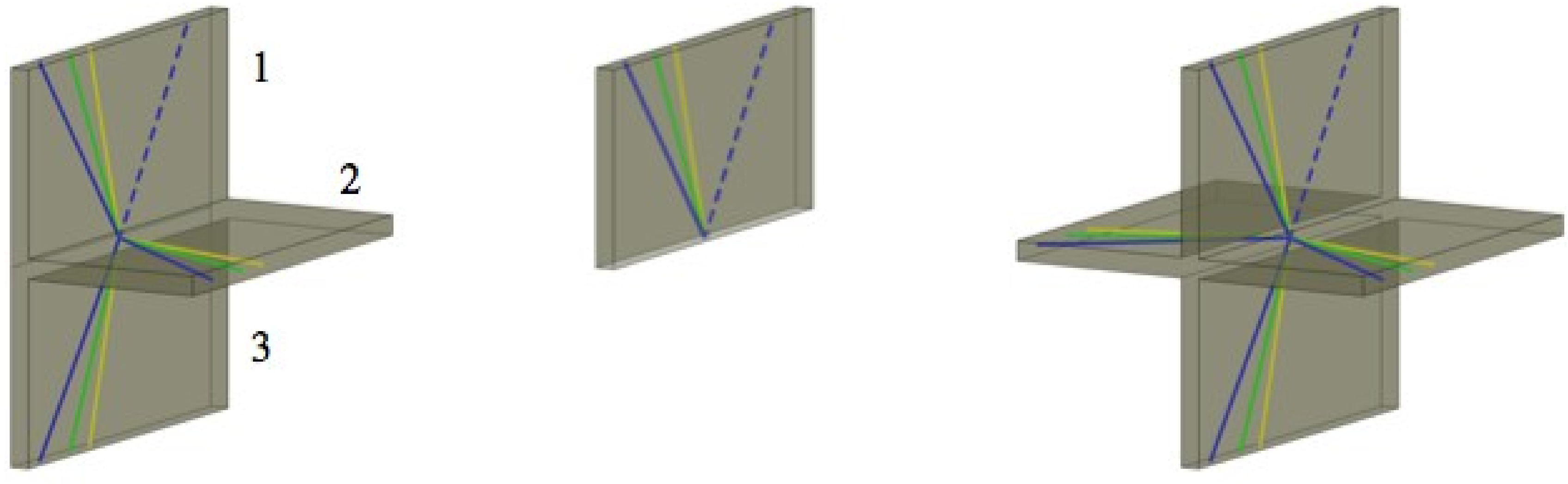
Germanischer Lloyd – Schiffsdeckhaus Model





Stifferners, T-joints, plates …

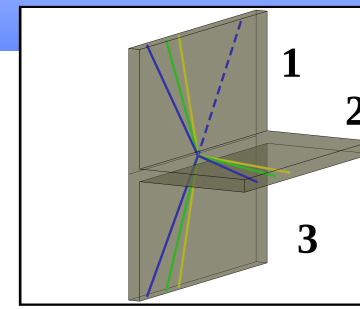
Junctions in the structure:



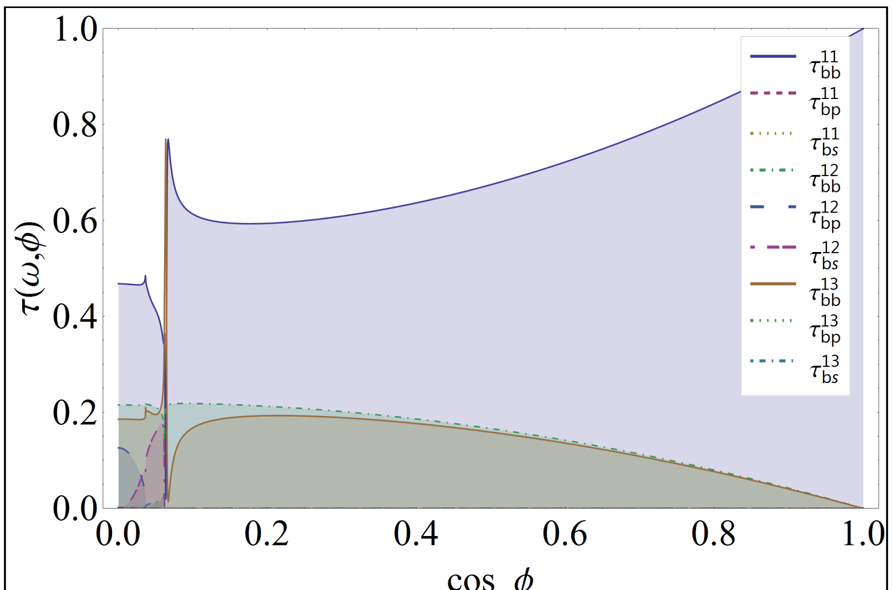
(a) T-junction (b) open end (c) X-junction

blue dashed: incoming s – ray; solid: outgoing s (blue), p (green) and b (yellow)
(s: shear-; p: pressure-; b: bending-wave)

