ESF Exploratory Workshop on MULTI-SCALE METHODS FOR WAVE AND TRANSPORT PROCESSES IN FUSION PLASMAS: THE LEGACY OF GRIGORY PEREVERZEV

Introduction to Transport in Fusion Devices and Transport Theory Agenda

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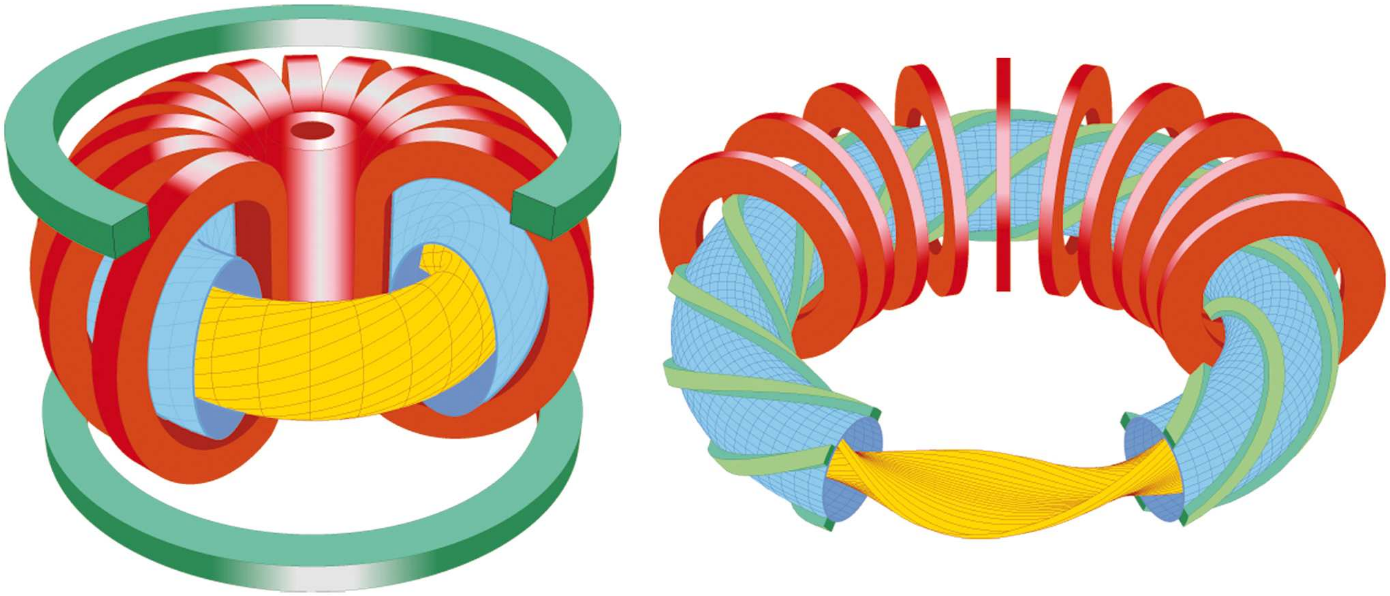
The problem of transport in fusion plasmas

