Practical course
at the Technische Universität München

Max Planck Institute for Plasma Physics

Plasmainterferometry

The experimental setup is located in building L5, room 328 in the MPI for Plasma Physics, Boltzmannstr. 2, 85748 Garching bei München

Responsible Officer:	Dr. R. Dux,	Phone: 3299 1256
Organization:	G. Dörsch,	Phone: 3299 1816
Technical support:	M. Huber,	Phone: 3299 1656

Meeting point: Building L6, 1st floor
Room 118
In front of Mrs. Dörsch's office

1 Introduction

To characterize a plasma discharge, plasma density and temperature measurements are the essential prerequisites. Probes (Langmuir probes) are often used to measure these plasma parameters in relatively cold plasmas, but the probe interferes with the plasma and changes the local plasma condition. Many other measurement techniques have also been developed to measure plasma temperature and density without disturbing the local plasma conditions. These are, Thomson scattering (light scattering from free electrons in the plasma), especially for local plasma density and temperature measurements, spectroscopic methods (Stark effect, Faraday effect etc.) and interferometry techniques for density measurements. In this practical experiment the method of interferometry will be introduced and discussed.

Interferometric methods generally use the dependence of the (complex) refractive index on the density of a transparent medium. This method is suitable especially for temporal changes in density in a plasma discharge. The disadvantage of this optical technique is that it determines the line integrated value along the line of sight. In practice, these procedures were rarely used before the availability of lasers, because the light sources available at that time had very short coherence lengths and therefore the adjustment of such an interferometer was very tiresome. Today this method is routinely used in all bigger plasmas related to fusion research and mostly in well known arrangements like Michelson, Mach-Zehnder or (sometimes) Fabry-Perot interferometers.

Since at a given plasma density the interferometric effect increases proportionally to the wavelength of the light, infrared lasers are used in preference. Some frequently used lasers (with typical output powers) are:

HCN	337 μm	approx. 150 mW
DCN	190 μm und 195 μm	approx. 250 mW
H_2O	119 μm	approx. 60 mW
CO_2	10.6 μm	up to kW in CW mode
HeNe	3.39 μm and 0.6328 μm	approx. 1 to 10 mW

To determine the plasma density an interesting and special interferometric procedure was developed by Ashby and Jephcott (D.E.T.F. Ashby, D.F. Jephcott, Appl. Phys. Lett. 3, 13 (1963), can be obtained from Mrs. Dörsch). Here, the plasma discharge to be examined is part of a Fabry-Perot interferometer, which is a component of a laser resonator. Hence, the tuning of the resonator is modulated by the change in optical wavelengths according to the variation of the plasma density. The procedure did not prevail in practice due to its high sensitivity to noise, however because of the attractive relation between laser and plasma physics this technique is used in this experiment for the density measurement. A He-Ne laser is used, the output is a complementary modulation (i.e. produced by electron transitions from the same exited state) at 3.39 μm and 0.6328 μm. The modulation in the infrared wavelength can be measured by detecting the modulation in visible wavelength.

Contents

2 Background

2.1 Hints for Preparation

The following basic aspects of the experiment should definitely be known by all students. It is impossible to accomplish this course successfully if you cannot answer to the following questions! Therefore you need to carefully take into account the following instructions.

1. Basic Theory

   - What is a plasma? Where do plasma form?
   - Maxwell's equations
   - How are the Debye length and the plasma frequency defined? Which dependencies they have and how can you derive them?
   - What is a dispersion relation and what is the expression for electromagnetic waves in plasma? Sketch
   - The wave equation and the plane wave description
   - How is the refractive index defined and what does it mean? What is the Brewster angle?
   - What is the interference and how do you calculate the path difference? What is a constructive or destructive interference? What is a Michelson, a Mach-Zehnder and a Fabry-Perot interferometer?
   - How do lasers work? What is the pump or the gas laser? What is the population inversion, stimulated emission?

2. Experimental Setup

   - What is the experimental setup? Where are the cavity mirrors? Where are the plasma discharge tube, the He-Ne-discharge tube, and the detector-photodiode placed relative to each other?
   - Which laser lines are used in the experiment? Which initial levels do they have in fig. 8?
   - What does a Germanium disc filter do? Where is it in the setup?
   - Which laser line passes the plasma? Which is detected by the photodiode?
   - How do we change the discharge current?
   - Which measurements should be realized in this course?
   - Which simplifications can be made in the evaluation?

2.2 Basics

2.2.1 Debye length and plasma frequency

The characteristic oscillation of electromagnetic radiation is modified if it passes through a medium. Gaseous plasmas are dominated by Coulomb collisions between charged particles like electrons and ions. The density distribution of the electrons, n, can be written for a potential Φ with the help of Boltzmann statistics as,

$$n(\Phi) = n \exp(e\Phi/k_B T) \tag{1}$$

where e is the electronic charge, k_B is the Boltzmann constant and T is the electron temperature of the plasma.

In hot plasmas, $e\Phi \ll k_B T$, and equation (1) can be written in a linear form as,

$$n(\Phi) \sim n(1 + \frac{e\Phi}{k_B T}) \tag{2}$$

For a hydrogen plasma, we consider the potential around a point charge at the origin of the coordinate system. The Poisson equation can be used to find the potential Φ assuming spherical symmetry.

$$\Delta\Phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) = -\rho/\epsilon_0 = -\frac{q\delta(r)}{\epsilon_0} + e\frac{n_e - n_i}{\epsilon_0} \tag{3}$$

In case of point charge in vacuum, the right hand side of equation (3) reduces to a δ-function at $r = 0$, so that for $r \neq 0$ we are left with $\Delta\Phi$=0. In this case, we obtain the Coulomb potential which decays radially with $1/r$. For the second term, the density distribution as a function of the potential energy $q\Phi$ is given by the linear approximation of equation (2).

$$n_e - n_i \approx n_{e,0}(1 + \frac{e\Phi}{k_B T} - (1 - \frac{e\Phi}{k_B T})) = 2n_{e,0}\frac{e\Phi}{k_B T} \tag{4}$$

Here, we assumed the quasi neutrality condition, i.e. $n_{e,0} = n_{i,0}$ for large distances to the point charge.

With these assumptions the Poisson equation becomes for $r \neq 0$

$$\Phi(r) = \frac{q}{4\pi\epsilon_0 r} e^{-\frac{\sqrt{2}r}{\lambda_D}} \tag{5}$$

where the Debey length λ_D is:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_{e,0} e^2}} \tag{6}$$

For $r << \lambda_D$, the exponential term can be approximated by ~ 1 and we have the usual Coulomb potential. On the contrary, for $r >> \lambda_D$, the potential vanishes exponentially, i.e. faster than $1/r$.

At the same time, electrons will oscillate around an inhomogeneous charge distribution with the so-called plasma frequency ω_P.

$$\omega_P = v_e/\lambda_D = \frac{\sqrt{k_B T/m_e}}{\lambda_D} = \sqrt{e^2 n/\epsilon_0 m_e} \tag{7}$$

where v_e is the thermal velocity, and m_e is the mass of an electron.

