Modern challenges in the gyrokinetic modeling of turbulent transport in tokamak plasmas

Yann Camenen

CNRS, Aix-Marseille Univ., Marseille, France

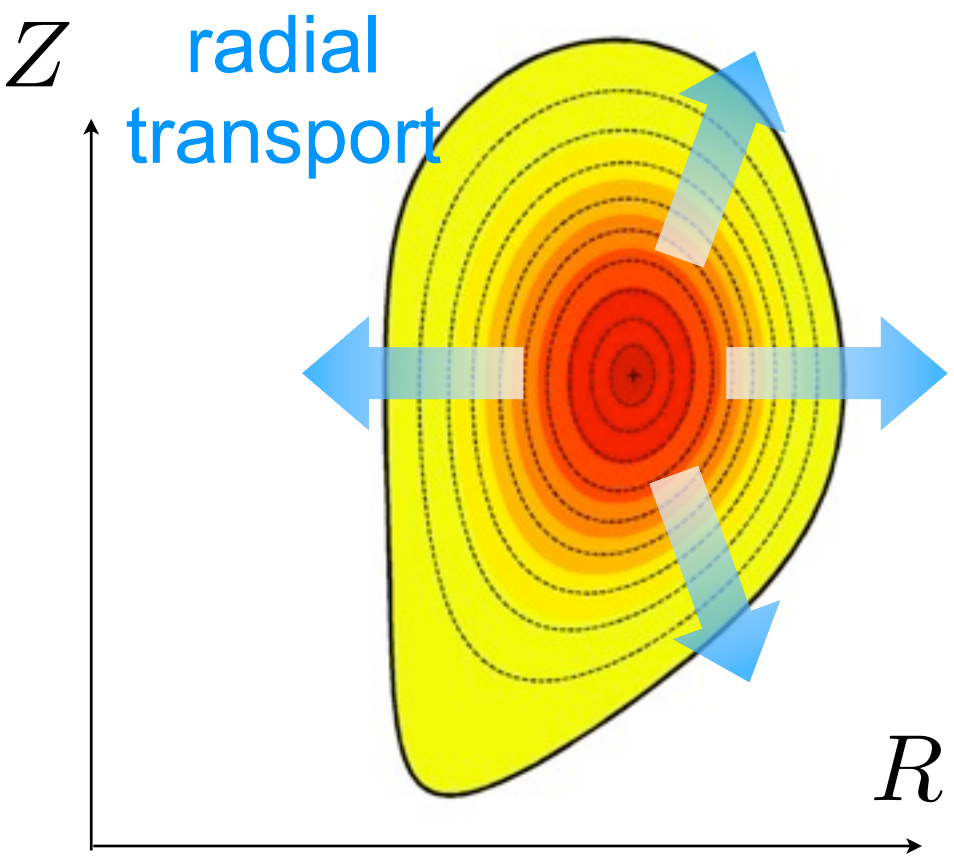
Foreword

- **What this talk is not:**
  - A review of gyrokinetics and tokamak turbulence
- **What this talk is:**
  - Focused on the practical application of gyrokinetics to the modelling of turbulent transport
  - List a few points that presently limit us (in my view)

Outline

- What is turbulent transport, why does it matter?
- Brief introduction to the gyrokinetic framework
- Standard simplifications
- Local delta-f simulations, typical results
- What would help?

The issue of transport in tokamaks



- **Desired:** hot, dense core → fusion?
- **Solution explored:** magnetic confinement in a toroidal device
- Nested magnetic flux surfaces: radial transport << parallel transport
- Core plasma evolution governed by the transport across flux surfaces: lower transport → better confinement

- **1D transport equation** (example of plasma density):

local conservation equation (3D): ⟶ *flux surface averaged version (1D):*

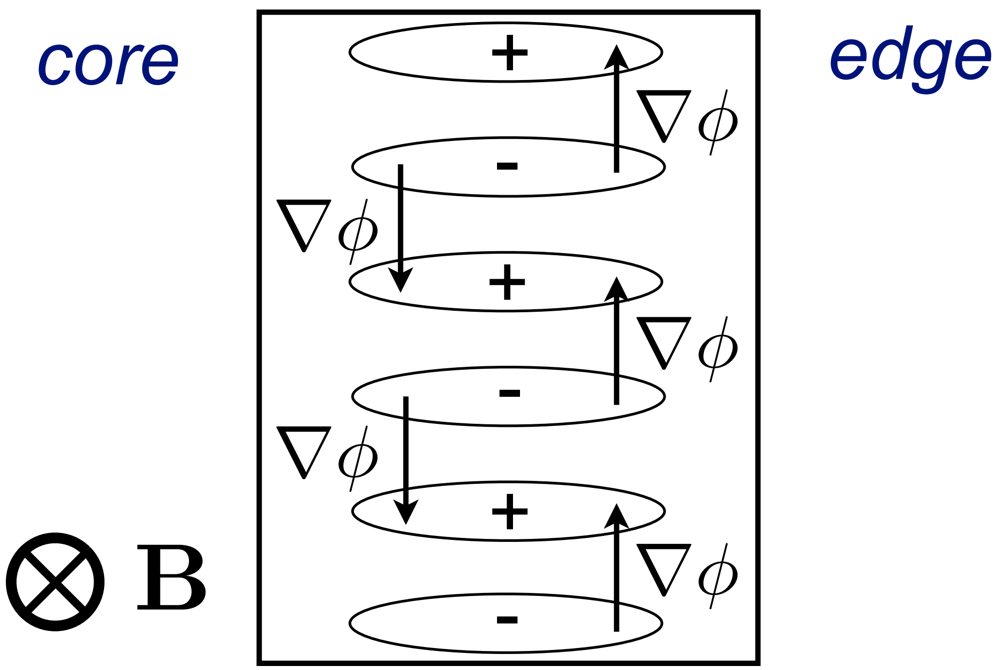
$$\frac{\partial n}{\partial t} + \nabla \cdot \underbrace{\mathbf{\Gamma}}_{\text{particle flux}} = \underbrace{\mathcal{S}}_{\text{source}}$$

$$\boxed{\frac{\partial < n >}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial r}\left[V' < \mathbf{\Gamma} \cdot \nabla r >\right] = < \mathcal{S} >}$$