Scattering matrix at edge – T junction



200 Hz



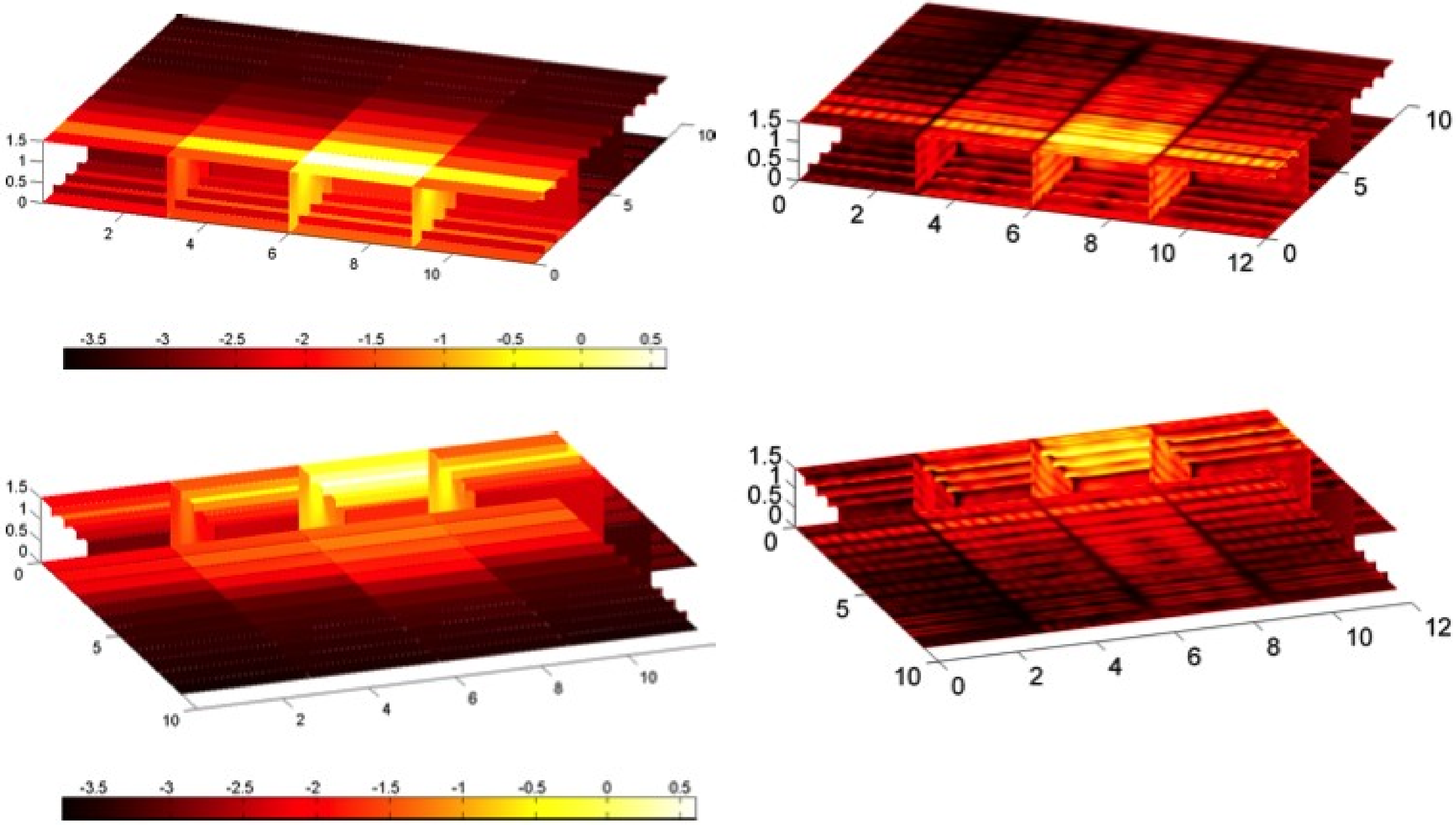
Bending → Bending, Pressure or Shear

mainly:
Bending → Bending

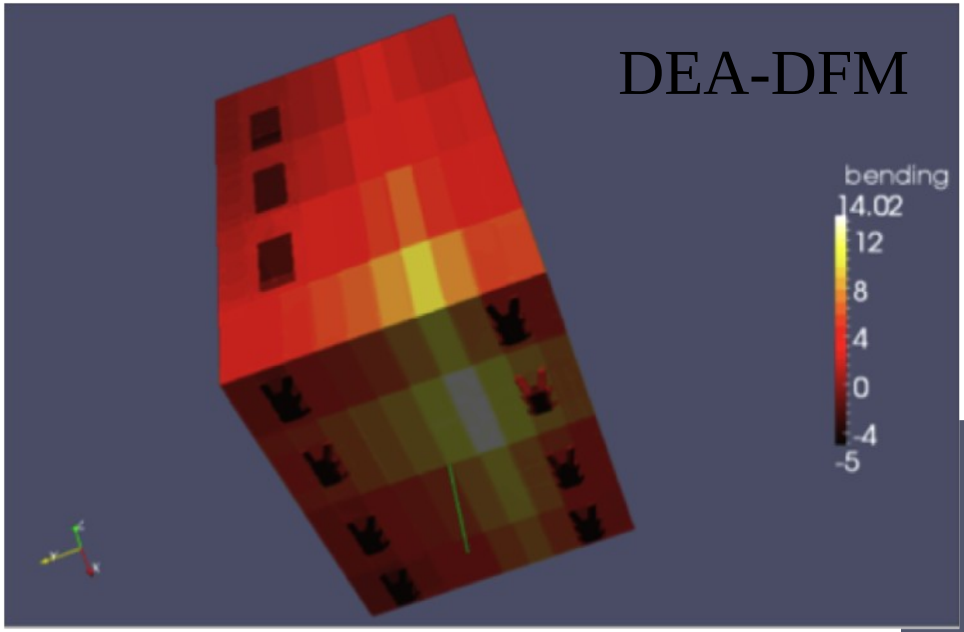
R S Langley and K H Heron, *Elastic wave transmission through plate/beam junctions*, JSV **143**, 241, 1990.