Time dependent potentials, e.g. an electromagnetic wave, can only be shielded by electrons if the frequency of the potential perturbation due to that electromagnetic wave is below the plasma frequency. Otherwise, the motion of the electrons is too slow to cause an effective shielding. Because of their higher mass, ions are practically not involved in shielding and can even be regarded as stationary in most cases.

This means in other words that only electromagnetic waves with frequencies above the plasma frequency can propagate through the plasma. Waves with lower frequencies are reflected by the plasma. They penetrate only a short distance into the plasma, which is on the order of the Debye length. Above the plasma frequency the influence of the plasma electrons on the potential of the wave decreases with increasing frequency. The higher the frequency of the electromagnetic wave, the smaller the deflection amplitude of the plasma electrons in its potential. If the influence of the plasma electrons on the potential of the wave shall be exploited in order to to measure the electron density in the plasma, it is advantageous to use a frequency as low as possible but "'sufficiently"' above the plasma frequency. In this instruction, for simplicity we assume that the electron density is the same as the plasma density, i.e. assuming singly ionized atoms.

2.2.2 Waves in the plasma

We now consider an electromagnetic wave in a plasma and start from the following Maxwell's equations.

$$\nabla \times \vec{E} = -\dot{\vec{B}} \quad | \quad \nabla \times \tag{8}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \tag{9}$$

where $\partial \vec{E}/\partial t$ denotes the displacement current of the wave and $\vec{j}$ is the current of the conducting electrons. When taking the curl of eq. (8) and using the time derivative of eq. (9), we arrive at the general form of the wave equation:

$$\nabla \times \nabla \times \vec{E} + \mu_0 \dot{\vec{j}} + \epsilon_0 \mu_0 \ddot{\vec{E}} = 0 \tag{10}$$

With the vector identity $\nabla \times \nabla \times \vec{E} \equiv \nabla(\nabla \cdot \vec{E}) - \Delta \vec{E}$ we get:

$$\Delta \vec{E} - \frac{1}{\epsilon_0} \nabla \rho - \mu_0 \dot{\vec{j}} = \frac{1}{c^2} \ddot{\vec{E}} \tag{11}$$

The term $-\nabla\rho/\epsilon_0$ indicates the change of the spatial charge distribution in the direction of wave propagation. For a transversely polarized wave (i.e. $\vec{E} \perp \vec{k}$), this term disappears. Lets consider a plane transversely polarized wave in $\vec{k}$ direction, $\vec{E} = \vec{E}_0 \exp(i\vec{k}\vec{r} - i\omega t)$. The same plane wave description is used for $\vec{j}$. With the wave number k and angular frequency ω, the local and time derivatives become:

$$\partial/\partial t \rightarrow -i\omega \tag{12}$$

$$\partial^2/\partial t^2 \rightarrow -\omega^2 \tag{13}$$

$$\Delta \rightarrow -k^2 \tag{14}$$

For transversally polarized waves with $\vec{E} \perp \vec{k}$, the wave equation is

$$-k^2\vec{E} + i\omega\mu_0\vec{j} = -\frac{\omega^2}{c^2}\vec{E} \tag{15}$$

We now use the generalized Ohm's Law (see lecture notes: Experimentelle Plasmaphysik, Prof. Günther), which is a result from the equation of motion for the electron fluid.

$$\vec{E} + \vec{u} \times \vec{B} = \eta\vec{j} + \frac{1}{en}(\vec{j} \times \vec{B}) - \frac{1}{en}\nabla p + \frac{m_e}{e^2 n}\frac{d\vec{j}}{dt} \tag{16}$$

Here $\vec{u}$ is the velocity of the plasma perpendicular to the magnetic field $\vec{B}$, η is the plasma resistivity due to collisions and ∇p is the gradient of the plasma pressure. The term proportional to the pressure gradient is only relevant for longitudinal waves ($\vec{E}\|\vec{k}$). In the simplest case, without any external magnetic field, Ohm's law reduces to

$$\vec{E} = \eta\vec{j} + \frac{m_e}{e^2 n}\frac{d\vec{j}}{dt} \tag{17}$$

Furthermore, the collisions shall be neglected ($\eta \rightarrow 0$), such that we are only left with the inertia of the electrons contributing to the resistivity of the plasma. We get:

$$\vec{j} = \frac{e^2 n}{m_e}\frac{i}{\omega}\vec{E} \tag{18}$$

The coefficient in front of $\vec{E}$ is now purely imaginary and indicates the high frequency resistivity. Alternatively, we could have derived it simply from the equation of motion for an electron in the electric field of the wave: $m_e\dot{\vec{u}}_e = -e\vec{E}$, while taking the ions to be stationary due to their much higher inertia. Replacing the current density in eq. (15) by eq. (18), we see that the wave equation only has a solution with non-vanishing electric field of the wave if

$$k^2 + \frac{\mu_0 e^2 n}{m_e} = \frac{\omega^2}{c^2} \tag{19}$$

With $c^2 = 1/(\mu_0\epsilon_0)$ and $\omega_p = e^2 n_e/(\epsilon_0 m_e)$, we receive the dispersion relation for a collision free plasma without magnetic field.

$$c^2k^2 + \omega_p^2 = \omega^2 \tag{20}$$

For $\omega_p \to 0$, we obtain the usual relationship for light waves in vacuum.

The phase velocity v_{ph} of the wave is given by $v_{ph} = \omega/k$, and the refractive index N of a medium is defined by the ratio $N = c/v_{ph}$, with c being the velocity of light in vacuum. Then the complex index N can be written as

$$N = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \tag{21}$$

The refractive index N in a plasma is always less than 1 and for sufficiently high frequencies a real number. The phase velocity is always above the spped of light, while the group velocity $v_{\text{gr}} = \text{d}\omega/\text{d}k$ is always smaller than the speed of light.

The refractive index decreases with increasing density and vanishes at the cutoff density n_{crit}. The phase velocity of the wave becomes infinity and the group velocity v_{gr} becomes zero. For $n > n_{\text{crit}}$, N is purely imaginary ($N^2 < 0$) and the electric field falls off exponentially. The displacement current due to the light wave is compensated by the electron current and the wave incident on the plasma is reflected. For $\omega \gg \omega_p$, relation (21) shows that the plasma has little influence on the propagation of the wave.