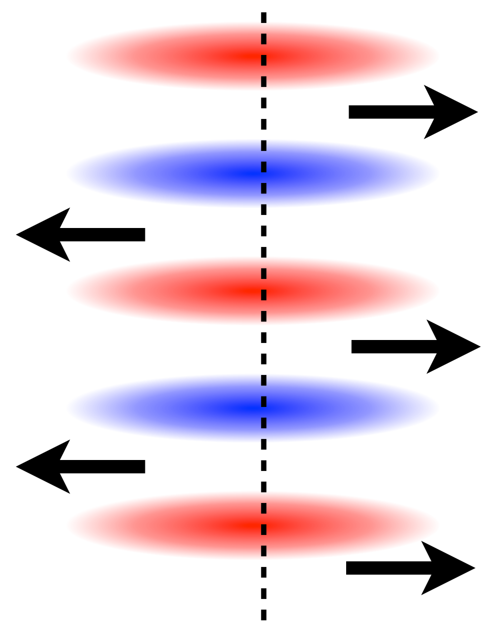
used to predict plasma profiles

Electrostatic turbulent transport

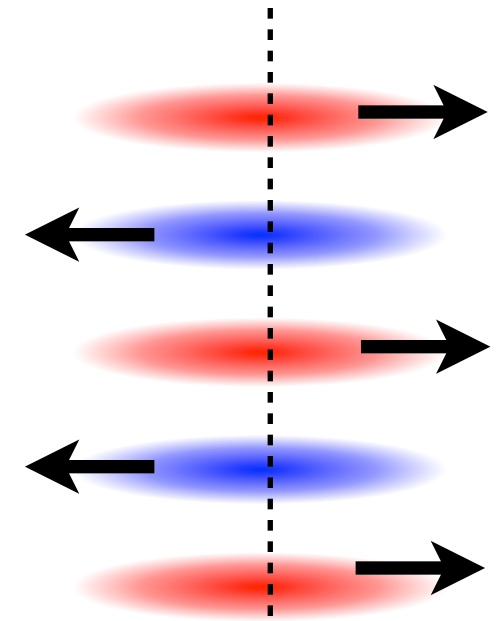
- Turbulence: small scale perturbations, electrostatic case



electrostatic
potential perturbation



density perturbation
in phase



density perturbation
out of phase

$$\mathbf{v}_{E\times B} = \frac{\mathbf{b}\times\nabla\delta\phi}{B}$$

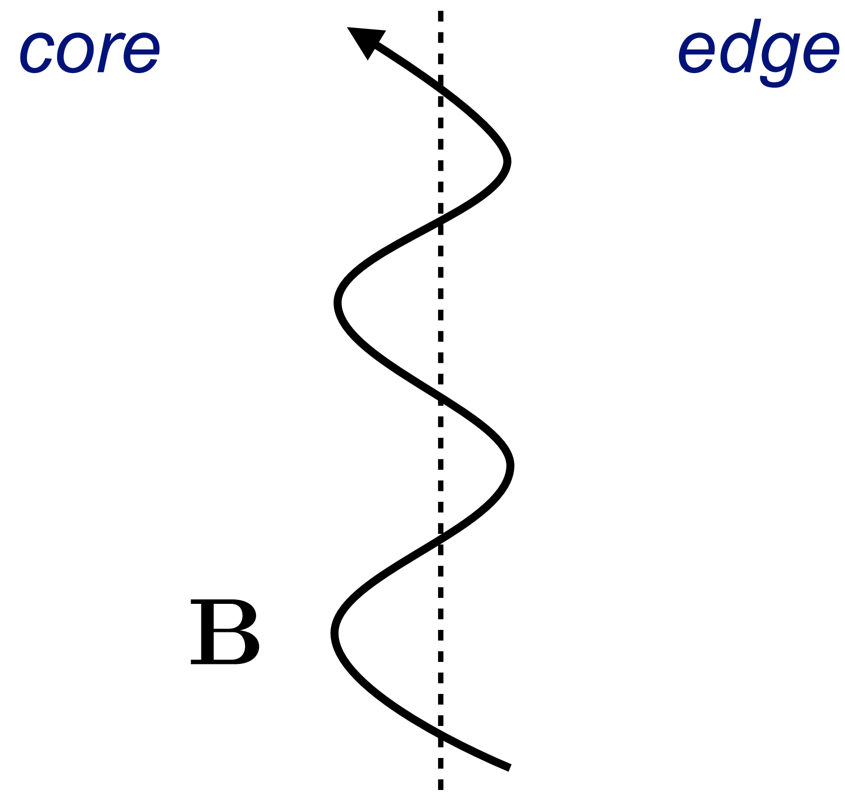
ExB drift motion

no net radial
transport

net radial
transport

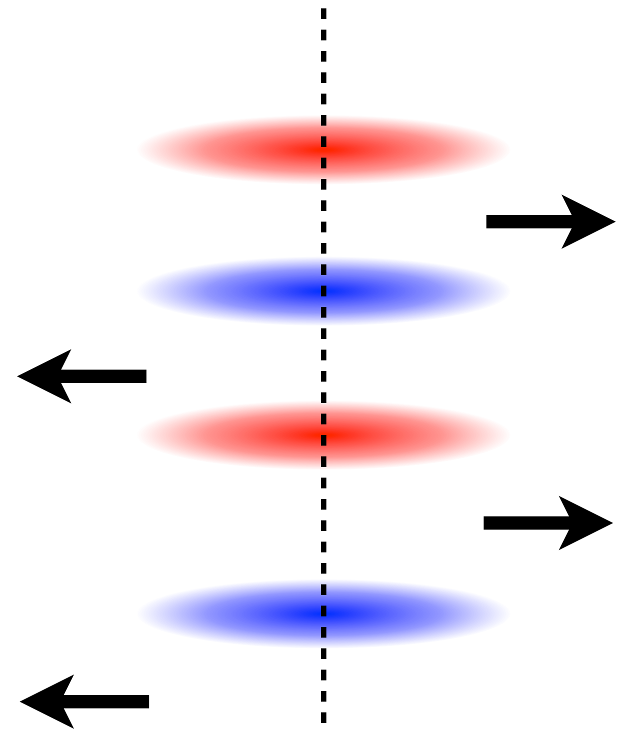
Electromagnetic turbulent transport

- Turbulence: small scale electromagnetic perturbations



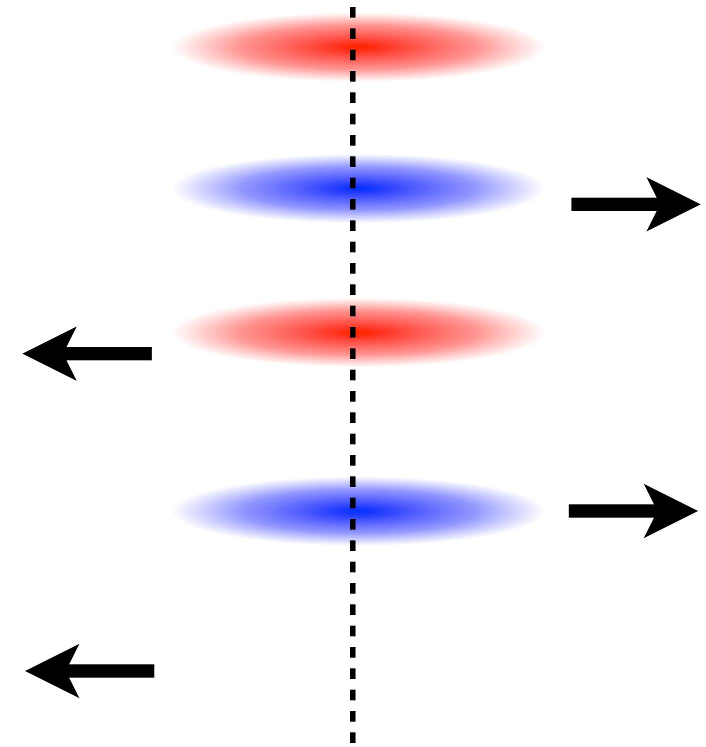
parallel motion along perturbed field line