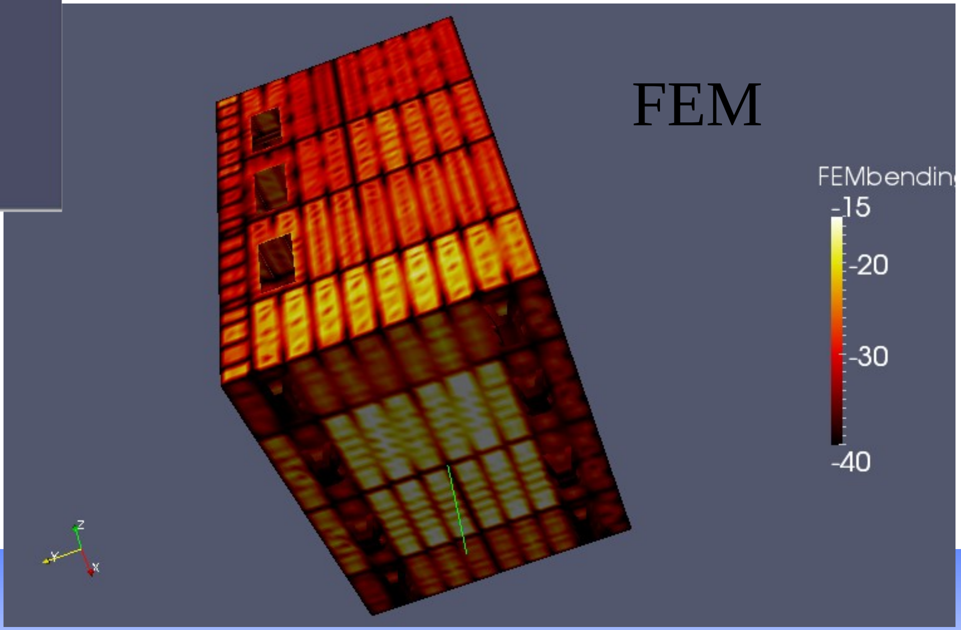
DEA - Bending

FEM-averaged

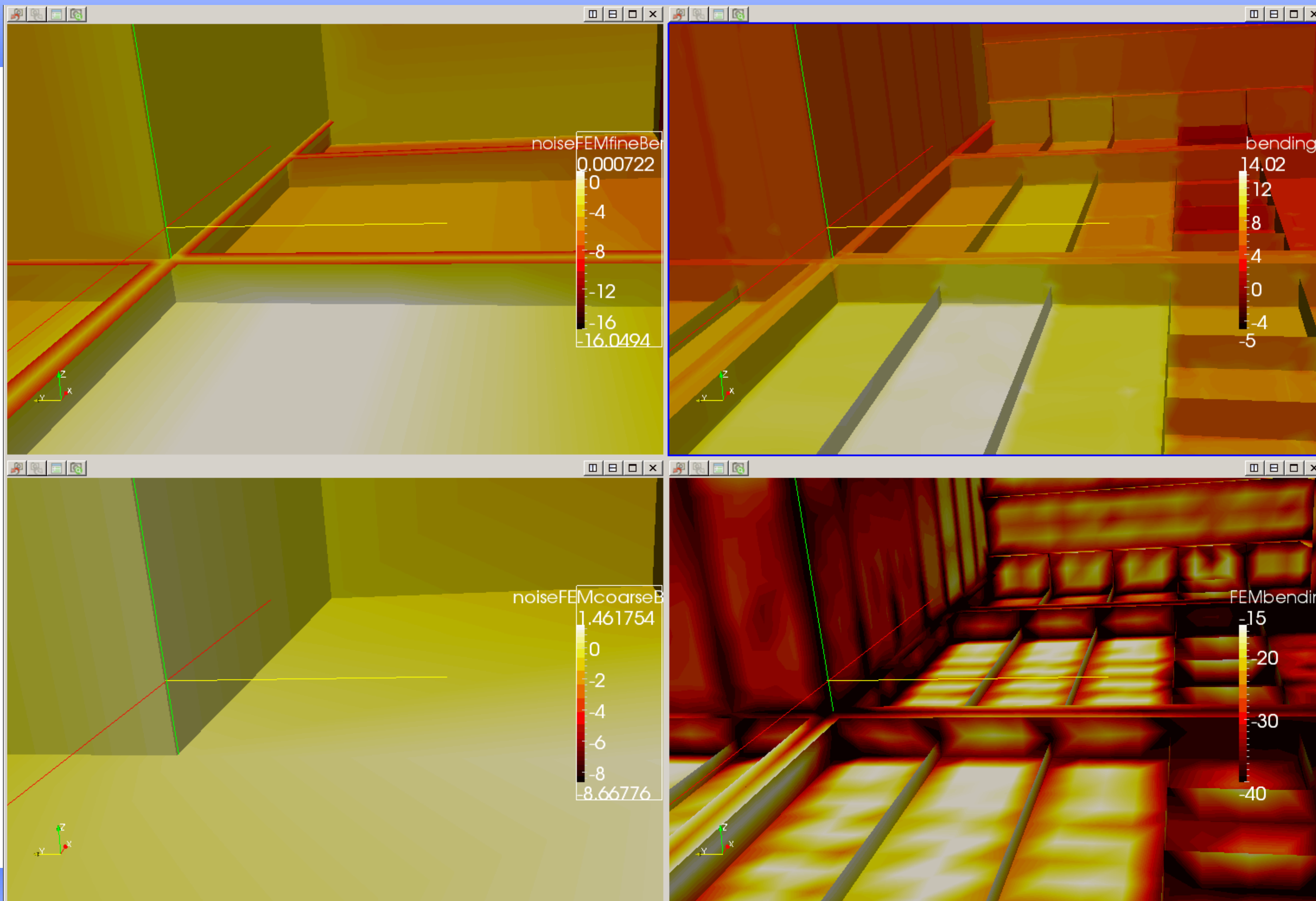


200 Hz

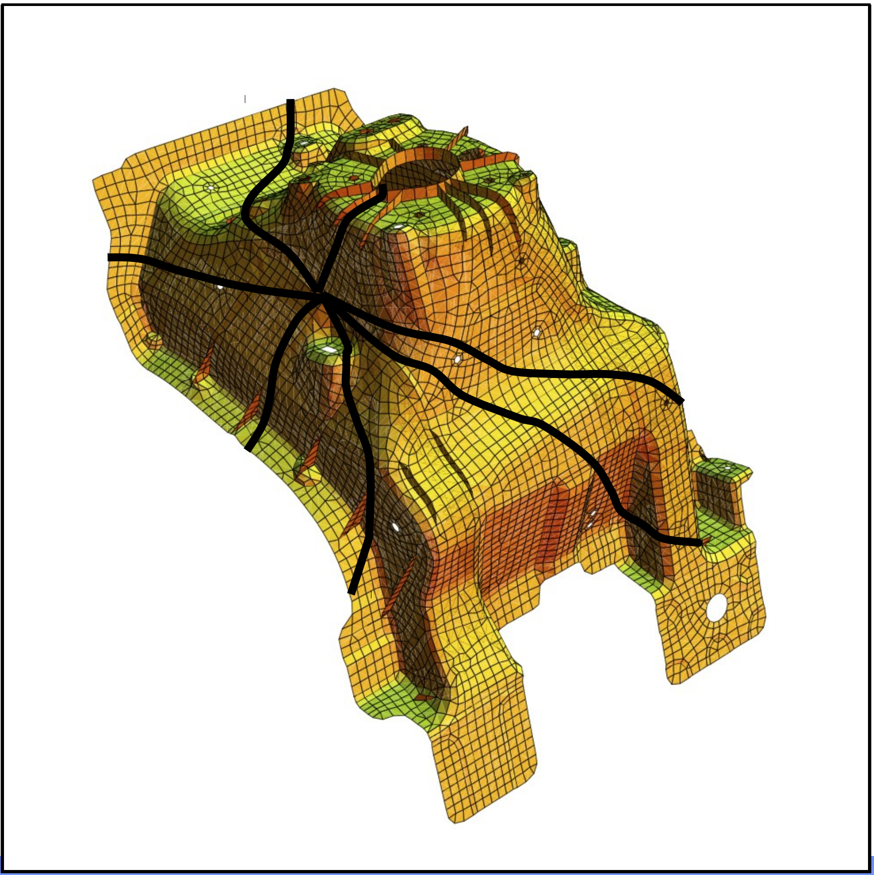




200 Hz


noiseFEMfineBe
0.000722
0
-4
-8