At the cutoff frequency, we have.

$$\omega = \left(\frac{e^2 n_{\text{crit}}}{\epsilon_0 m_e}\right)^{1/2} \tag{22}$$

and the cutoff density n_c is

$$n_c = \frac{4\pi^2 c^2}{\lambda^2}\frac{\epsilon_0 m_e}{e^2} = 1.1148 \cdot 10^{15}\text{m}^{-3}\,\lambda^{-2} \tag{23}$$

The refractive index in terms of the cutoff density can be written as

$$N = \left(1 - \frac{n}{n_c}\right)^{1/2} \tag{24}$$

In practice, the laser frequency usually is sufficiently large against the plasma frequency and equation (24) can be used in linearized form.

$$N \simeq 1 - \frac{n}{2n_c} = 1 - \frac{\omega_p^2}{2\omega^2} \tag{25}$$

2.2.3 Interferometry

Interferometric Technique The method of interferometry makes use of the modulation of an electromagnetic wave due to the phase difference of two or several superimposed waves in order to measure wavelengths or characteristic properties of a medium through which the rays pass. For example in a Mach-Zehnder-Interferometer (see Fig. 1), the phase difference between a reference ray and the probe ray (passing through the plasma) is measured. This phase difference φ is created by the plasma due to the path length $(z_1 - z_2)$ of the probe ray inside the plasma: then the path difference is given by:

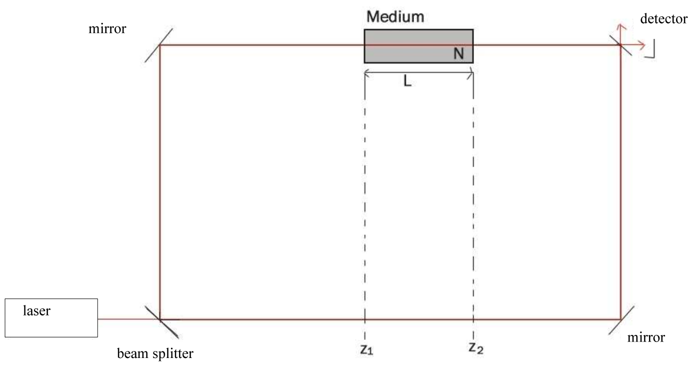

Figure 1: *Beam path in the Mach-Zehnder interferometer.*

$$\Delta s = \int_{z_1}^{z_2} N_{\text{vac}} \, \mathrm{d}z - \int_{z_1}^{z_2} N(z) \, \mathrm{d}z \tag{26}$$

With $\varphi/(2\pi) = \Delta s/\lambda$ we get:

$$\varphi = \frac{2\pi}{\lambda} \int_{z_1}^{z_2} \left(N_{\text{vac}} - N(z)\right) dz = \frac{\pi}{\lambda n_c} \int_{z_1}^{z_2} n(z) \, dz \tag{27}$$

where in the second part of the equation $N_{vac} = 1$ is an approximation for the relevant refractive index for the reference ray, which is passing through air. This gives,

$$\varphi = \frac{\lambda e^2}{4\pi c^2 \epsilon_0 m_e} \int_{z_1}^{z_2} n(z) \, dz = 2.82 \cdot 10^{-15} \, \lambda \int_{z_1}^{z_2} n(z) \, dz \tag{28}$$

This means the phase change of the interferometric signal is proportional to the product of wavelength of the probe ray and the line integrated density along the ray path. Dividing the phase change φ by 2π gives the number of the fringes N_f

$$N_f = \frac{\varphi}{2\pi} = 4.49 \cdot 10^{-16}\,\lambda \int_{z_1}^{z_2} n(z)\,dz \tag{29}$$

To record the data, detectors whose signal level is proportional to the electric field strength of the light wave (square-law detectors) are used in general. Since the detector cannot resolve the frequencies ω_1, ω_2 of the light wave, a constant signal with a temporal change in amplitude modulation due to the phase difference is obtained (E_1, E_2 are the electric field strengths of the two rays).

$$\bar{E}^2 = \frac{1}{2}E_1^2 + \frac{1}{2}E_2^2 + E_1 E_2 \cos\varphi \tag{30}$$

In this interferometric technique, it is not simple in practice to distinguish the variations of amplitudes due to other causes (vibrations, absorption, diffraction), especially when the involved frequencies are similar. With this kind of power measurement it is impossible to distinguish between increasing and decreasing densities in a discharge because the directions of the phase shift are not recorded.

In plasma physics, common setups are frequently used for the interferometric density measurement: Mach-Zehnder interferometers are preferably used if there is enough space for the reference ray to be guided around the plasma apparatus. If there is not enough space, a Michelson type setup is used, where the probe ray is reflected back from a mirror, thus passing the plasma twice.

In this practical experiment, a type of Fabry-Perot interferometer is used, where interference is achieved by superimposing the probe ray with itself in between two closely spaced and highly reflecting (sometimes plane-parallel) glass or quartz plates having reflection coefficient R and transmission coefficient $T \simeq 1 - R$. Geometrically, only one ray is necessary. The transmitted signal is not modulated sinusoidally with phase, but as the sum of all transmitted rays after each reflection at the same phase. This can be described by the Airy formula

$$I_T = I_0 \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\varphi/2)} \tag{31}$$

with phase $\varphi = 2klN$ (l is the distance between the two reflecting surfaces)[1]. With an increase of the number of the interfering rays (i.e. in a given arrangement with increasing reflection coefficient) very sharp maxima in the transmitted power can be achieved (see fig. 2).

[1]For mirrors with reflectivity R_1 and R_2 the numerator of the Airy-formula becomes $(1 - R_1)(1 - R_2)$ and in the denominator the factor R is given by $\sqrt{R_1 R_2}$.

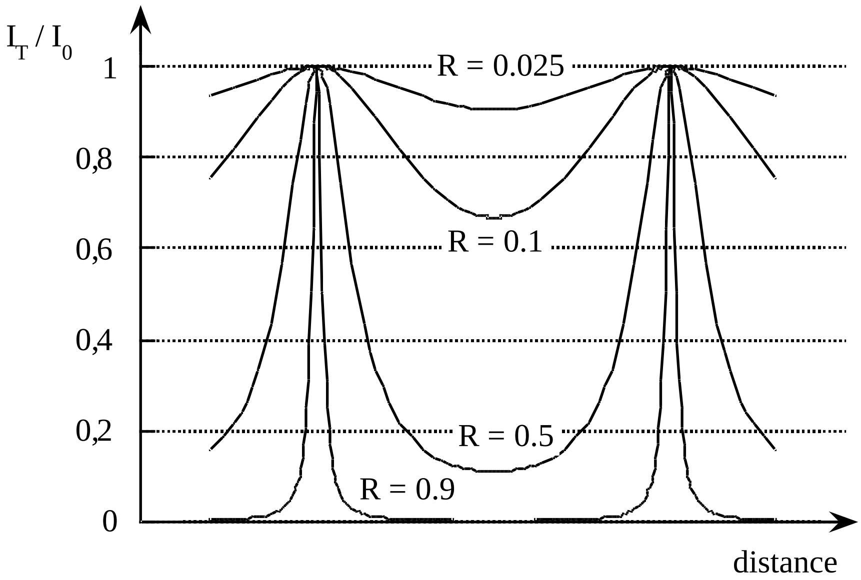

Figure 2: Transmission coefficient $T = I_T/I_0$ of a Fabry-Perot interferometer vs. mirror distance l for different reflection coefficients R of the mirror for a given wave vector $k = 2\pi/\lambda$.