$$v_{||}\nabla\delta A_{||}\times\mathbf{b}_0$$



density perturbation in phase

no net radial transport



density perturbation out of phase

net radial transport

Gyrokinetic description of turbulent transport

- **Desired:** self-consistent evolution of density, momentum and temperature perturbations + electromagnetic perturbations
- **Caveat:** first principle approach based on Vlasov-Maxwell equations is very costly (6D) and rather inefficient
- **Gyrokinetic ordering:**
  - Magnetised plasma $\rho_i \ll L \sim a$
  - Small perturbations $$\frac{\delta f}{F} \sim \frac{e\delta\phi}{T_e} \sim \frac{\delta B}{B} \ll 1$$
  - Spatial scales: gyro-radius $k_\perp \rho_i \sim 1$
  - Time scales:
    - Slow compared to the gyro-period $\omega \ll \Omega_{ci}$
    - Keep wave-particle interactions $\omega \sim k_\parallel v_{thi}$
- **Gyro-averaged equations:**
  - Drop the gyrophase: 6D → 5D
  - Parallel/perpendicular scale separation $k_\parallel / k_\perp \ll 1$

The gyrokinetic Vlasov-Maxwell system

$$\frac{\partial F}{\partial t} + \frac{d\mathbf{X}}{dt} \cdot \frac{\partial F}{\partial \mathbf{X}} + \frac{d\mathbf{V}}{dt} \cdot \frac{\partial F}{\partial \mathbf{V}} = 0$$

evolution of the gyrocenters distribution function

$$\nabla^2(\phi_0 + \delta\phi) = -\frac{1}{\epsilon_0}(\bar{\rho} + \rho_{\mathrm{pol}})$$

electrostatic potential

$$\nabla \times (\mathbf{B}_0 + \delta\mathbf{B}) = \mu_0(\bar{\mathbf{J}} + \mathbf{J}_{\mathrm{pol}} + \mathbf{J}_{\mathrm{mag}})$$

magnetic field

[see e.g. Brizard RMP'07]

- This **non-linear** system yields the turbulent radial fluxes we are looking for:

$$< \mathbf{\Gamma} \cdot \nabla r >= \left\langle \int F \frac{\mathbf{b} \times \nabla \delta\bar{\phi}}{B} \cdot \nabla r \mathrm{d}\,\mathbf{v} \right\rangle - \left\langle \int F v_{\parallel} \frac{\mathbf{b} \times \nabla \delta \bar{A}_{\parallel}}{B} \cdot \nabla r \mathrm{d}\,\mathbf{v} \right\rangle$$

Radial flux of gyrocenters

Good but too expensive...simplifying further

- Standard simplifications:
  - Frozen magnetic equilibrium $\partial \mathbf{B}_0 / \partial t = 0$

>200,000,000
CPU hours/sim.

Good but too expensive...simplifying further

- Standard simplifications:
  - Frozen magnetic equilibrium $\partial \mathbf{B}_0/\partial t = 0$

    >200,000,000 CPU hours/sim.

  - Frozen background (δf approximation) $\partial F_0/\partial t = 0$ with $F = F_0 + \delta f$

The delta-f approximation

- Equilibrium given by the stationary lowest order gyro-kinetic equation:

$$\left.\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t}\right|_0 \cdot \frac{\partial F_0}{\partial \mathbf{X}} + \left.\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\right|_0 \cdot \frac{\partial F_0}{\partial \mathbf{V}} = 0 \qquad F = F_0 + \delta f$$

- Solution is a canonical Maxwellian, constant on the unperturbed trajectories

- Then solves for the perturbed distribution function only:

$$\frac{\partial \delta f}{\partial t} + \frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} \cdot \frac{\partial \delta f}{\partial \mathbf{X}} + \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} \cdot \frac{\partial \delta f}{\partial \mathbf{V}} = \underbrace{\left.\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t}\right|_1 \cdot \frac{\partial F_0}{\partial \mathbf{X}} + \left.\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\right|_1 \cdot \frac{\partial F_0}{\partial \mathbf{V}}}_{\textit{source}}$$

- Valid provided the feedback of turbulence on the equilibrium can be discarded: gradient-driven simulations

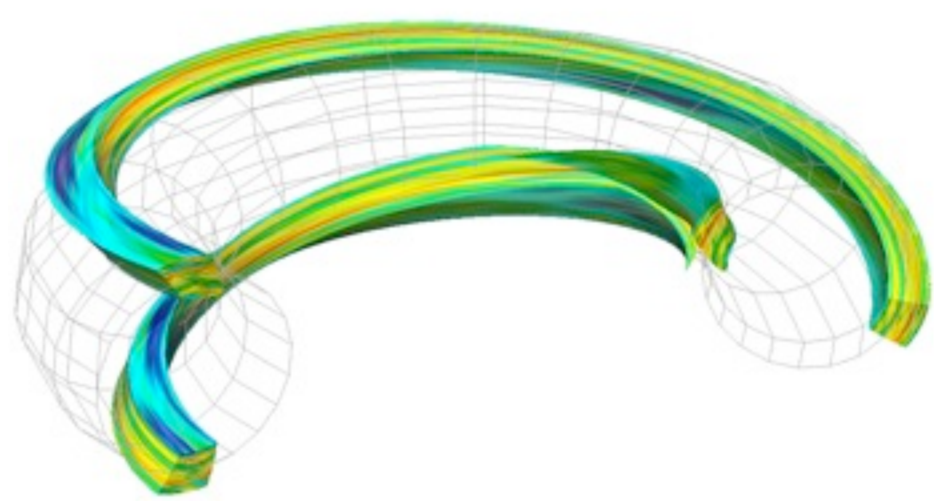
Good but too expensive...simplifying further

- Standard simplifications:
  - Frozen magnetic equilibrium $\partial \mathbf{B}_0/\partial t = 0$

    >200,000,000 CPU hours/sim.
  - Frozen background (δf approximation) $\partial F_0/\partial t = 0$ with $F = F_0 + \delta f$
  - Local approximation $F_0(r) = F_0(r_0)$ and $\nabla F_0(r) = \nabla F_0(r_0)$

The local approximation

- A single flux-tube is simulated rather than the full tokamak
- Profiles are linearised and evaluated at the center of the flux-tube:

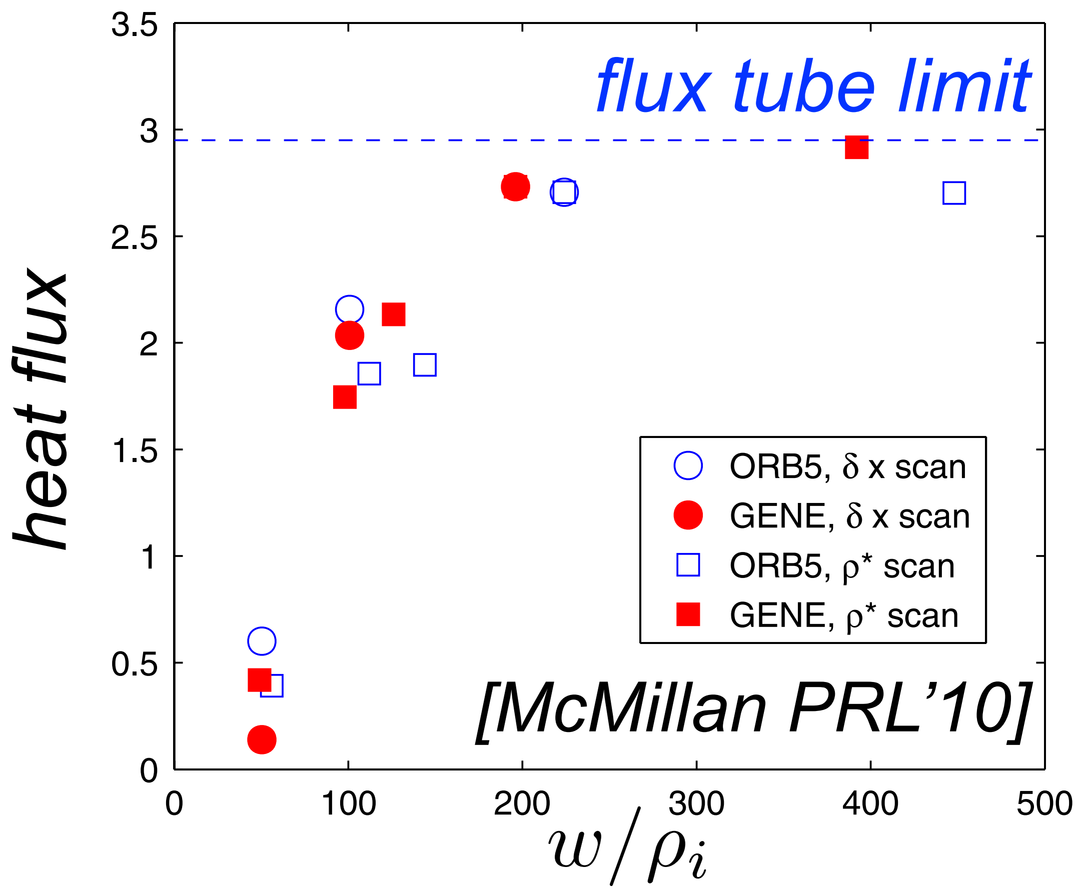


$$F_0(r) = F_0(r_0) + (r - r_0)\frac{\partial F_0}{\partial r}[r_0]$$

$$F_0(r) = F_0(r_0)$$

$$\nabla F_0(r) = \nabla F_0(r_0)$$

- Valid provided the largest turbulent structures (~10-20 Larmor radii) do not feel the variation of the equilibrium



Good but too expensive...simplifying further

- Standard simplifications:
  - Frozen magnetic equilibrium $\partial \mathbf{B}_0 / \partial t = 0$

    >200,000,000 CPU hours/sim.
  - Frozen background (δf approximation) $\partial F_0 / \partial t = 0$ with $F = F_0 + \delta f$
  - Local approximation $F_0(r) = F_0(r_0)$ and $\nabla F_0(r) = \nabla F_0(r_0)$
  - Quasi-linear approximation
    - Cross-phase is assumed to be given by the linear response
    - Saturation amplitude is modelled

$$< \mathbf{\Gamma} \cdot \nabla r >= \left\langle \int F \frac{\mathbf{b} \times \nabla \delta \bar{\phi}}{B} \cdot \nabla r \mathrm{d}\, \mathbf{v} \right\rangle - \left\langle \int F v_{\parallel} \frac{\mathbf{b} \times \nabla \delta \bar{A}_{\parallel}}{B} \cdot \nabla r \mathrm{d}\, \mathbf{v} \right\rangle$$

$$F \delta \phi \sim A^{\mathrm{model}} \cos \varphi^{\mathrm{linear}}$$

Back to the NL local delta-f approximation

- Standard simplifications:
  - Frozen magnetic equilibrium $\partial \mathbf{B}_0/\partial t = 0$
  - Frozen background (δf approximation) $\partial F_0/\partial t = 0$ with $F = F_0 + \delta f$
  - Local approximation $F_0(r) = F_0(r_0)$ and $\nabla F_0(r) = \nabla F_0(r_0)$

>200,000,000 CPU hours/sim.

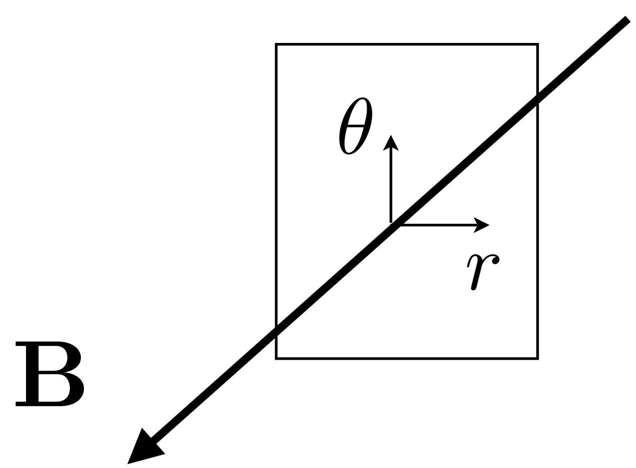
Back to the NL local delta-f approximation

- Standard simplifications:
  - Frozen magnetic equilibrium $\partial \mathbf{B}_0 / \partial t = 0$

    >200,000,000 CPU hours/sim.