-12
-16
-16.0494
bending
14.02
12
8
4
0
-4
-5
noiseFEMcoarseB
1.461754
0
-2
-4
-6
-8
-8.66776
FEMbendin
-15
-20
-30
-40

Complex structures: Example - Range Rover Shock Tower:



Material: Aluminum
Thickness: 2 – 8 mm
FEM: 40000 elements

Ray-Tracing on curved surfaces:

$\lambda \ll R$ (radius of curvature): rays along geodesics

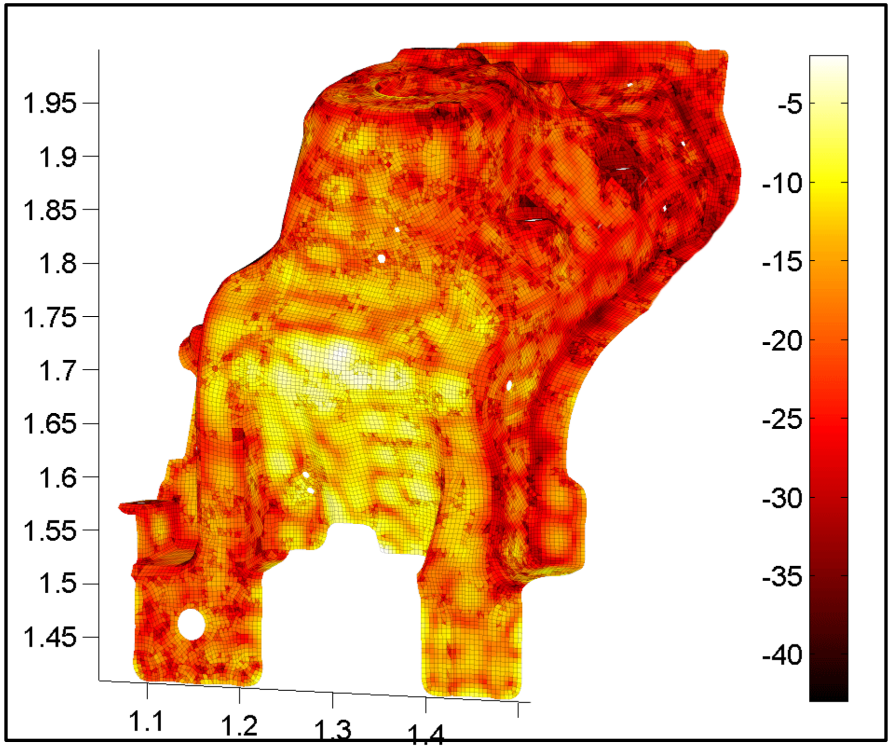
$\lambda \sim R$ – curvature corrections

