Modern challenges in the gyrokinetic modeling of turbulent transport in tokamak plasmas

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Foreword

‣ What this talk is not:
  ➤ A review of gyrokinetics and tokamak turbulence

‣ What this talk is:
  ➤ Focused on the practical application of gyrokinetics to the modelling of turbulent transport
  ➤ List a few points that presently limit us (in my view)
Outline

- What is turbulent transport, why does it matter?
- Brief introduction to the gyrokinetic framework
- Standard simplifications
- Local delta-f simulations, typical results
- What would help?
The issue of transport in tokamaks

- **Desired:** hot, dense core $\rightarrow$ fusion?
- **Solution explored:** magnetic confinement in a toroidal device
- Nested magnetic flux surfaces: radial transport $\ll$ parallel transport
- Core plasma evolution governed by the transport across flux surfaces: lower transport $\rightarrow$ better confinement

**1D transport equation** (example of plasma density):

*local conservation equation (3D):*

\[
\frac{\partial n}{\partial t} + \nabla \cdot \Gamma = S
\]

*source*

*particle flux*

*flux surface averaged version (1D):*

\[
\frac{\partial <n>}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial r} [V' < \Gamma \cdot \nabla r>] = <S>
\]

*used to predict plasma profiles*
Electrostatic turbulent transport

- Turbulence: small scale perturbations, electrostatic case

**core**

**edge**

Electrostatic potential perturbation

$$\nabla \phi$$

density perturbation in phase

$$\nabla \delta \phi$$

density perturbation out of phase

$$\mathbf{v}_{E \times B} = \frac{\mathbf{b} \times \nabla \delta \phi}{B}$$

ExB drift motion

no net radial transport

net radial transport

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Electromagnetic turbulent transport

- Turbulence: small scale electromagnetic perturbations

$$v_{\parallel} \nabla \delta A_{\parallel} \times b_0$$

- Density perturbation in phase: no net radial transport
- Density perturbation out of phase: net radial transport
Gyrokinetic description of turbulent transport

- **Desired:** self-consistent evolution of density, momentum and temperature perturbations + electromagnetic perturbations

- **Caveat:** first principle approach based on Vlasov-Maxwell equations is very costly (6D) and rather inefficient

**Gyrokinetic ordering:**
- Magnetised plasma \( \rho_i \ll L \sim a \)
- Small perturbations
- Spatial scales: gyro-radius \( k_{\perp} \rho_i \sim 1 \)
- Time scales:
  - Slow compared to the gyro-period \( \omega \ll \Omega_{ci} \)
  - Keep wave-particle interactions \( \omega \sim k_{\parallel} v_{thi} \)

**Gyro-averaged equations:**
- Drop the gyrophase: 6D → 5D
- Parallel/perpendicular scale separation \( k_{\parallel} / k_{\perp} \ll 1 \)
The gyrokinetic Vlasov-Maxwell system

\[
\frac{\partial F}{\partial t} + \frac{dX}{dt} \cdot \frac{\partial F}{\partial X} + \frac{dV}{dt} \cdot \frac{\partial F}{\partial V} = 0
\]

\[\nabla^2 (\phi_0 + \delta \phi) = -\frac{1}{\epsilon_0} (\bar{\rho} + \rho_{pol})\]

\[\nabla \times (B_0 + \delta B) = \mu_0 (\bar{J} + J_{pol} + J_{mag})\]

[see e.g. Brizard RMP'07]

- This **non-linear** system yields the turbulent radial fluxes we are looking for:

\[
< \Gamma \cdot \nabla r >= \left\langle \int F \frac{b \times \nabla \delta \phi}{B} \cdot \nabla rd v \right\rangle - \left\langle \int F v_{||} \frac{b \times \nabla \delta A_{||}}{B} \cdot \nabla rd v \right\rangle
\]

Radial flux of gyrocenters
Good but too expensive...simplifying further

- Standard simplifications:
  - Frozen magnetic equilibrium $\frac{\partial B_0}{\partial t} = 0$

>200,000,000 CPU hours/sim.
Good but too expensive...simplifying further

- Standard simplifications:
  - Frozen magnetic equilibrium: \( \partial B_0 / \partial t = 0 \)
  - Frozen background (\( \delta f \) approximation): \( \partial F_0 / \partial t = 0 \) with \( F = F_0 + \delta f \)
The delta-f approximation