  - Frozen background (δf approximation) $\partial F_0 / \partial t = 0$ with $F = F_0 + \delta f$
  - Local approximation $F_0(r) = F_0(r_0)$ and $\nabla F_0(r) = \nabla F_0(r_0)$

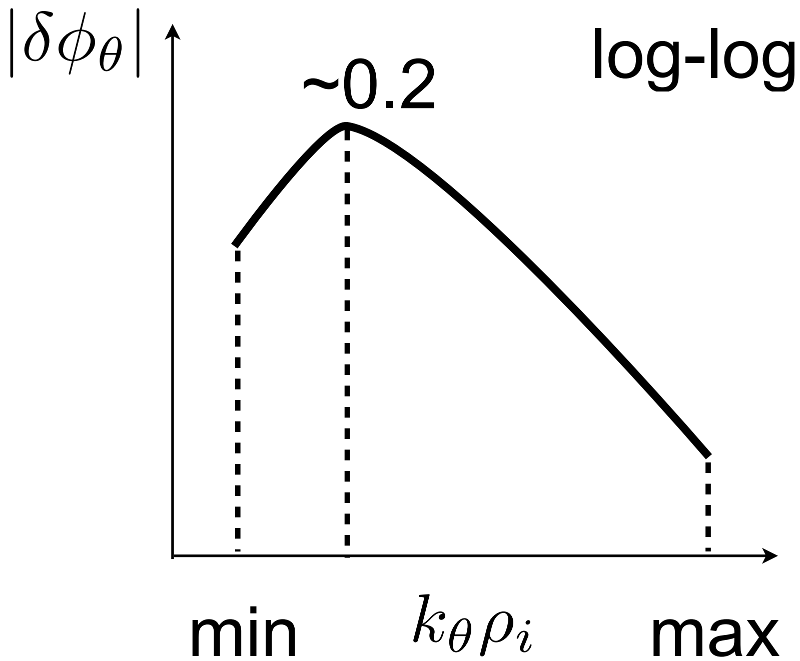
- Practical choices:
  - Parallel/perp dynamics → field aligned coordinates (r, θ, s)
  - Turbulence homogeneous in the perpendicular plane (local approximation) → spectral representation

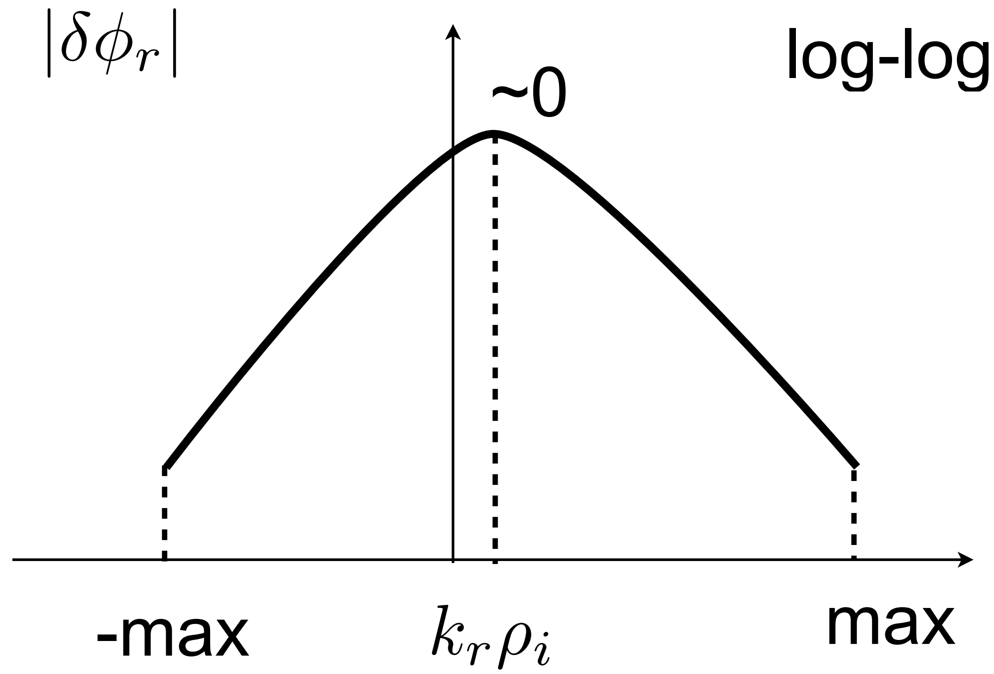


$$\delta f(\mathbf{X}) \longrightarrow \delta \hat{f}(k_r, k_\theta, s)$$
$$\delta \phi(\mathbf{X}) \longrightarrow \delta \hat{\phi}(k_r, k_\theta, s)$$
$$A_{||}(\mathbf{X}) \longrightarrow \delta \hat{A}_{||}(k_r, k_\theta, s)$$