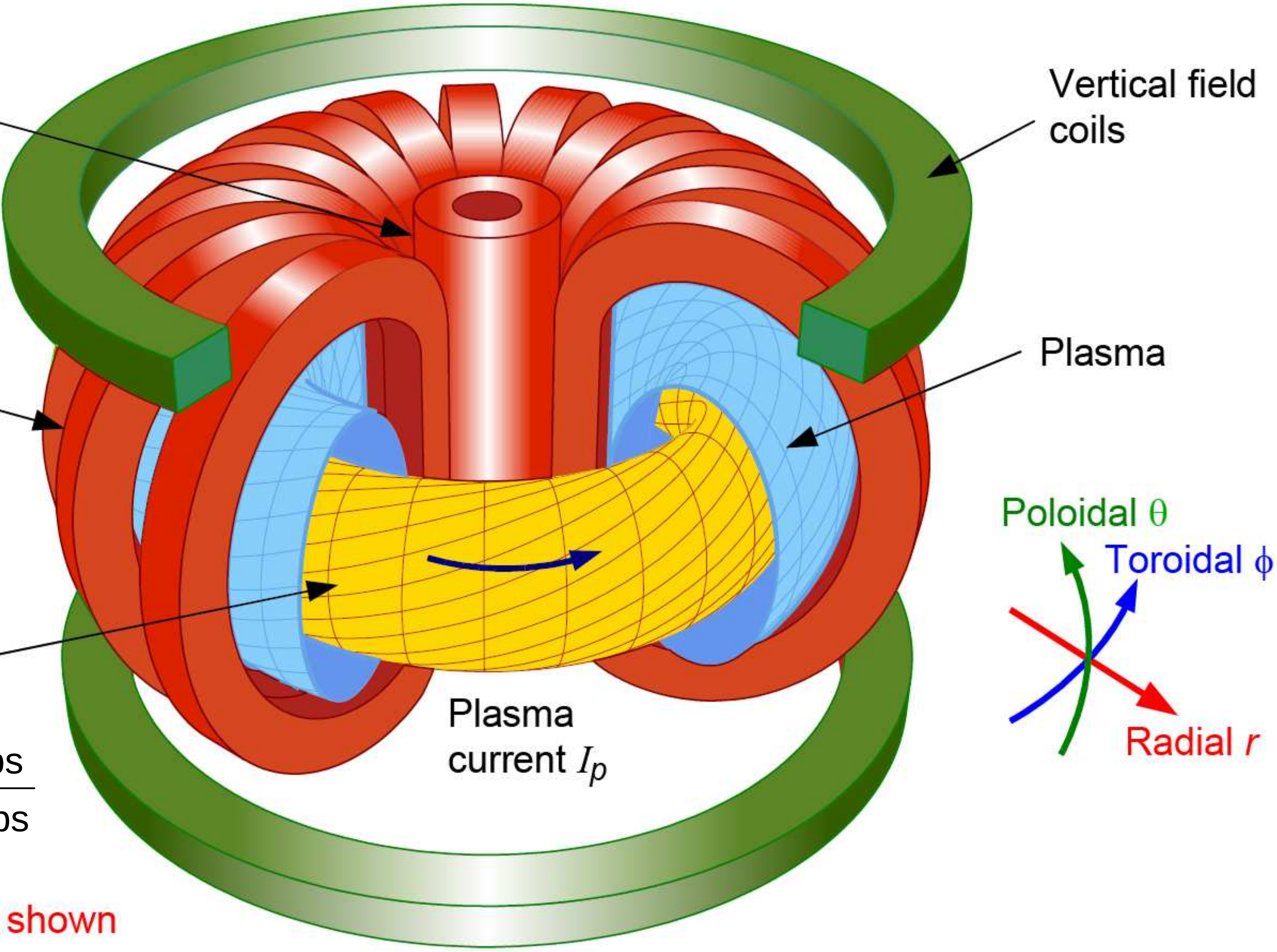
- Due to the different behaviour of charged particles along and across a magnetic field, transport in fusion plasmas is strongly anisotropic. It is very large in the direction parallel to the magnetic field, and much slower in the direction perpendicular to the magnetic field [talk by C. Negulescu]
- Parallel transport is large enough to strongly limit the variation of density and pressure along the field lines (on the flux surfaces)
- How large is the perpendicular (radial) transport is what determines the confinement properties of the plasma
- Collisions provide an unavoidable mechanism of transport (classical transport), which is enhanced by the toroidal geometry of the plasma (neoclassical transport)
- Experimentally, it is observed that while parallel transport is largely consistent with collisional transport theory, perpendicular transport is significantly larger than the predicted levels (up to two orders of magnitude for the electron heat transport)
- Turbulence at micro- (Larmor radius) scales is responsible for the larger amount of radial transport in most conditions

Toroidal geometry

- **The plasma is confined by a system of nested magnetic flux surfaces**
- **Toroidal geometry is chosen to remove open ends (like in magnetic bottle)**
- **However, toroidal magnetic field alone is not sufficient to confine the plasma, due to the charge dependent drifts generated by magnetic field gradient and curvature**
- **Helical magnetic field lines allow the compensation of the drifts (additional poloidal component)**
- **Most successful configurations are the Tokamak and the Stellarator**