Equilibrium given by the stationary lowest order gyro-kinetic equation:

\[
\frac{dX}{dt} \left|_0 \right. \cdot \frac{\partial F_0}{\partial X} + \frac{dV}{dt} \left|_0 \right. \cdot \frac{\partial F_0}{\partial V} = 0 \quad F = F_0 + \delta f
\]

Solution is a canonical Maxwellian, constant on the unperturbed trajectories

Then solves for the perturbed distribution function only:

\[
\frac{\partial \delta f}{\partial t} + \frac{dX}{dt} \cdot \frac{\partial \delta f}{\partial X} + \frac{dV}{dt} \cdot \frac{\partial \delta f}{\partial V} = \frac{dX}{dt} \left|_1 \right. \cdot \frac{\partial F_0}{\partial X} + \frac{dV}{dt} \left|_1 \right. \cdot \frac{\partial F_0}{\partial V}
\]

Valid provided the feedback of turbulence on the equilibrium can be discarded: gradient-driven simulations
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  - Frozen background (\( \delta f \) approximation) \( \partial F_0 / \partial t = 0 \) with \( F = F_0 + \delta f \)
  - Local approximation \( F_0(r) = F_0(r_0) \) and \( \nabla F_0(r) = \nabla F_0(r_0) \)

> 200,000,000 CPU hours/sim.
The local approximation

- A single flux-tube is simulated rather than the full tokamak
- Profiles are linearised and evaluated at the center of the flux-tube:

\[
F_0(r) = F_0(r_0) + (r - r_0) \frac{\partial F_0}{\partial r} [r_0] \\
F_0(r) = F_0(r_0) \\
\nabla F_0(r) = \nabla F_0(r_0)
\]

- Valid provided the largest turbulent structures (~10-20 Larmor radii) do not feel the variation of the equilibrium

[Fig: Heat flux vs. \(w/\rho_i\).]
Good but too expensive...simplifying further

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  - Local approximation: \( F_0(r) = F_0(r_0) \) and \( \nabla F_0(r) = \nabla F_0(r_0) \)
  - Quasi-linear approximation
    - Cross-phase is assumed to be given by the linear response
    - Saturation amplitude is modelled

\[
< \Gamma \cdot \nabla r > = \left\langle \int F \frac{b \times \nabla \delta \phi}{B} \cdot \nabla r d v \right\rangle - \left\langle \int F v_\parallel \frac{b \times \nabla \delta A_\parallel}{B} \cdot \nabla r d v \right\rangle
\]

\[ F \delta \phi \sim A_{\text{model}} \cos \varphi_{\text{linear}} \]
Back to the NL local delta-f approximation

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- Practical choices:
  - Parallel/perp dynamics → field aligned coordinates \((r, \theta, s)\)
  - Turbulence homogeneous in the perpendicular plane (local approximation) → spectral representation

\[
\delta f(X) \rightarrow \delta \hat{f}(k_r, k_\theta, s) \\
\delta \phi(X) \rightarrow \delta \hat{\phi}(k_r, k_\theta, s) \\
A_{\parallel}(X) \rightarrow \delta \hat{A}_{\parallel}(k_r, k_\theta, s)
\]
A typical non-linear local delta-f simulation

Electrostatic potential spectra

- Electrostatic potential spectrum: \( |\delta \phi_\theta| \)
  - Log-log scale:
    - Minimum: \( \min \)
    - Maximum: \( \max \)
    - \( k\theta \rho_i \)

- Electrostatic potential spectrum: \( |\delta \phi_r| \)
  - Log-log scale:
    - Minimum: \( -\max \)
    - Maximum: \( \max \)
    - \( k_r \rho_i \)

Fluxes

- Temporal evolution:
  - Ion heat flux
  - Parallel velocity flux

- Spectra:
  - NL fluxes peak in a narrow spectral region

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Spectral grids and simulation cost

- Large grids = high computational cost
  → what physically determines the grid sizes?