Range Rover Shock Tower: Including curvature corrections

10 KHz

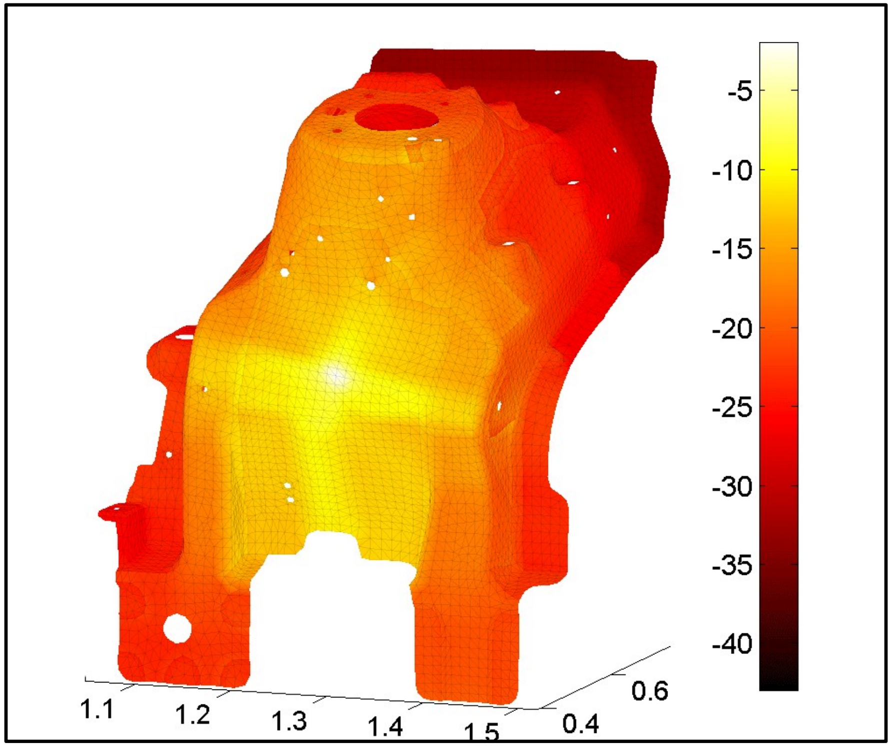
FEM

Δf = 1 kHz



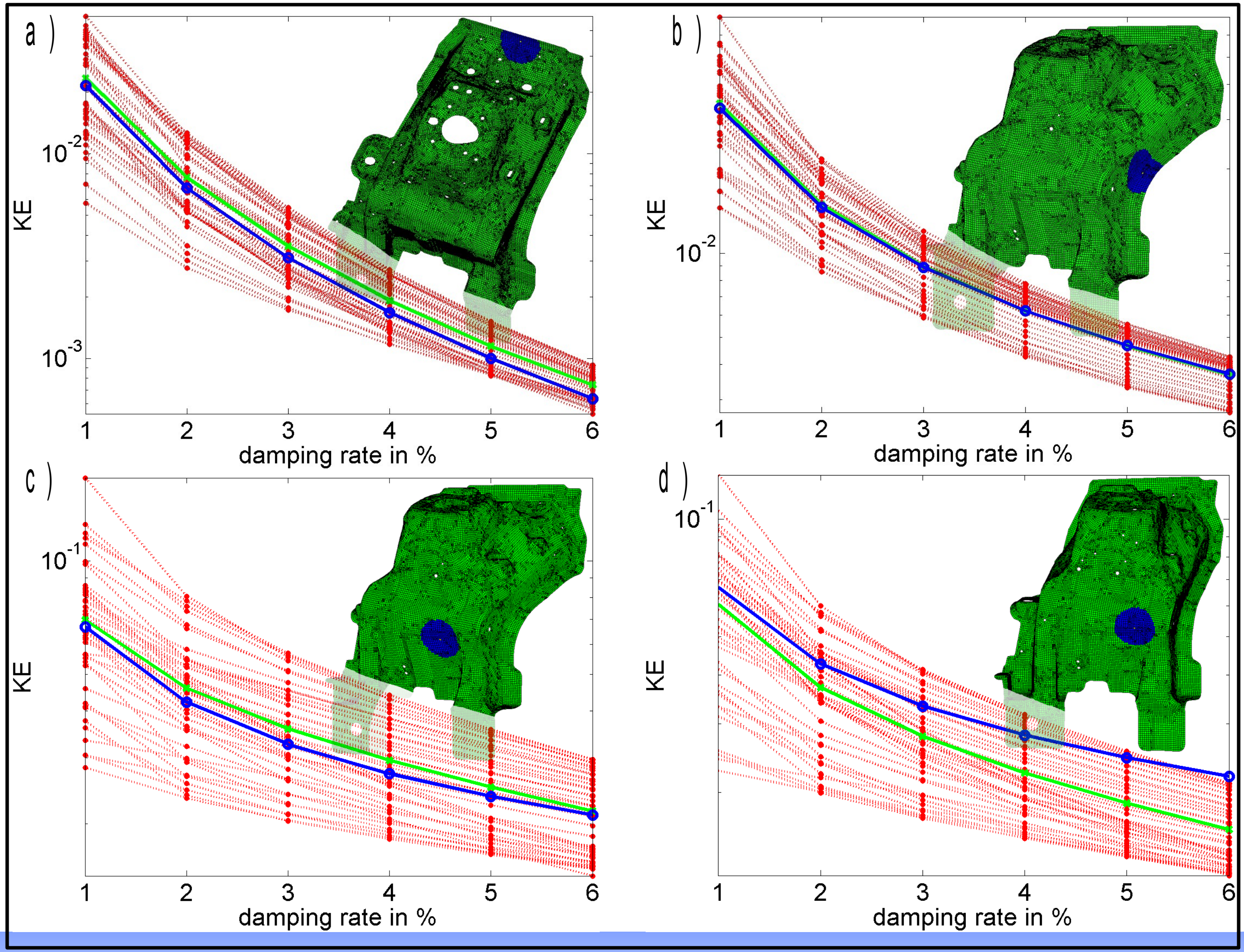