The tokamak, toroidal, poloidal and radial directions



Central solenoid

Loop Voltage

$E_{||}$

Toroidal field coils

Helical magnetic field lines B

$$q = \frac{\text{nb toroidal loops}}{\text{nb poloidal loops}}$$

Vacuum vessel not shown

Vertical field coils

Plasma

Plasma current I_p

Poloidal θ

Toroidal ϕ

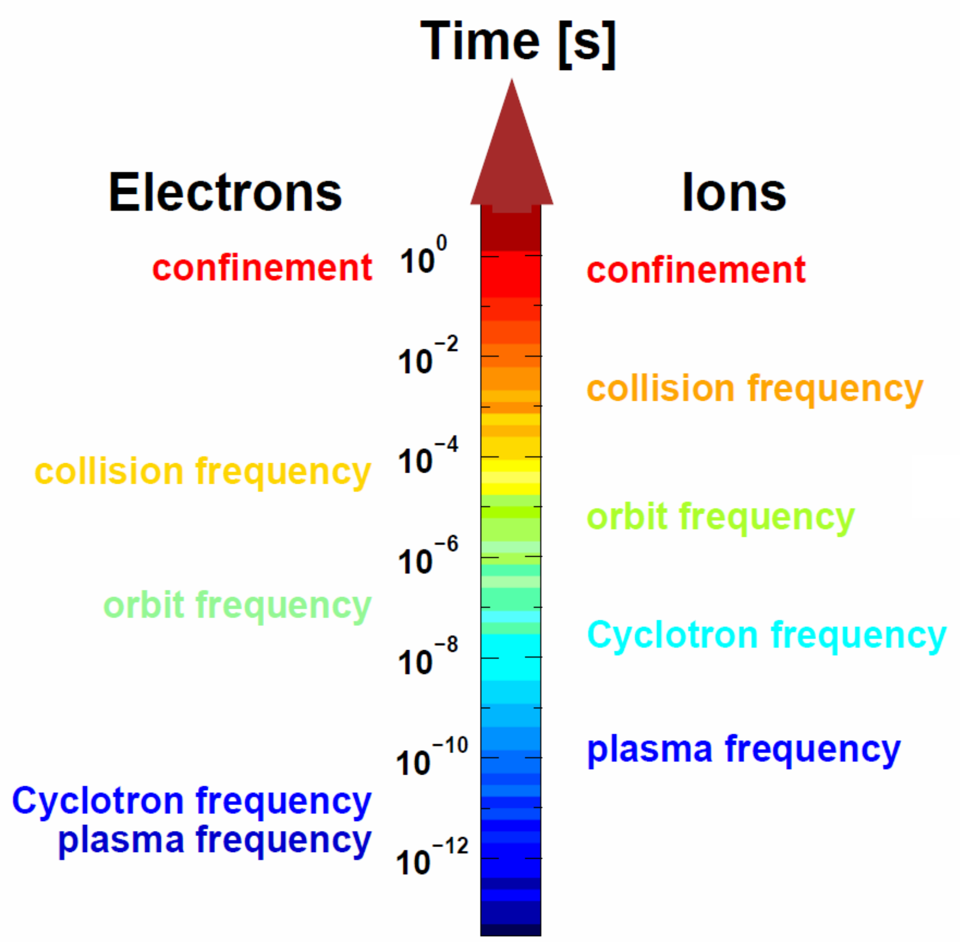
Radial r

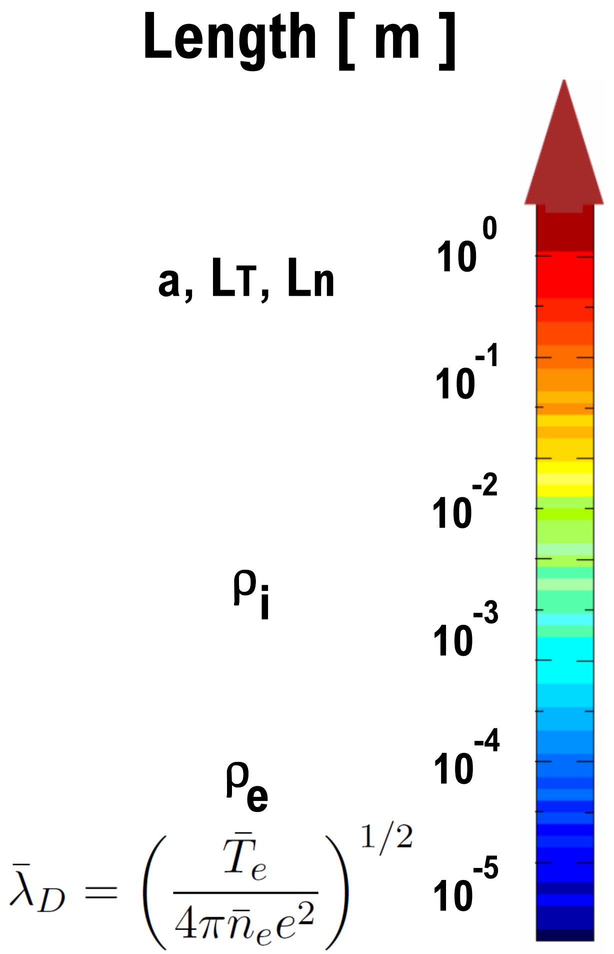
Goals and challenges of transport theory and modelling