- Box size > largest physical structures
  → gives the step size in spectral space

- Small scales needed for dissipation

![log-log graph]

In practice, this implies about 600x600 spectral modes!!

\[ k_{\perp}^{\min} = \frac{2\pi}{L_{\perp}} < \frac{2\pi}{L_{\text{phys}}} \]

\[ k_{\perp}^{\min} \rho_i \sim 0.05 - 0.1 \]

\[ k_{\perp}^{\max} \rho_e \sim 1 \quad \implies k_{\perp}^{\max} \rho_i \sim 60 \]
Getting rid of small scales?

- Fluxes peak at large scale, why bother with the small scales?
- Small scales suppressed → unphysical energy pile up

Can be cured by imposing numerical dissipation
Caveat: difficult to assess the impact on the solution...

- Collisions help to avoid the pile-up

More refined approach: use a simpler model for the small scales

Used in fluid turbulence: Large Eddy Simulations
Interesting, but not so easy to in practice (the model parameters tend to depend on the turbulence type)

[see e.g. Morel PoP'12]
**What is done in practice**

- **Large scales**: increase box size until convergence of the solution
  - Not always possible: if box size larger than plasma size, global simulations are required
- **Small scales**: cut the smallest scales in the poloidal direction (saves a factor ~20-60)
  - Not totally satisfactory: only valid if electron scales are not active

Even like this, one arrives at about 10,000 to 100,000 CPU hours per simulation.

Saving an extra factor 10-100 would change our life!! Any idea?

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**References**

1. Akers R J et al. [Roach PPCF’09](http://doi.org/10.1088/0741-3335/51/35/124020)
Reduced spectral resolution at small scales?

- Spectral space:

\[ k_{\perp} \rho_i \]

| large scales | small scales |

- Non-uniform Fourier transform?
Adding more physics to the model

- Two examples, still in the local and delta-f approximations
  - Collisions
  - Modification of the background:
    - Anisotropic distribution functions, example of ICRH
Collisions

- In tokamaks, collisionality is small but non-zero:
  - Collisions determine the equilibrium state
  - Collisions affect turbulence (modify particle orbits)
- Desired: a collision operator which
  - Relaxes towards a Maxwellian
  - Conserves particle, energy and momentum
- Linearised Landau collision operator meets these requirements:

\[
C(F_a, F_b) = C(F_{0a}, F_{0b}) + C(\delta f_a, F_{0b}) + C(F_{0a}, \delta f_b)
\]

zero for a Maxwellian background

- Gyro-averaged version: see e.g. Brizard PoP’04
- Widely used in delta-f gyrokinetic codes, with various degree of approximation
Collisions - a recent development

- Gyrocenters are not particles, the difference gives rise to the polarisation density for instance
- Collisions apply between particles, not gyrocenters
- Consequently, the collision operator applied to a Maxwellian of gyrocenters does not vanish for fluctuating fields:

\[
C(F_{Ma}, F_{Ma}) = C(F_{Ma} \frac{q\delta\phi}{T}, F_{Ma}) + C(F_{Ma}, F_{Ma} \frac{q\delta\phi}{T})
\]

vanishes only for \(\delta\phi = 0\)  

[\text{Madsen PRE’13}]

- So far, no delta-f gyrokinetic code keeps this term... (at least to my knowledge)
A purely Maxwellian background is not always a good representation of the reality

Example of Ion Cyclotron Resonant Heating:

- Resonance between the gyro-motion and the wave → increase perpendicular energy \( T_\perp > T_\parallel \)

- Increased trapping → poloidal density asymmetries
- Poloidal density asymmetries modify turbulent impurity transport
- Included in the modelling by considering an anisotropic \( F_0 \)

\[ b_c = 0.97 \]

[Image of a circular cross-section with ICRH and the equation for the density variation on the flux surface.]

[Kazakov PPCF’12]
The delta-f approximation

- Equilibrium given by the stationary lowest order gyro-kinetic equation:

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Conclusions

‣ Any idea to decrease the numerical cost of NL local delta-f simulations would be very welcome!
  ‣ It seems to me that there may be some room for improvement in the spatial discretization

‣ Gyrokinetic theory lies on robust foundations
  ‣ Not obvious the extension of simplified model is as robust. Would be worth giving it a go?