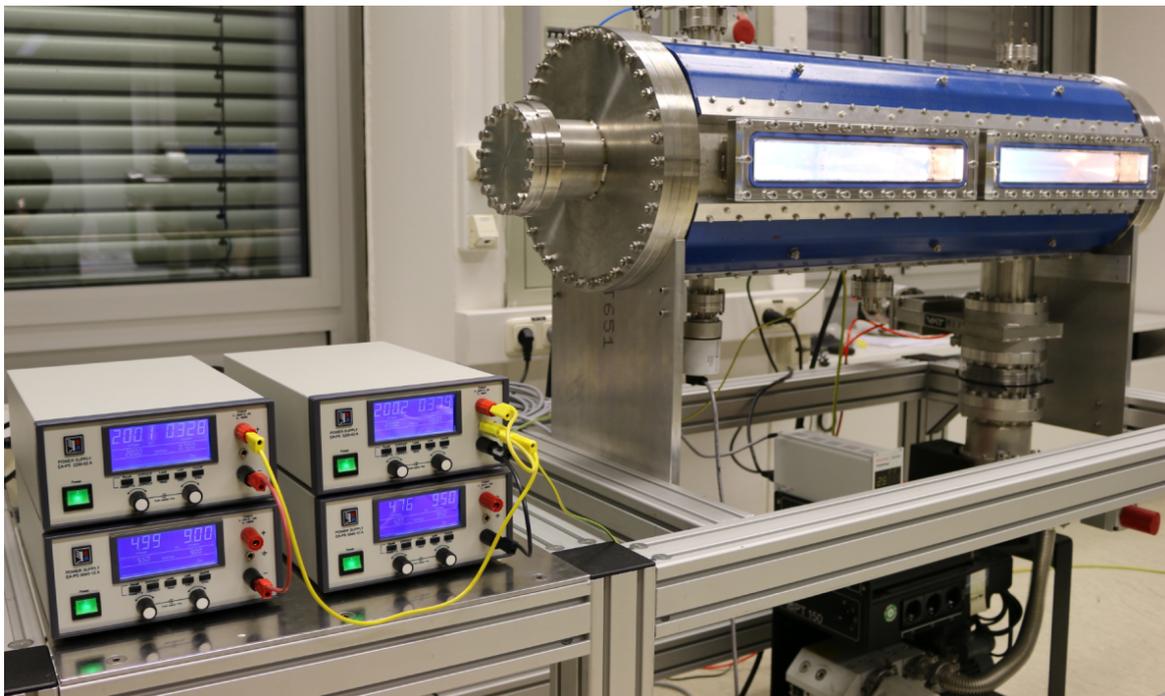




Wave Phenomena in a Double Plasma Experiment

Fortgeschrittenen-Praktikum der TUM



Max-Planck-Institut für Plasmaphysik
and
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Linear and non-linear waves in a double plasma experiment

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Abstract

A double plasma device consists of two plasma volumes, electrically separated by a conducting grid and biased to different plasma potentials. By manipulating the potential difference, a controlled plasma flow from one volume into the other can be generated. With the help of a harmonically modulated potential difference, plasma waves can be excited. Using different modulation frequencies allows measuring the wave's dispersion relation and by increasing the amplitude a continuous transition from a linear wave to a non-linear behavior can be studied including wave steepening and solitons.

For the diagnostic of the plasma and the wave parameters Langmuir probes will be used. The Langmuir probe is a standard diagnostic applied in most plasma experiments to measure the electron temperature, the plasma density and the plasma potential. It consists of an electrically conducting filament connected to a power supply. The plasma parameters are deduced from measured current-voltage characteristics.

A first step of the experiment is to understand plasma generation by a thermionic discharge in the vacuum chamber of the double plasma device, which provides additional plasma confinement through a magnetic multipole field generated by rows of permanent magnets. The accessible plasma parameter range will be explored by varying the discharge parameters. The plasma density will be measured by Langmuir probes and by an oscillation method exciting the plasma frequency. The next step is to excite plasma waves and to deduce the phase velocity which gives an estimate for the electron temperature that will be compared with the Langmuir probe result, while the damping of the wave amplitude allows estimating the neutral particle background. Finally larger amplitude waves will be excited and the transition to a non-linear regime is studied.

The experiment gives the student an excellent insight in some characteristic plasma properties and experimental techniques. The study carried out on the plasma wave is exemplary for wave physics in general and for non-linear effects in wave propagation.

1 Introduction

A plasma is a gas or a fluid at high temperature where matter becomes ionized. The constituents of a plasma therefore are ions, electrons and a background of neutrals. If the degree of ionization is high enough, the plasma develops collective dynamics unknown for neutral gases. The new collective behavior can be very well studied by wave phenomena as they can be excited in the present double-plasma device.