- **Main goal of theory-based transport modelling is to predict plasma kinetic profiles (density, temperatures, rotation)**
- **For given conditions at a single time slice or over the entire evolution of a plasma discharge (up to 100 s in some devices) (leads to so-called integrated scenario modelling, talk by I. Voitsekhovitch)**
- **Plasma self- (OH and fusion reactions) and external heating sources determine the heat fluxes (similarly particle sources and torques are also present)**
- **Plasma temperature profiles adjust in order to transport out the heat (similarly density and angular velocity profiles …)**
- **The modelling of such a system is confronted with the difference in the time and spatial scales which are relevant at the microscopic level (determining the micro-turbulence) and at the macroscopic level (determined by the global confinement and the time evolution of the plasma discharge)**
- **Example of challenges provided by multi-scale physics and the consequent development of different regimes in M.-H. Vignal talk**

Time and spatial scales at play

- **There are 7 to 8 order of magnitudes in time scales and 5 orders of magnitude in space scales**





$$\bar{\lambda}_D = \left(\frac{\bar{T}_e}{4\pi \bar{n}_e e^2} \right)^{1/2}$$

Transport modelling approaches in the magnetic fusion community

- Two main (complementary) approaches can be identified in the fusion community, with present attempts in trying to merge them

- The first one is provided by the conventional transport modelling approach, through the solution of a system of time dependent diffusion and heat conduction equations, aims at modelling the entire plasma discharge

- This is performed by transport codes (one of the most popular ASTRA was developed by G. V. Pereverzev) [talks by I. Voitsekhovitch and E. Fable]

- Within a theory-based modelling approach, transport coefficients are provided by theory-based models, which are usually quasi-linear fluid or kinetic models which describe plasma micro-turbulence

- The second approach is provided by the direct description of the turbulence in a local portion of the plasma, or over extended radial domains, by non-linear fluid or kinetic models

- Kinetic description particularly important to describe wave-particle interactions in velocity space [talk by T. Fülop]

- Gyrokinetic models and codes are developed and applied to this purpose [talks by N. Tronko, Y. Camenen, S. Leerink]

Transport codes and models, rationale

- **A transport code evolves densities, temperatures, toroidal momentum of the plasma species (sort of mean-field approach) through a usually coupled system of diffusion and heat conduction equations (ex. from ASTRA manual, G. Pereverzev)**

$$\begin{cases} \dfrac{1}{V'}\left(\dfrac{\partial}{\partial t} - \dfrac{\dot{B}_0}{2B_0}\dfrac{\partial}{\partial \rho}\rho\right)(V' n_e) + \dfrac{1}{V'}\dfrac{\partial}{\partial \rho}\Gamma_e = S_e, \\[2ex] \dfrac{3}{2}(V')^{-5/3}\left(\dfrac{\partial}{\partial t} - \dfrac{\dot{B}_0}{2B_0}\dfrac{\partial}{\partial \rho}\rho\right)\left[(V')^{5/3} n_e T_e\right] + \dfrac{1}{V'}\dfrac{\partial}{\partial \rho}\left(q_e + \dfrac{5}{2}T_e\Gamma_e\right) = P_e, \\[2ex] \dfrac{3}{2}(V')^{-5/3}\left(\dfrac{\partial}{\partial t} - \dfrac{\dot{B}_0}{2B_0}\dfrac{\partial}{\partial \rho}\rho\right)\left[(V')^{5/3} n_i T_i\right] + \dfrac{1}{V'}\dfrac{\partial}{\partial \rho}\left(q_i + \dfrac{5}{2}T_i\Gamma_i\right) = P_i, \\[2ex] \sigma_{\parallel}\left(\dfrac{\partial \psi}{\partial t} - \dfrac{\rho \dot{B}_0}{2B_0}\dfrac{\partial \psi}{\partial \rho}\right) = \dfrac{J^2 R_0}{\mu_0 \rho}\dfrac{\partial}{\partial \rho}\left(\dfrac{G_2}{J}\dfrac{\partial \psi}{\partial \rho}\right) - \dfrac{V'}{2\pi\rho}(j_{BS} + j_{CD}) . \end{cases}$$

- **Within a theory-based modelling approach, transport coefficients are provided by theory-based models, which are usually quasi-linear fluid or kinetic models**