Generally it is well-known that in a Fabry-Perot interferometer, interference between rays with the same inclination angle produces concentric rings. In this experiment interferences with the same thickness l are used, where the ray is guided parallel to the optical axis and the optical path length in the interferometer is changed.

In our experiment, the path difference changes with time since $n = n(t)$. This means, that the path difference runs through all values between Δs till 0.

Now, we like to derive the correlation between the plasma density and the number of maxima, which one can see during the discharge. To this end, we have to determine the change of the path difference between ray 1 (reflected in mirror 2) and ray 2 (traversed the plasma and reflected in mirror 3) but not the path difference between ray 1 and 2 itself. See also Fig. 3, and Chap. 2.3:

The mirrors of this experiment have been designed so that at the end, when the plasma is again neutral ($N = 1$), the phase difference between beam 1 and 2 is equal to an odd multiple of π, i.e. there is a destructive interference in the IR between beam 1 and 2. This is the case when for a round-trip through the Fabry-Perot interferometer, the phase φ is an integer multiple of π and the transmission has a maximum or the reflection a minimum (see equation 31).

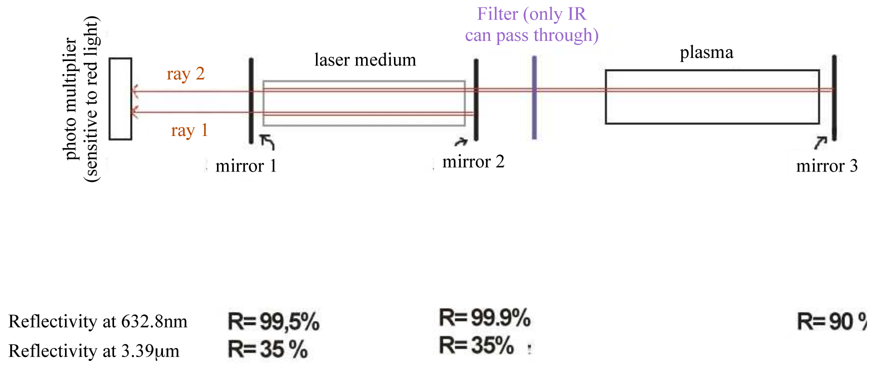

Figure 3: *Beam path in the experimental setup with specification of the mirror reflectivity.*

Then you can best see the maxima during the plasma discharge. [2]

Path Difference Δs:
At the beginning, then at the maximum of the plasma ionisation, the path difference Δs is different with respect to the end, with Δs given by:

$$\Delta s = s_2(\text{full recombination}) - s_2(\text{max. plasma ionisation}) \tag{32}$$

i.e. comparing the optical path length of the beam 2 through the plasma with the one though the gas ('vacuum')

$$\Delta s = N_{\text{Vac}} \cdot L - N \cdot L \tag{33}$$
$$\Delta s = \int (1 - N) \mathrm{d}L \tag{34}$$

Assuming a refractive index $N \approx 1 - n/(2n_{\text{crit}})$
and $N \neq N(L)$ we get:

$$\Delta s = \frac{n}{2n_{\text{crit}}} \cdot 2L \tag{35}$$

[2]there will be still some intensity at the minimum, because ray 1 and ray 2 have not the same intensity due to the reflectiivity of the mirrors and thus both rays are not completely cancelling out at destructive interference.

Note that we have considered the distance L from the laser beam twice ($2L$)!

$$\Delta s = \frac{n}{n_{\mathrm{crit}}} \cdot L \tag{36}$$

Phase Shift $\Delta\varphi$:
The path difference can be written in terms of phase shift using:

$$\Delta\varphi = 2\pi \cdot \tfrac{\Delta s}{\lambda} \tag{37}$$

$$\Delta\varphi = \tfrac{2\pi}{\lambda} \tfrac{n}{n_{\mathrm{crit}}} L \tag{38}$$

Constructive interference occurs if φ is an integer multiple of 2π. It will pass through the maximum of ionization up to the complete recombination of all the phase differences from $\Delta\varphi$ to 0. Therefore there are (in the maximum ionization observation period and recombination):

$$N_{\mathrm{f}} = \mathrm{MOD}(\frac{\Delta\varphi}{2\pi})$$

You can either have a maximum intensity (constructive interference) or low intensity (destructive interference). Since the plasma density changes much slower than the $\vec{E}$ vector, we only see much or little light, we can not see the maxima and minima of the E-field vector. (this is different in experiments like the Michelson interferometer, in which one counts the maxima and minima in space as rings. Only then we must have the number N_f of the rings.)

$$N_f = \frac{\Delta\varphi}{2\pi} \tag{39}$$

$$N_f = \frac{n}{\lambda n_{crit}} L \tag{40}$$

where the critical density n_{crit} is given by:

$$n = 1.1148 \cdot 10^{15} m^{-3} \frac{N_f}{\lambda[m] L[m]} \tag{41}$$

Fabry-Perot interferometers are of high relevance in laser physics. In classical applications and during the production of interference filters, the length of the interferometers (i.e. the distance d between the mirrors) is much smaller than the radius of the mirrors a. Then the Fresnel number $N_{\mathrm{Fresnel}} = a^2/(\lambda d)$ is very large and diffraction losses play a minor role in comparison to transmission losses.

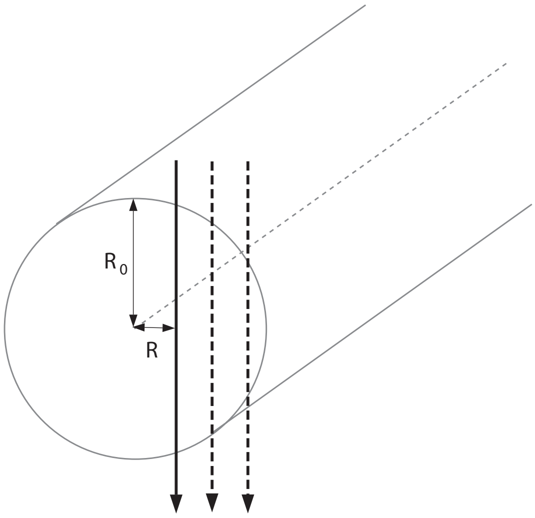

Figure 4: Deconvolution, geometry 1

If the Fabry-Perot interferometer is used as a laser resonator and also for the interferometer in this practical experiment, it has a small Fresnel number (between 4 and 30 with respect to the laser wave lengths in use). Therefore refraction losses mainly determine the number of rays which effectively interfere and consequently the finesse of the arrangement.

Let us make some comments on the practical part of interferometry measurements in large plasma experiments.