A typical non-linear local delta-f simulation

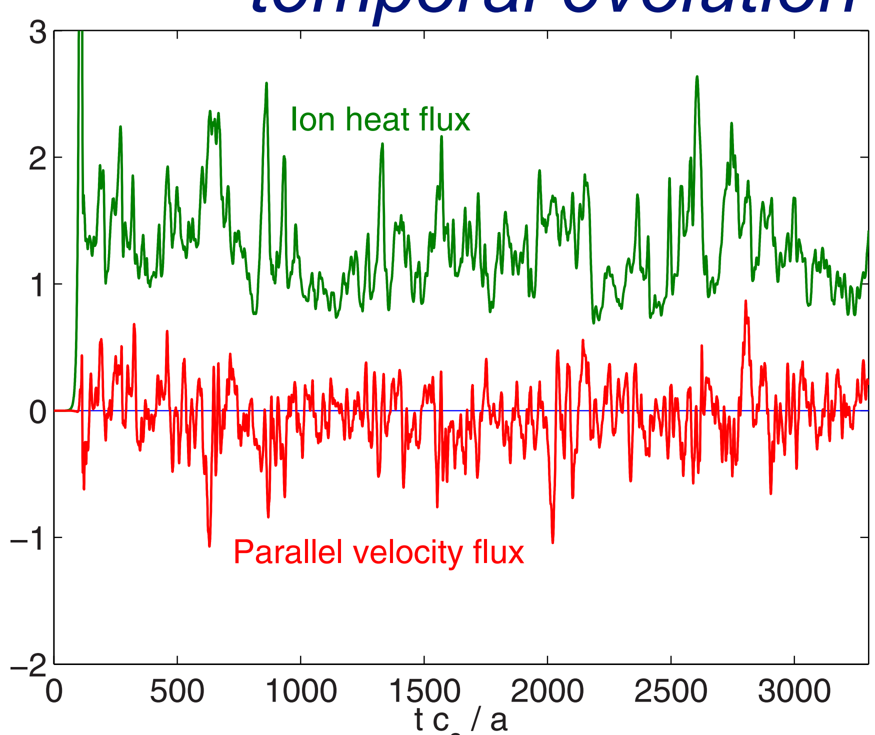
Electrostatic potential spectra



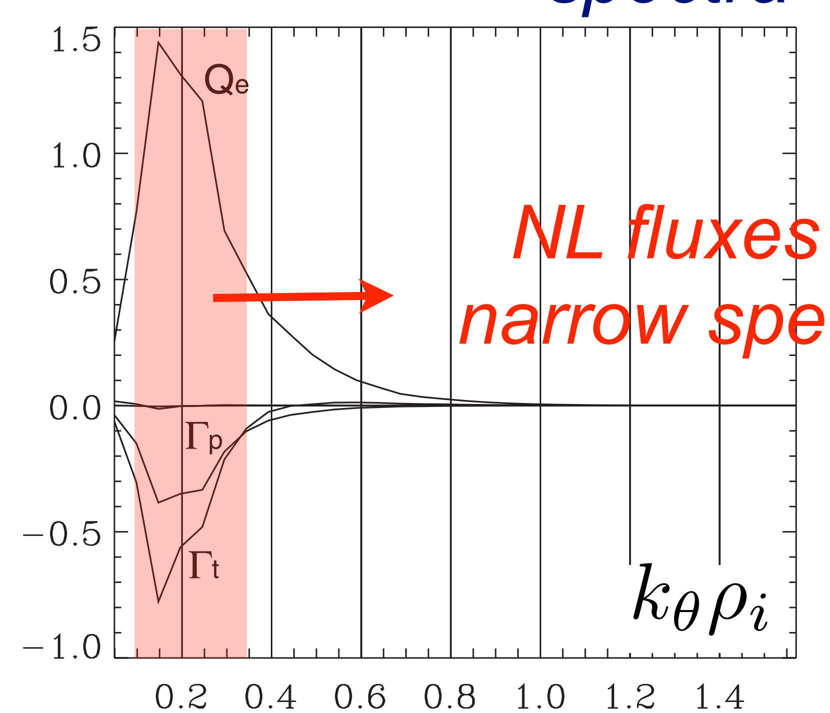


Fluxes

temporal evolution



spectra



NL fluxes peak in a narrow spectral region

Spectral grids and simulation cost

- Large grids = high computational cost
  $\rightarrow$ what physically determines the grid sizes?

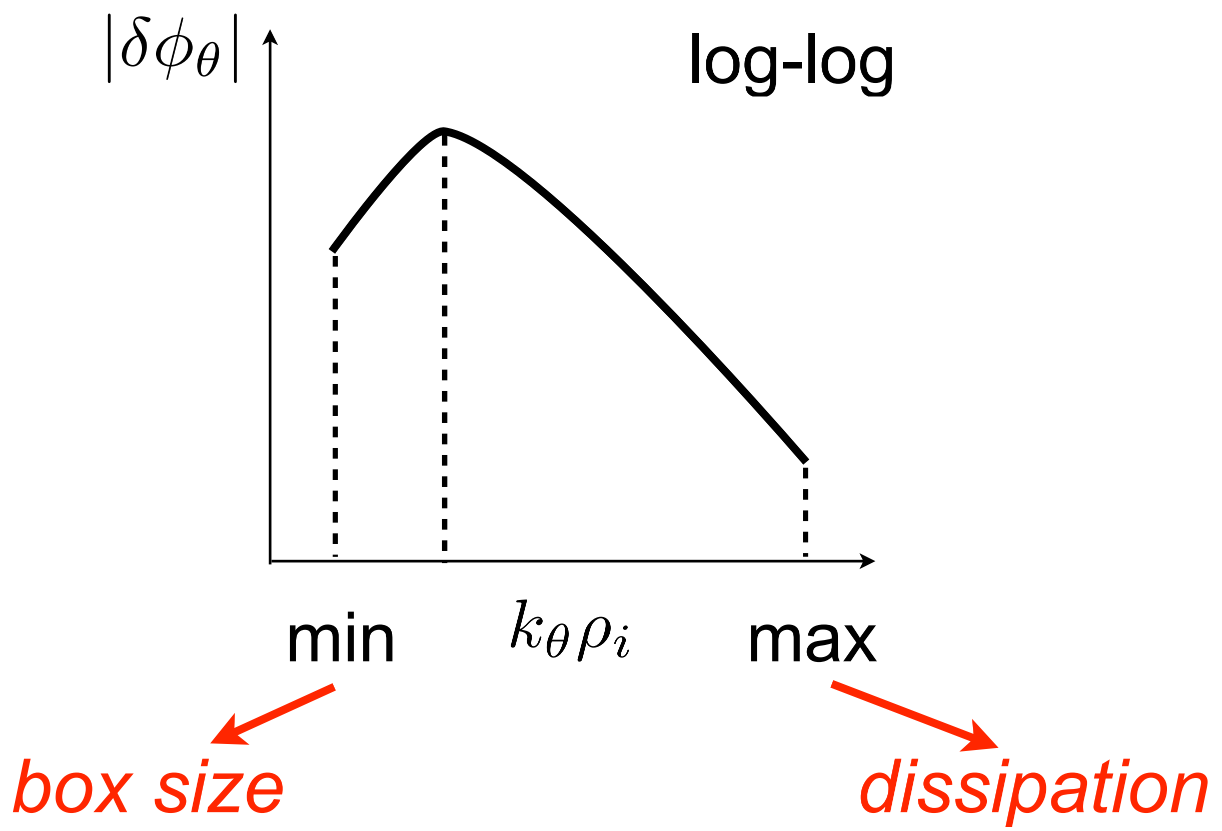
- Box size > largest physical structures
  $\rightarrow$ gives the step size in spectral space

$$k_\perp^{\min} = \frac{2\pi}{L_\perp} < \frac{2\pi}{L_{\mathrm{phys}}}$$

$$k_\perp^{\min}\rho_i \sim 0.05 - 0.1$$

- Small scales needed for dissipation

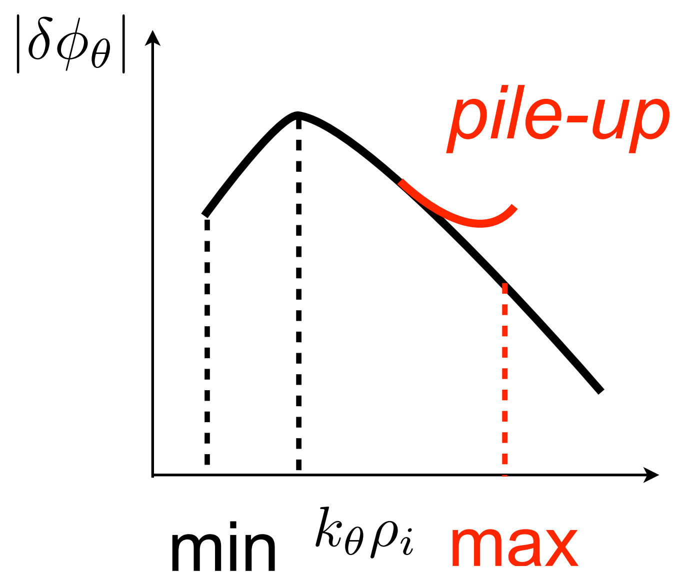
$$k_\perp^{\max}\rho_e \sim 1 \longrightarrow k_\perp^{\max}\rho_i \sim 60$$



In practice, this implies about 600x600 spectral modes!!