Transport codes and models, challenges

- One of the main difficulties faced here are provided by the **strong non-linearity of the flux vs gradient relationship** provided by turbulent transport [talk by E. Fable]

- A **non-diagonal transport matrix** (presence of strong convective components in some transport channels) (example of transport matrix implemented in ASTRA)

$$\begin{pmatrix} \dfrac{\Gamma_e}{n_e} \\ \dfrac{q_e}{n_e T_e} \\ \dfrac{q_i}{n_i T_i} \\ V'G_1 \dfrac{\mu_0 j_{BS}}{B_p} \end{pmatrix} = -V'G_1 \begin{pmatrix} D_n & D_e & D_i & D_E \\ \chi_n^e & \chi_e & \chi_i^e & \chi_E^e \\ \chi_n^i & \chi_e^i & \chi_i & \chi_E^i \\ C_n & C_e & C_i & 0 \end{pmatrix} \cdot \begin{pmatrix} \dfrac{1}{n_e}\dfrac{\partial n_e}{\partial \rho} \\ \dfrac{1}{T_e}\dfrac{\partial T_e}{\partial \rho} \\ \dfrac{1}{T_i}\dfrac{\partial T_i}{\partial \rho} \\ \dfrac{E_\parallel}{B_p} \end{pmatrix}$$

- Additionally, difference in parallel and perpendicular transport allows a **1D treatment** of the transport code **in the core**, but **at the edge a 2D treatment** is required (connection between edge and core) [talk by C. Negulescu]

Theory-based transport models for the description of turbulent transport

- **Transport coefficients** are computed by external modules which solve **linearised (gyro-)fluid or (gyro-)kinetic equations**. The module is called by the transport code, takes as inputs all the plasma and geometry profiles at a given time step, and gives as outputs the transport coefficients to be used in the next time step

- Linearisation implies that the impact of the fluctuating electromagnetic potentials on the dynamics of the perturbed distribution function is not retained

- An expansion in Fourier modes, combined with some assumptions on the functional shape of the mode along the field line, leads to reduce the problem to an **algebraic homogeneous system**, where modes with different toroidal mode numbers are not coupled (in contrast to the nonlinear system).

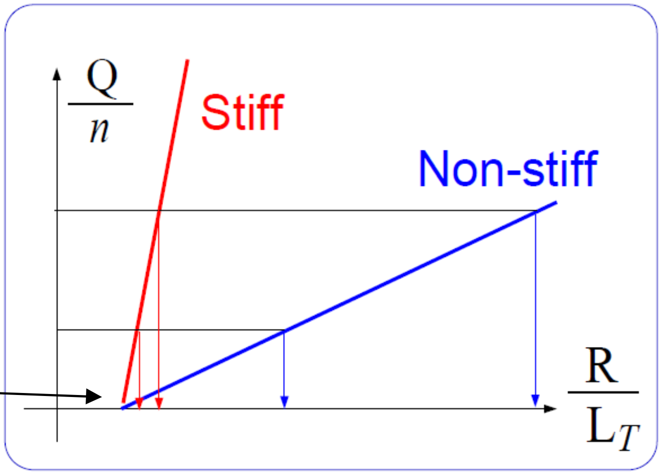
- Unknowns of the algebraic system are the complex amplitudes of density, temperature, momentum and e.m. potentials fluctuations, eigenvalues provide **growth rates and propagation frequencies** of the linear modes, eigenvectors allow the **density, temperature and momentum fluctuations to be expressed in terms of the e.m. potentials**

- **Quasi-linear transport** is provided by the product of the perturbed density, temperature and momentum fluctuations times the fluctuating e.m. potentials, and is **proportional to the amplitude squared** of the e.m. potential fluctuations

General properties of turbulent transport, threshold, stiffness, multi-channel, off-diagonal terms

- The computation of the quasi-linear turbulent transport requires a (mixing length) rule for the saturation amplitude of the fluctuating e.m. potentials (usually determined by matching one or a set of NL simulations)