Deconvolution In general, the density along the probe ray varies, however, the line integrated density $\int n(l)\,dl$ is actually measured. To determine the local plasma density further information is required. Either additional measurements must be done or model assumptions must be made. Frequently, interferometric measurements at a cylindrical plasma column are carried out such that the probe ray crosses the cylindrical plasma of radius R_0 perpendicularly to the cylindrical axis. The smallest distance of the probe ray from the axis R can be measured. Several measurements with different R can be made by parallel shift of the interferometer or with several probe rays. The phase change due to the shift in position R is,

$$\varphi(R) = \frac{2\pi}{\lambda n_c} \int_R^{R_0} \frac{n(r)}{(r^2 - R^2)^{1/2}} r\,dr \tag{42}$$

This relation can be inverted (Abel inversion) to give the plasma density,

$$n(r) = \frac{-\lambda n_c}{\pi^2} \int_r^{R_0} \frac{d\varphi(R)}{dR} \frac{dR}{(R^2 - r^2)^{1/2}} \tag{43}$$

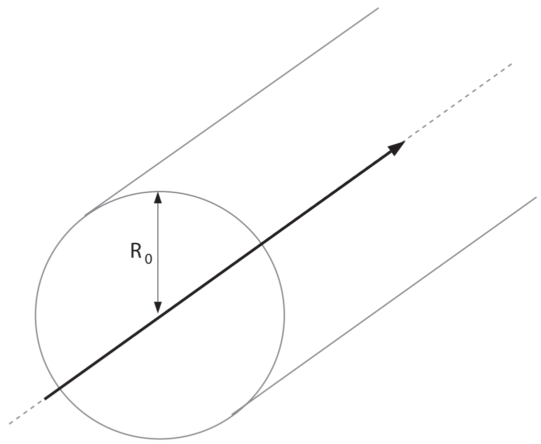

Figure 5: Deconvolution, Geometrie 2

With a sufficient number of measurements with different R the density profile can be determined.

The assumption of axisymmetric density profiles is used here! If axisymmetry is not a valid assumption, or if e.g. a parallel shift of the interferometer is not possible, but only a rotation, the density can be evaluated with tomographic techniques. In this practical experiment, a linear discharge is placed parallel to the optical axis, i.e. the probing rays pass axially through the discharge. Abel inversion is therefore not necessary. At both ends of the discharge the plasma density is axially inhomogeneous. As the spatial dimensions of these inhomogeneities are comparatively small related corrections are neglected, and we can find the electron density directly from the line integrated density:

$$n = \frac{1}{L} \int_0^L n(l)\, dl \tag{44}$$

Refraction In practice, ray deflection represents an essential problem for interferometric measurements and is not avoidable because most of the rays are not perpendicular to the gradient of the density and therefore to the gradient of the refractive index. In the framework of geometrical optics, the beam path of a light ray is described by the following equation.

$$\frac{d}{ds}\left(N\vec{e}_{\parallel}\right) = \nabla N \tag{45}$$

Here, ds is an element of length along the beam path and $\vec{e}_{\parallel}$ is the unit vector in the direction of the beam bath. For the local curvature radius ρ of the beam bath, we can derive from eq. (45).

$$\frac{dN}{ds}\vec{e}_{\parallel} + N\frac{d\vec{e}_{\parallel}}{ds} = \nabla_{\parallel} N + N\frac{\vec{e}_{\perp}}{\rho} = \nabla N \quad \rightarrow \quad \frac{\vec{e}_{\perp}}{\rho} = \frac{\nabla_{\perp} N}{N} \tag{46}$$

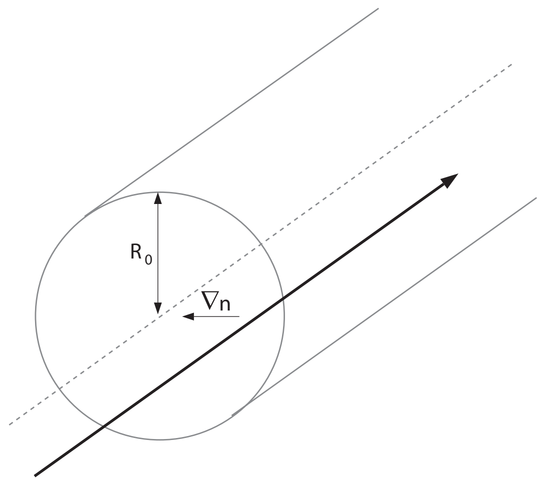

Figure 6: Refraction, geometry 1

Here, $\vec{e}_\perp$ is orthogonal to the path and points towards the center of the curvature circle. Thus, the light ray is bent into the direction of increasing refractive index. The infinitesimal deflection angle $d\alpha$ is ds/ρ.

We consider an axially homogeneous plasma column of the length z_0, the radius R_0, the central density $n(r=0)$ and the edge density $n=0$. A non-central, axial probe ray passes perpendicularly to the refractive index gradient (see fig. 6) the angle of deflection can be written as:

$$\alpha = \int_0^{z_0} \frac{\nabla_\perp N}{N}\, dz \tag{47}$$

for $\alpha \ll 1$. If the probe ray passes perpendicularly to the axis of the plasma column of radius R_0 and a parabolic, axisymmetric density profile is assumed, $n(r) = n(r=0) \cdot (1 - r^2/R_0^2)$, we obtain for the maximum deflection angle

$$\alpha_{\max} = \sin^{-1}\left(\frac{n(r=0)}{n_c}\right) \approx \frac{n_0}{n_c} = \frac{e^2\lambda^2 n_0}{4\pi^2 c^2 \epsilon_0 m_e} = 8.97 \cdot 10^{-16}\, n_0 \lambda^2 \tag{48}$$

Apart from ray deflection, we also have to consider ray expansion. The natural expansion of a Gaussian beam (diffraction limitation, Helmholtz invariant) is

$$d = 2\sqrt{\frac{\lambda z_p}{\pi}} \tag{49}$$

where z_p is the plasma radius and it is assumed that the beam is focussed in the center of the plasma. In practice limitations due to refraction and diffraction are more serious than those due to the cutoff density.