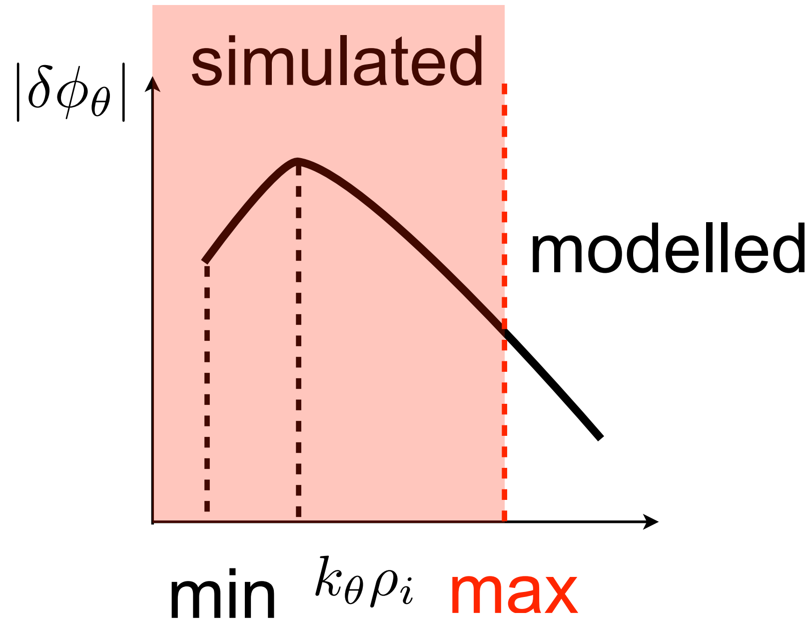
Getting rid of small scales?

- Fluxes peak at large scale, why bother with the small scales?
- Small scales suppressed → unphysical energy pile up



- Can be cured by imposing numerical dissipation
  Caveat: difficult to assess the impact on the solution...
- Collisions help to avoid the pile-up

- More refined approach: use a simpler model for the small scales

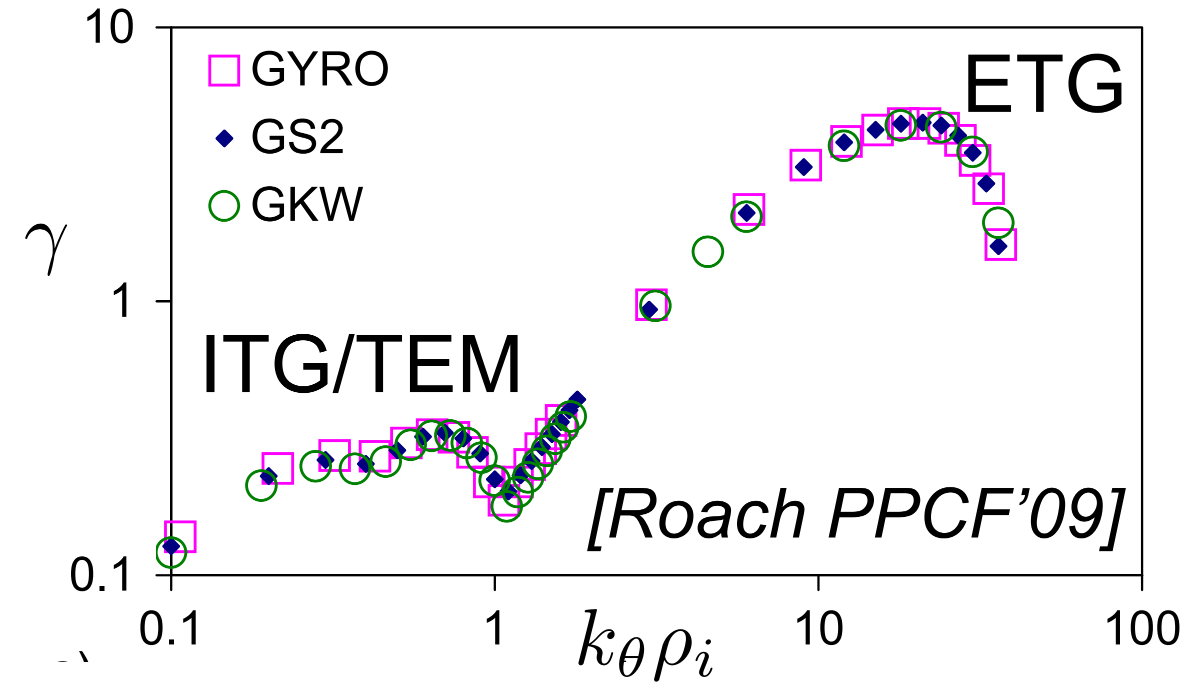


- Used in fluid turbulence: Large Eddy Simulations
- Interesting, but not so easy to in practice (the model parameters tend to depend on the turbulence type)

[see e.g. Morel PoP'12]

What is done in practice

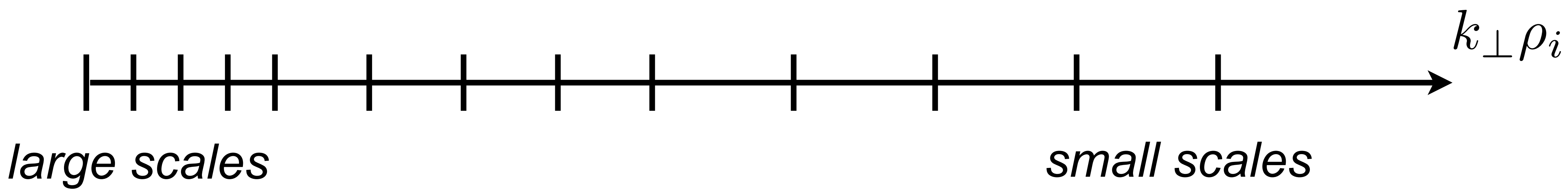
- **Large scales:** increase box size until convergence of the solution
  - Not always possible: if box size larger than plasma size, global simulations are required
- **Small scales:** cut the smallest scales in the poloidal direction (saves a factor ~20-60)
  - Not totally satisfactory: only valid if electron scales are not active



- Even like this, one arrives at about 10,000 to 100,000 CPU hours per simulation.
- Saving an extra factor 10-100 would change our life!! Any idea?

Reduced spectral resolution at small scales?

- Spectral space:



- Non-uniform Fourier transform?