- Turbulent transport involves thresholds in the driving (logarithmic) gradients

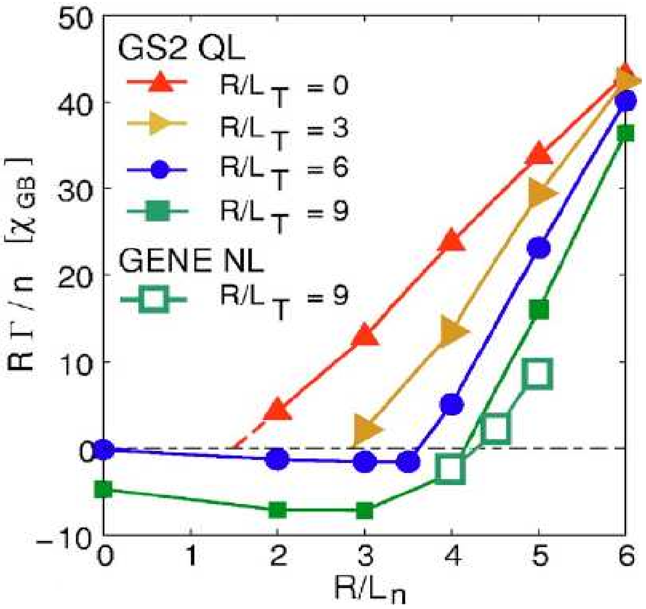


- Fluxes vary by large amounts with small changes in the driving gradients (profile stiffness)

[talk by E. Fable]

- Transport is produced on multiple transport channels (particle, momentum, heat)

- Flux versus gradient dependence (or flux as function of other plasma parameters) can be highly nonlinear, and involve convective components (off-diagonal terms)



Gyrokinetic models and codes

- **Gyrokinetic models** are derived from the complete kinetic model by considering that particle gyro-motion occurs on time scales which are faster than those which are relevant for turbulence and transport [talk by N. Tronko]

- **Phase space becomes 5D**, 3 spatial coordinates and 2 velocity coordinates, gyrating particles are transformed in rings, actual knowledge of position of particle on the ring is lost, but ring size (Larmor radius) is retained

- **Multiple approaches**, local or global, gradient driven or flux driven, δf or full f, [talk by Y. Camenen]

- **Local**, gradient driven approach, computes turbulent perturbation over a static background, computes fluxes for given input gradients
- **Global**, profile driven approach, computes turbulent perturbation over static background, computes fluxes over extended radial domains for given profiles in input
- **Global, full f**, flux driven approach, computes plasma profiles from given fluxes, without making any separation between equilibrium and fluctuating quantities (in principle should be able to replace the transport modelling approach, but 4 to 5 orders of magnitude computationally more expensive)

Combination of conventional transport modelling with nonlinear local transport codes

- More recent developments consider the coupling of local non-linear gyrokinetic codes within a master code (a transport code) which evolves plasma profiles over the transport time scale

- Local gyrokinetic codes are run over a reduced radial grid of the transport code (usually 8 or 10 points), and compute the fluxes which are then used by the transport code to evolve the profiles. The new profiles are then utilized to produce the inputs of the local non-linear simulations

- Reduced computational time with respect of a full global simulation (one order of magnitude), but does not include real global effects

- Is this approach really justified with respect to 4 orders of magnitude faster transport modelling with quasi-linear transport models, or more effort should be put in properly capturing NL effects in QL models on one side, and in actual global treatments on the other side ?

Transport Modelling Sessions Agenda

- **Monday: 2 Sessions**
  - Numerical methods to describe anistropic and multi-scale nature of transport in plasmas (**Claudia Negulescu** and **Marie-Hélène Vignal**)
  - Transport codes for the description of tokamak plasmas, scope, applications and schemes for stiff transport (**Irina Voitsekhovitch** and **Emiliano Fable**)
- **Tuesday: 1 Session**
  - Gyrokinetic models for the description of turbulent transport, general properties, hierarchy of models and present challenges (**Natalia Tronko** and **Yann Camenen**)
- **Wednesday: 1 Session**
  - Applications of kinetic models, wave-particle interactions, treatment of multi-scale physics in global gyrokinetics (**Tünde Fülöp** and **Susan Leerink**)

Discussion Session, Transport Modelling I & II

- Challenges for transport codes, diffusion and heat conduction equations with theory-based transport models
  - Transport coefficients are strongly nonlinear functions of log gradients (threshold and stiffness) and depend nonlinearly on many plasma parameters
  - Fluxes include diffusive and convective components in nonlinear flux versus gradient functional dependences, how to properly treat these terms
- Direct use of turbulent fluxes in transport codes: Is it of interest (more also in next days sessions) and how to avoid too short time steps ?
- Connecting edge (2D, both parallel and perpendicular transport) with core (1D, only radial transport, parallel transport infinitely fast) ?
- There is the need of schemes which allow the stable transition from one domain to the other, without the requirement of introducing an artificial core-edge boundary where the grid is changed from 2D to 1D
- Additional points …