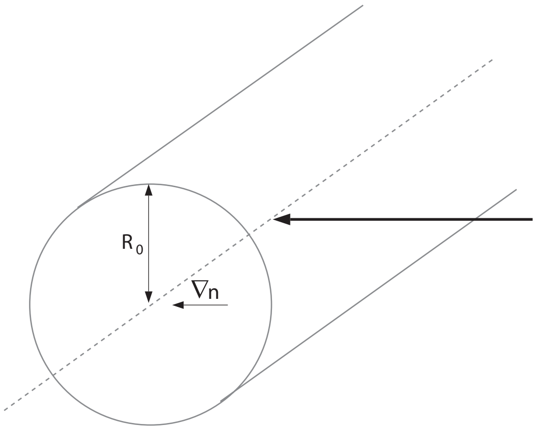

Figure 7: Refraction, geometry 2

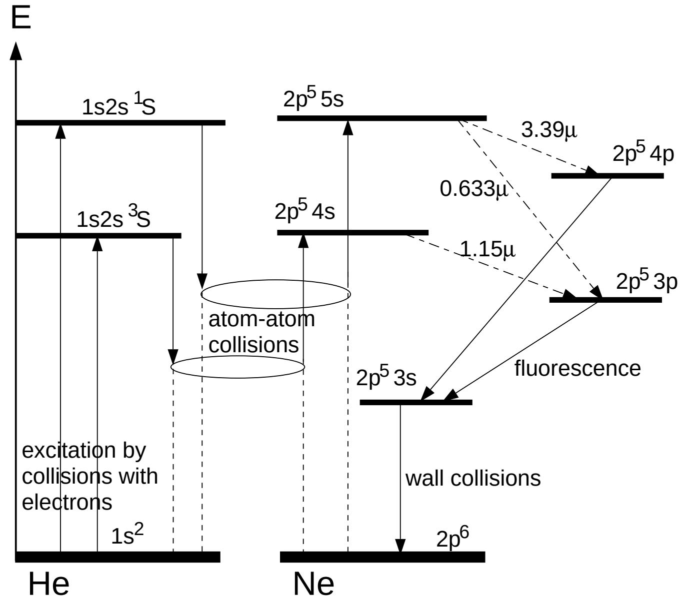

Figure 8: Transitions in the He-Ne laser

2.2.4 The He-Ne-Laser

The active medium in a He-Ne laser is Neon; The metastable Helium states are excited via electron collisions in the discharge. As these states are energetically at the same level as states of Neon with electron configuration $2p^55s$ and $2p^54s$ the latter can selectively be populated via collisions between He and Ne. Population inversion is thus achieved in Ne. The lower laser levels relax radiatively into the states of the $2p^53s$ configuration. From there, depopulation to the ground level takes place predominantly via collisions with the wall. Therefore, He-Ne lasers generally have (unlike CO_2 lasers) very thin capillary tubes. The strongest lasing transitions are at 3.39 μm ($3S_2 \rightarrow 3P_4$), in a group at 1.15 μm ($2S_2 \rightarrow 2P_4$) and at 0.6328 μm ($3S_2 \rightarrow 2P_4$). The amplification for transitions in the same atom are proportional to the third power of the wavelength, that is why the line at 3.39 μm with approx. 40 dB per meter is stronger than the red line in 0.6328 μm (approx. 0.4 dB / m). It is to be noted that those two lines are generated from the same upper excited level. Any modulation in the infrared light field causes a complementary modulation in the red light field – with very high amplification of the infrared line the laser threshold of the red line is not reached. Therefore the reflection coefficients in the resonator must be higher for the red line than for the infrared one.

2.3 Experimental setup

The experimental setup (see fig. 9) consists of the following components:

- Storage oscilloscope connected to a printer.
- Photodiode and/or Photomultiplier with IF-filters 632.8 ± 1 nm.
- He-Ne laser discharge tube model 120 S (Spectra Physics) with nominal 6 mW power at 632.8 nm wavelength in multi mode (in the experimental arrangement a power of approx. 0.7 mW is available in single mode). For the experiment, relevant wavelengths of the laser are (in vacuum): $\lambda = 632.8$nm (red) and $\lambda = 3.39\mu$m (IR).
- Laser resonator mirrors
  - mirror 1: concave (Herasil), 3m radius of curvature, reflectivity 99.5% (632.8 nm), 35% (3.39 μm)
  - mirror 2: planar (Herasil), reflectivity 99.9% (632.8 nm), 35% (3.39 μm)
  - mirror 3: planar, metal surface mirror, reflectivity approx. 90%, wideband

  The mirrors are very sensitive. Cleaning of these mirrors and the Brewster window is only to be done by the supervisor!
- Germanium filter (which is not transparent to the red spectral region).
- Plasma discharge tubes with He (5 Torr), Xe (1.5 Torr), and Ar (2 Torr)
- Capacitor discharge circuit with a power supply 5 mA, 10 kV
- Rogowski coil: N_S=92, internal diameter $r_i = 16$ mm, external diameter r_a=30 mm, length z=15 mm.
- Fabry-Perot interferometer.

2.3.1 Laser

The He-Ne laser used in the experiment consists of a discharge tube of length 35 cm with a capillary diameter of 2 mm. The discharge is run with a current of 5.5 - 7 mA and a high voltage power supply of 2.9 kV over the tube and two resistors (80 kΩ in total) for current limitation.

In order to keep the radiation losses small in the resonator, the laser tube (also the plasma discharge tube) is terminated at the ends with quartz disks mounted at the Brewster angle. Linearly polarized light with the electric field vector in the plane of incidence suffers no reflection losses at the Brewster angle. In a He-Ne laser, a large number of laser transitions occur (see fig.8)).

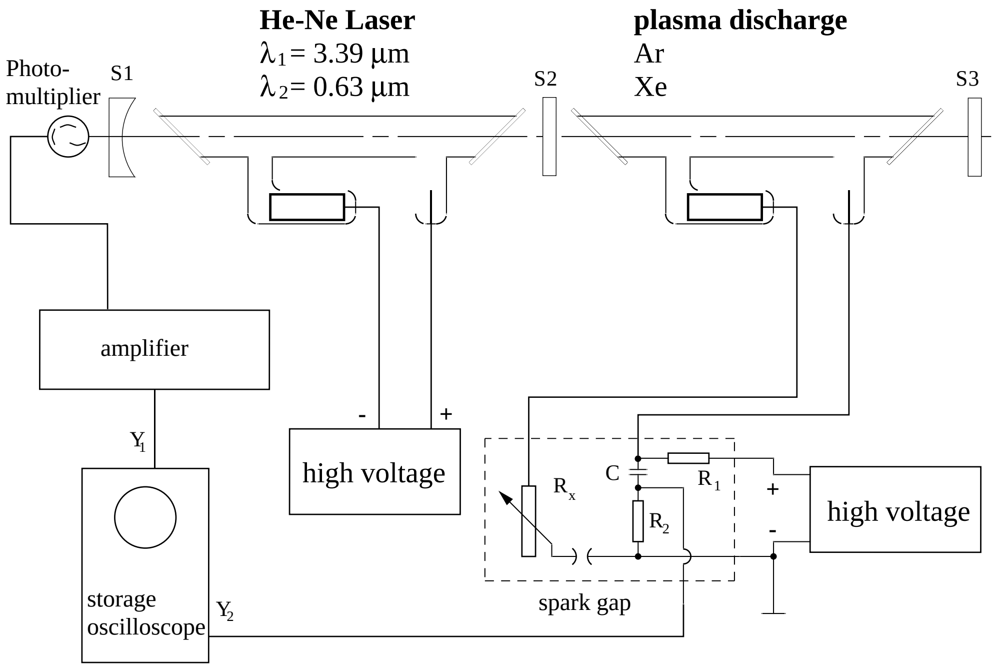

Figure 9: Experimental Setup

By means of the concave and the planar mirror, a hemispherical resonator is set up for the laser tube with a length of 40 - 45 cm. The mirrors consist of Herasil (trade mark of the quartz substrates) and are coated with interference layers (layers with alternating low and high refraction index having the optical thickness $\lambda/4$) to obtain a certain reflectivity at the laser wavelengths. In this resonator, the resulting mode diameter (for the TEM_{00} mode) is approx. 1.2 mm, and this is the fundamental transverse mode which oscillates easily in the hemispherical resonator. If other mirrors with different radii of curvature (or two planar mirrors) are used for the resonator, the laser will operate as a multi mode laser (several overlapping transverse modes) and will have a higher output power related to the larger mode volume. In the following however, we ignore these higher modes.