Adding more physics to the model

- Two examples, still in the local and delta-f approximations
  - Collisions
  - Modification of the background:
    - Anisotropic distribution functions, example of ICRH

Collisions

- In tokamaks, collisionality is small but non-zero:
  - Collisions determine the equilibrium state
  - Collisions affect turbulence (modify particle orbits)
- Desired: a collision operator which
  - Relaxes towards a Maxwellian
  - Conserves particle, energy and momentum
- Linearised Landau collision operator meets these requirements:

$$\mathcal{C}(F_a, F_b) = \cancel{\mathcal{C}(F_{0a}, F_{0b})} + \mathcal{C}(\delta f_a, F_{0b}) + \mathcal{C}(F_{0a}, \delta f_b)$$

zero for a Maxwellian background

- Gyro-averaged version: see e.g. Brizard PoP'04
- Widely used in delta-f gyrokinetic codes, with various degree of approximation

Collisions - a recent development

- Gyrocenters are not particles, the difference gives rise to the polarisation density for instance
- Collisions apply between particles, not gyrocenters
- Consequently, the collision operator applied to a Maxwellian of gyrocenters does not vanish for fluctuating fields:

$$\mathcal{C}(F_{Ma}, F_{Ma}) = \mathcal{C}(F_{Ma}\frac{q\delta\phi}{T}, F_{Ma}) + \mathcal{C}(F_{Ma}, F_{Ma}\frac{q\delta\phi}{T})$$

vanishes only for $\delta\phi = 0$

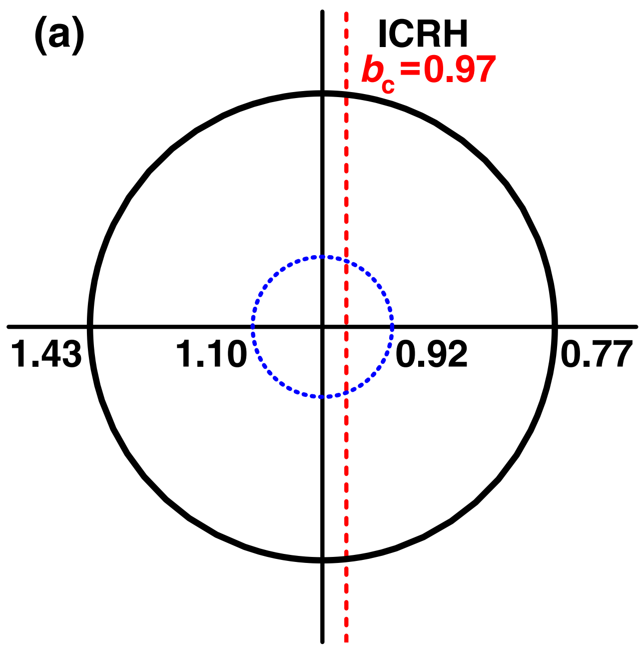
[Madsen PRE'13]

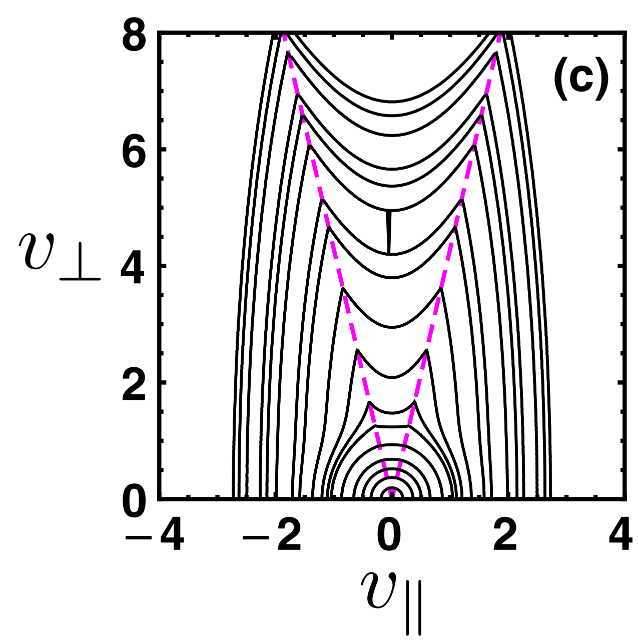
- So far, no delta-f gyrokinetic code keeps this term... (at least to my knowledge)

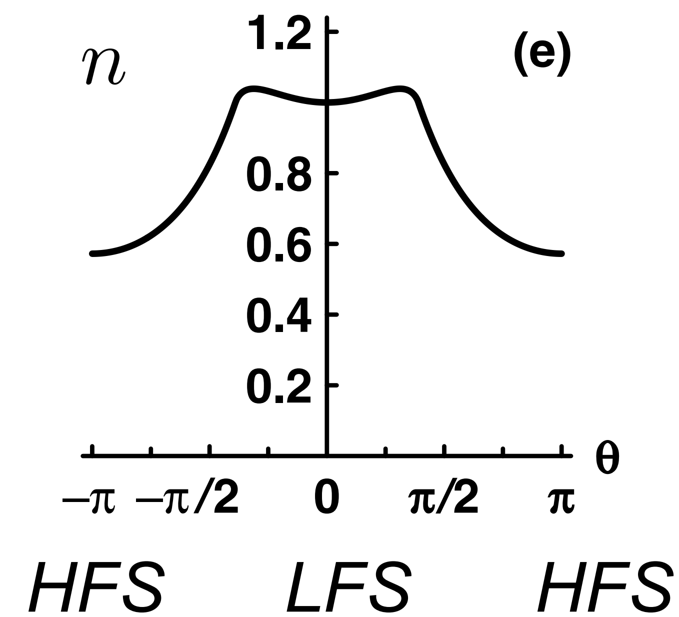
Anistropic distribution function

- A purely Maxwellian background is not always a good representation of the reality
- Example of Ion Cyclotron Resonant Heating:
  - Resonance between the gyro-motion and the wave → increase perpendicular energy $T_\perp > T_\|$

[Kazakov PPCF'12]







  - Increased trapping → poloidal density asymmetries
- Poloidal density asymmetries modify turbulent impurity transport
- Included in the modelling by considering an anisotropic $\mathbf{F_0}$

The delta-f approximation

- Equilibrium given by the stationary lowest order gyro-kinetic equation:

$$\left.\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t}\right|_0 \cdot \frac{\partial F_0}{\partial \mathbf{X}} + \left.\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\right|_0 \cdot \frac{\partial F_0}{\partial \mathbf{V}} = 0 \qquad F = F_0 + \delta f$$

- Solution is a canonical Maxwellian, constant on the unperturbed trajectories

- Then solves for the perturbed distribution function only:

$$\frac{\partial \delta f}{\partial t} + \frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} \cdot \frac{\partial \delta f}{\partial \mathbf{X}} + \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} \cdot \frac{\partial \delta f}{\partial \mathbf{V}} = \underbrace{\left.\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t}\right|_1 \cdot \frac{\partial F_0}{\partial \mathbf{X}} + \left.\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\right|_1 \cdot \frac{\partial F_0}{\partial \mathbf{V}}}_{\textit{source}}$$

- Valid provided the feedback of turbulence on the equilibrium can be discarded: gradient-driven simulations

Conclusions

- Any idea to decrease the numerical cost of NL local delta-f simulations would be very welcome!
  - It seems to me that there may be some room for improvement in the spatial discretization
- Gyrokinetic theory lies on robust foundations
  - Not obvious the extension of simplified model is as robust. Would be worth giving it a go?