DEA

Including curvature



Computation in 7 min on Laptop – 700 000 unknowns!

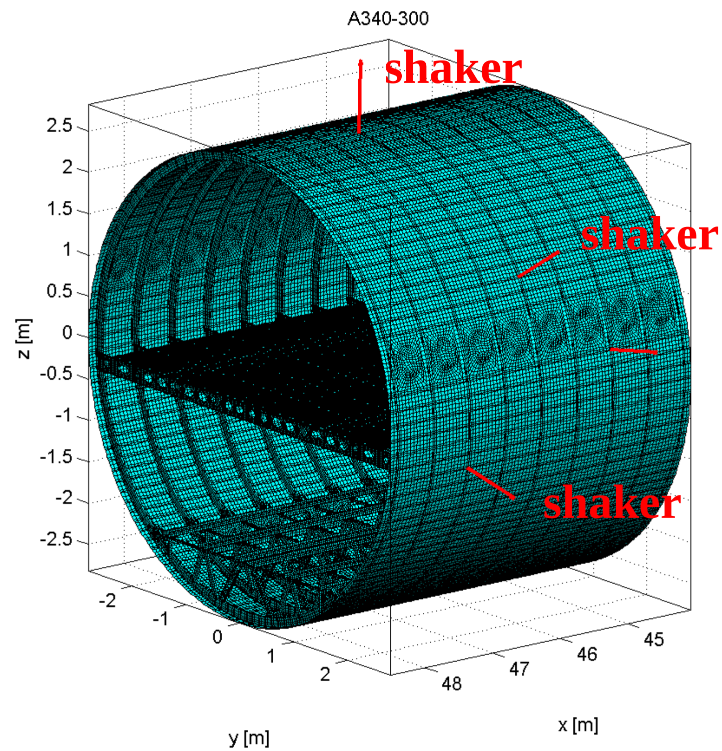
Validation:



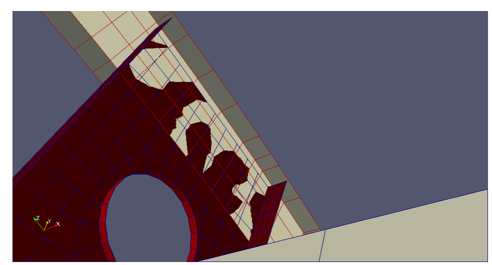
a)
b)
c)
d)
KE
10^-2
10^-3
10^-1
1 2 3 4 5 6
damping rate in %

FEM: f1 … fn
FEM: mean
DEA-DFM

Segment of an A340-300:



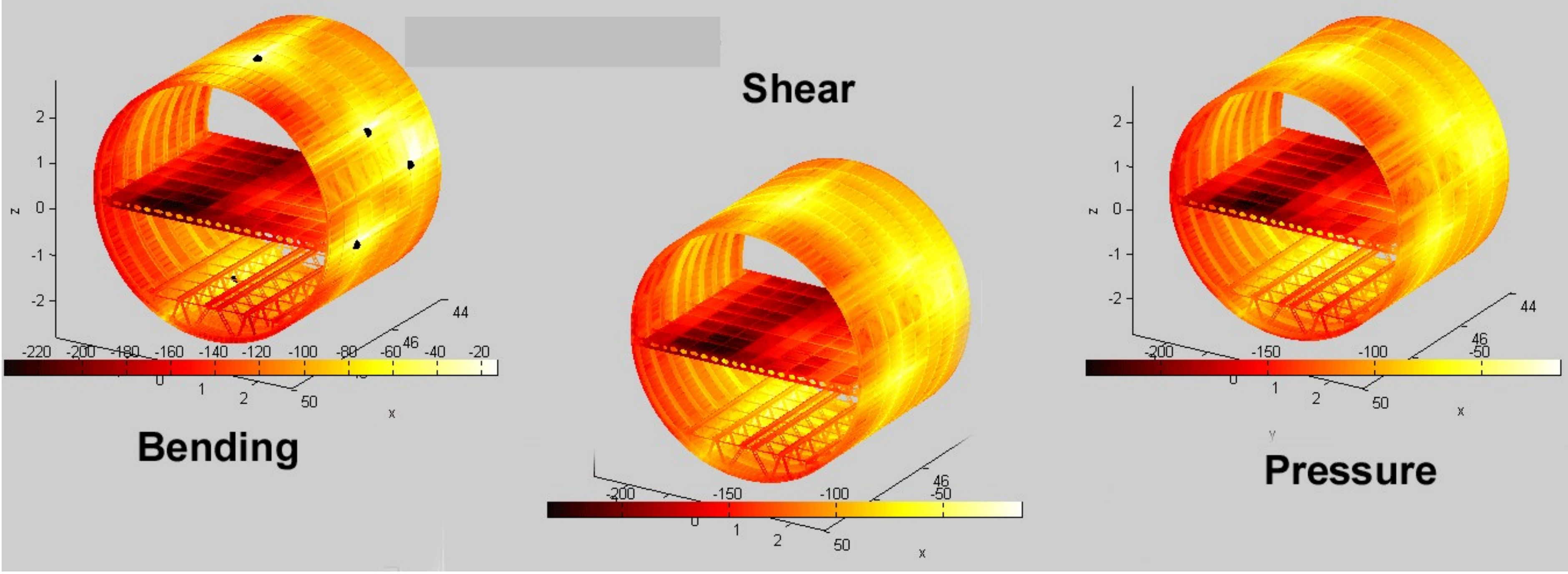
DEA analysis of complex structure
- FEM meshes glued together
  (RBAR – Nastran)
- Anisotropic floor panels



Courtesy: Innovation Works - EADS

Segment of an A340-300:

200 Hz



Not validated yet

Dynamical Energy Analysis – Discrete Flow Mapping
-
vibro-acoustic modeling using a mesh based approach!

- **applicable in the mid- to high frequency regime – extending SEA.**
- **can be integrated into existing meshes → compatibility with standard FEM.**
- **can involve high degree of complexity and structural details.**
- **Transfer paths can be easily visualized and detected.**

Dynamical Energy Analysis – Discrete Flow Mapping
-
vibro-acoustic modeling using a mesh based approach!

References:

- *Discrete flow mapping: transport of phase space densities on triangulated surfaces*, D. J. Chappell1, G. Tanner, D. L öchel, and N. Søndergaard, Proc. Royal Soc. A 469 2155 20130153 (2013).
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