Similar to hohlraum resonators, the characteristic oscillations in laser resonators can be characterized by three numbers. The transverse field strength distribution is described by two of these numbers (TEM_{00} has a radial Gaussian profile). The third number indicates the number of the half waves between the two mirrors, and is therefore the longitudinal mode number. In gas lasers, that number is of the order of millions and is mostly not indicated. In our experimental setup, it corresponds to a frequency range of the single longitudinal mode of approx. 350 MHz. The width of the (passive) mode is given by the geometric dimensions and the reflectivity of the resonators. Several longitudinal resonator modes lie in the Doppler broadened excitation profile of the laser medium (approx. 1.5 GHz as compared to a natural line width of approx. 100 MHz). By introduction of the amplifying medium, the resonator is undamped, consequently the quality of the resonator (q-factor) increases, and the line width decreases drastically. For lasers with high stability, bandwidths are reached of the order of 100 Hz or less. In our case, several longitudinal modes oscillate simultaneously in the laser, whose intensities depend on the (net) laser amplification at the respective frequency (see fig. 10).

2.3.2 Interferometer

With a third (plane) mirror S3, the Fabry-Perot interferometer is coupled to the laser containing a plasma discharge tube.

3 Experimental procedure

Caution! When the laser is operating, a high voltage of 2.8 kV is applied between the anode and the cathode of the laser. The plasma discharge tube is operated with 5-10 kV. Touching the high voltage parts is dangerous!

Touch conducting parts only if the manual grounding pole is positioned and **grounding is visible**! Never rely on automatic grounding!

Caution! The radiation intensity of the He-Ne laser in the coupled resonator and especially that of the alignment laser is sufficient to damage the retina if the laser beam is viewed directly and without precaution!

Never look directly into the beam! Avoid careless reflections on metal or mirror surfaces, e.g. watches!

3.1 Measurement principle

The plasma discharge in a tube filled with He or Xe is used in pulsed mode. Via a charging resistor a capacitor (0.62 μF) is charged. After reaching the breakdown voltage of a spark gap the capacitor is discharged across the the plasma tube. By changing the distance between the electrodes of the spark gap the breakdown voltage and consequently the discharge current can be varied. Moreover, the discharge current can also be changed via a variable resistor in the discharge circuit.

The current is measured by means of a Rogowski coil, which is put around the current carrying cable to the plasma discharge tubes. The temporal derivation of the closed line integral of the azimuthal magnetic field is measured, which is proportional to the current in the cable.

When the capacitor discharges, the gas inside the plasma discharge tube is ionized and the optical path length in the interferometer changes. Consequently, the intensity of the light in the interferometer and the reflected light is modulated. Note, that the light of the He-Ne laser is reflected by the Fabry-Perot interferometer, i.e. its reflection characteristics change. The modulation of the intensity of laser light is recoupled into the active laser section.

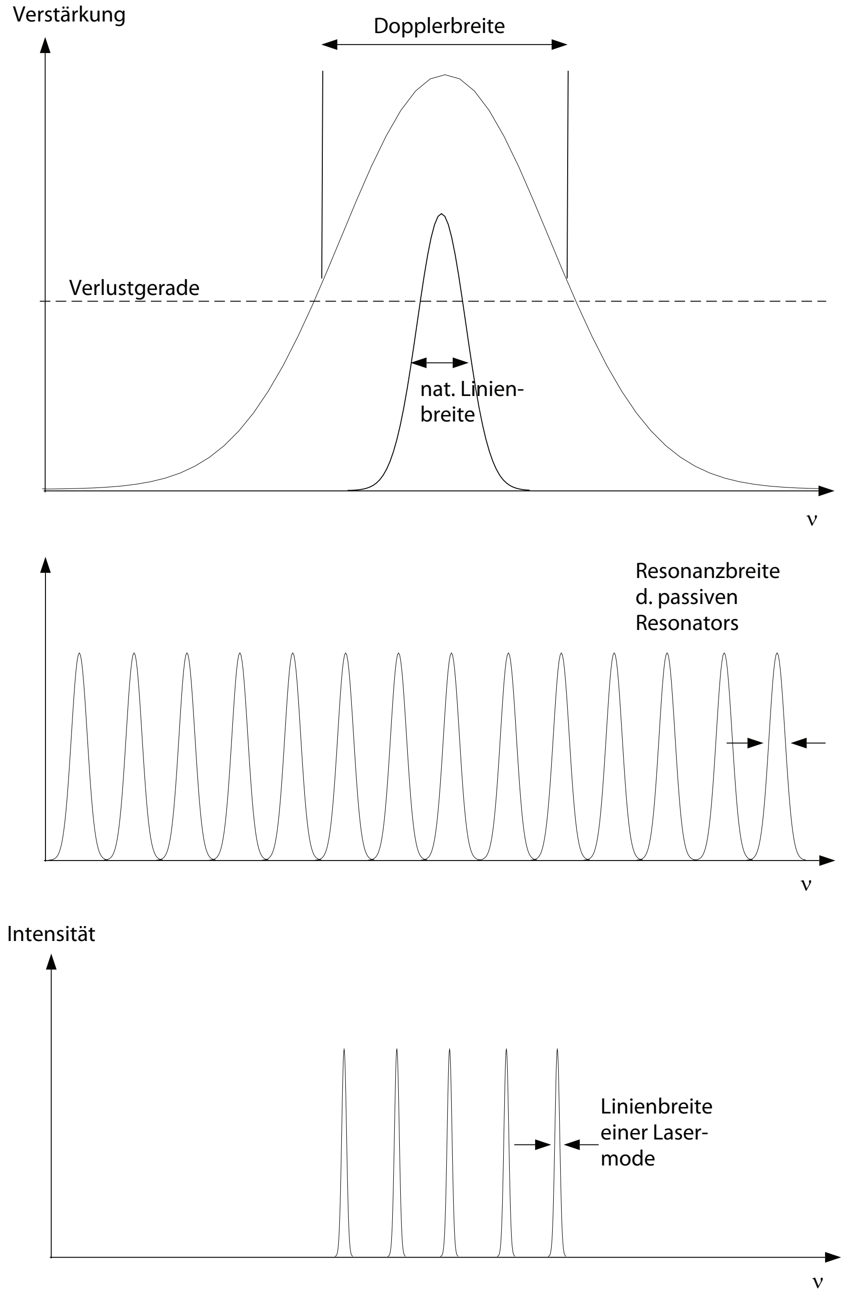

Figure 10: Illustration of the different line widths in the laser system

The modulation of the intensity of 3.39 μm radiation causes a corresponding modulation of the stimulated emission of the complementary 632.8 nm laser line. This visible line at 632.8 nm is detected because this is easier and the detector is more sensitive at that wavelength.

The red laser light is measured by a fast photodiode with an integrated amplifier. The signal from the photodiode is then transmitted to one channel of a storage oscilloscope. The glowing of the plasma discharge is measured on a second channel by a further photo diode. At the same time the integrated signal of the Rogowski coil is recorded on the third channel.

3.2 Measurement

For three different gas discharge tubes (gas species and pressures are written on the tubes), the electron density has to be determined for different discharge currents.

Since in this experiment one always has to work with two laser wavelengths, it is necessary to control which effect is related to the infrared and which to the visible beam.

As explained earlier, the interferometric effect has to be measured with the infrared line. This can be tested by placing a Germanium disc, which is coated with an anti-reflection coating at 3.39 μm, into the beam path between S2 and the plasma tube. Germanium is only transparent for wavelengths larger than 2 μm, thus completely suppressing all lines in the visible. So the interferometric effect is only caused by the infrared line. All effects caused by e.g. adjustment of S3, breathing in the interferometer or the gas discharge can now be attributed solely to the infrared line. The electron density measurements should be done with the Germanium disc inserted.

Another possibility is to suppress the infrared line by inserting in the interferometer a plane-parallel plate of Bor-Kron glass: this glass is transparent in the visible but it completely blocks the light at 3.39 μm. Check that, by inserting this disk, interferometric effects should completely disappear.

After these initial tests, measurements of the intensity modulations together with the temporal development of the discharge current should be measured for each discharge tube at three different discharge currents. To this aim, turn on the high voltage power supply of the capacitor discharge, which will lead to a discharge in the tube dependent on the spark gap. With the storage oscilloscope you can store the current and the oscillations in the detected laser signal caused by the change in electron density. Make sure that you record all maxima. For the current measurement, the Rogowski coil with a well known sensitivity is used.

During the plasma discharge, the electron density increases quickly but decreases subsequently with a slower time constant. Only in this second phase, the measurement succeeds because the time resolution of the oscilloscope is not high enough to resolve the rising density. Moreover, in the starting phase of the plasma discharge nonlinear effects disturb the interferometric signal. The electron density decreases roughly exponentially due to recombination and wall collisions, which can be seen in the increase of the duration of the oscillation periods. The turning point of the oscillations (the density maximum) is delayed as compared to the maximum of the discharge current, because more atoms are still ionized by collisions with free electrons than are neutralized by recombination. About the same temporal development as the electron density can be seen in the glow of the discharge, represented in the background of the interferometer signal. This is based on the fact that the glow is essentially due to recombination radiation, which is at least proportional to the density of the free electrons. We ignore a possible influence of the electron temperature. With an interference filter for the laser line at 632.8 nm, the intensity of discharge light can be reduced to a tolerable value. The glow stemming from the discharge can be recorded on its own by a second photodiode, so that the density maximum can be determined temporally.

4 Analysis

4.1 Tasks

Starting from the number of intensity maxima or minima, the electron density of the plasma can be estimated: $n = \text{const.} \cdot N_f$ where N_f is the number of the maxima of the fringes. Derive this formula (using equation 27 of the guide) and determine the constant factor.

Analyse for each gas species the following problems:

- Calculate electron density n, plasma frequency ω_p, and refractive index N for each spark gap (2 mm, 4 mm, 6 mm). Hint for the calculation of errors: the error of the length of the discharge tube is to be determined by a measurement at the setup. The error of N_f is to be estimated individually. There is no random error to be analysed, because of the small amount of measurements.

- Calculate the maximal discharge current I in the measurement of the Rogowski coil for each spark gap. Hint: The signal is terminated with 50Ω? and is integrated with $R = 10\text{k}\Omega$ and $C = 100\text{nF}$ (see fig. 11). The Rogowski coil has the following data: Number of turns = 92, inner diameter = 16 mm, outer diameter = 30mm, length = 15 mm.

- Calculate the degree of ionization of the plasma at the point of maximal discharge current for each spark gap, assuming singly ionized gas.

Show graphically the dependence of the plasma data n, ω_p, and N on the discharge current. Error bars in the x and y directions are to be plotted. Plot the different gas species into one diagram.

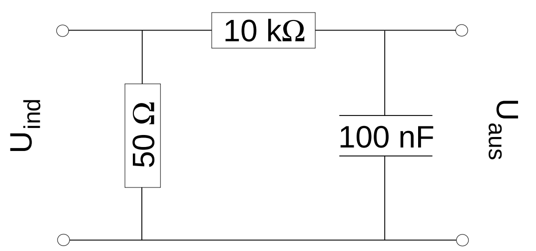

Figure 11: Circuit diagram for the Rogowski coil.

4.2 Instructions for the Minute

The minute should include the following information:

0. Title:

    - include title and number of the experiment

      TUM Fortgeschrittenenpraktikum
      im Studiengang ...
      Versuch 40: "'Plasmainterferometrie"',

    - and the group number,
    - the names of the group members, their e-mail adresses, the matriculation numbers and the dates of birth!

1. Theoretical Background:

    - Definition of Plasma.
    - The aim of the experiment.
    - Derivation of important plasma parameters (plasma frequency, Debye length...).
    - Sketch the dispersion relation and derive the equation.
    - Derive the refractive index N (linearised).
    - Equation for the density n. (Caution: the laser beam crosses the plasma of lenght L, 2 times).

2. Experimental Setup:

    - Sketch of the experimental setup with the Laser, Fabry-Perot interferometer,
      Rogowski-coil.
    - Derive the Airy's formula (eq.(31)). Where does the refractive index N come into play?
    - Rogowski-coil: design (scheme with r_i, r_a, z), formula and dependences. Why and how does the measured discharge current curve deviate from a $1/e$ function?
    - Laser: type, functional principle (keywords: energy levels, population inversion, stimulated emission), wavelength.

- Measuring arrangement and procedure (complementary), motivations, tests (2 filters).
- Approximations (n constant, no refraction...). Draw qualitatively the path of a laser beam in a plasma column with a radial density gradient. The incoming beam is parallel to the axis, but does not start centrally.
- Discuss the advantages and disadvantages of the use of such an interferometer for a large plasma e.g. at ASDEX Upgrade.

3. Measurements:

   - Which quantities must be measured?
     Other important parameters (λ, Rogowski-values)
   - Where (oscilloscope channels) and in which units are they measured?
   - Table with the measures and their errors (mean of 5 values): the number of maxima N_f, the voltage U and the plasma discharge length L

4. Analysis:

   - Table of the results: discharge current I, plasma density n, plasma frequency ω_p, refractive index N, degree of ionization and the errors for all the estimates.
   - Discuss the results and their errors.
   - Graphical analysis including error bars.

Since the printed measurements are not personally handed with the minute (because this is send to the tutor by e-mail) they should be handed at the follow-up meeting.