With this experiment, students can learn about basic plasma properties and fundamental plasma dynamical phenomena such as oscillations at the plasma frequency, the ion acoustic waves and solitons. The plasma is created by filaments in a double-plasma device. A multi-cusp magnetic field, generated by permanent magnets mounted on the outside of the vacuum vessel, leads to relative high plasma densities in a magnetic field-free region in the plasma bulk. The experiment offers great flexibility in the external control parameters in order to offer maximum practical experience for the students.

2 Theoretical background

A brief overview of the theoretical background of the topics addressed in the experiments is given here.

2.1 Plasma definition

Plasma is often referred to as the fourth state of matter. This classification is not without problems as there is no clear phase transition into the plasma state. Plasma consists of a globally neutral gas or fluid of electrons and ions and is generated by ionizing in general a neutral gas. The ionization degree gives the ratio of the electron density to the density of the neutrals, which are present as a background. In order to exhibit the typical collective phenomena of a plasma, the degree of ionization must be sufficiently high. A more rigorous plasma definition needs the following parameters: The plasma frequency ω_{pe} refers to the characteristic time constant at which the electron fluid of the plasma collectively oscillates against the ion fluid

$$\omega_{pe} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}}. \quad (1)$$

The Debye length is the distance over which a plasma can shield an electrical charge by an internal polarization;

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n_e}}. \quad (2)$$

The volume with a radius of λ_D is called *Debye sphere* and the average number of electrons inside the Debye sphere is given by

$$N_D = \frac{4\pi}{3} \lambda_D^3 n_e. \quad (3)$$

The different quantities entering these definitions are listed at the end.

In order to be a plasma, the gas or fluid has to fulfill the following criteria:

- $\lambda_D \ll L$, with L the spatial dimension of the plasma. This is needed for the plasma in order to stay globally close to neutral, called *quasi neutrality*,
- $N_D \gg 1$, in order to exhibit collective behavior.

2.2 Plasma generation

The double plasma device consists of two plasma volumes, electrically separated by a conducting grid. The plasma in the two sections of the vacuum vessel is created by thermionic discharges. A tungsten filament is strongly heated by an electric current to temperatures large enough to generate electron emission leading to an electron current from the filament into the volume given by *Richardson's law* [11]¹:

$$j(T) = A_R T_f^2 \exp\left(-\frac{W_e}{k_B T_f}\right), \quad (4)$$

with A_R the Richardson constant, W_e the work function and T_f the temperature of the filament. For tungsten it is $W_e = 4.54$ eV [12]. In its general form, the Richardson constant can be calculated from the following formula: $A_R = 4\pi m_e k_B^2 e / h^3$, with h the Planck constant. It turns out, however, that A_R depends on the material and for tungsten it is $A_R = 6 \cdot 10^5$ A/(m²K²) [12]. According to Eq. (4), there is already a significant emission current even for an average temperature of the tungsten wire well below the corresponding work function.

In order to gain enough energy to ionize the gas, the emitted electrons have to be accelerated by an externally applied electric potential which is obtained by biasing the filament against the vacuum vessel, see Fig. 4. Due to the low neutral gas pressure of the order of a few mPa and the resulting large mean free path lengths of up to one meter, a magnetic cusp configuration like the one described in Sec. 3 is required to confine the primary electrons and thus enhance

¹Richardson received the Nobel price in 1928 *for his work on the thermionic phenomenon and especially for the discovery of the law named after him* [13].

the plasma production. In this regime, the plasma is also referred to as *non-self-sustained discharge*. The discharge current I_d first increases linearly with the applied voltage U_d until it saturates. As soon as the source of energetic electrons is turned off, the discharge current drops to zero and the discharge is terminated. The important parameters for describing a plasma are the temperatures of the electron and ion components, T_e and T_i , the plasma density n which is in a plasma with single charged ions the same for electrons and ions, and the electric potential inside the plasma, called plasma potential ϕ_p . In low temperature plasmas, as in the present experiment, the ions are at about room temperature and one can use $T_i = 0$.

2.3 Langmuir probes

A direct approach to diagnose a plasma is to insert an electrically conducting probe connected to a power supply and to measure its current-voltage characteristic. The *Langmuir probe* is a realization of this idea; it has been used since the very beginning of plasma physics [22]. It consists of a small electrode, usually a tungsten wire, that can withstand high temperatures. From the current-voltage characteristic of the probe, the plasma density, the electron temperature, and both the plasma and floating potentials, ϕ_p and ϕ_{fl} , can be deduced. In Fig. (1) a typical characteristics it depicted, described by the following function:

$$I = I_{e,sat} \exp \left\{ -\frac{e(\phi_p - U)}{k_B T_e} \right\} + I_{i,sat}. \quad (5)$$

The current as a function of the bias voltage of the probe, U , exhibits an exponential shape, where the logarithmic slope is determined by the electron temperature. By fitting the characteristics with this function, T_e , n_e , and ϕ_p can be obtained.

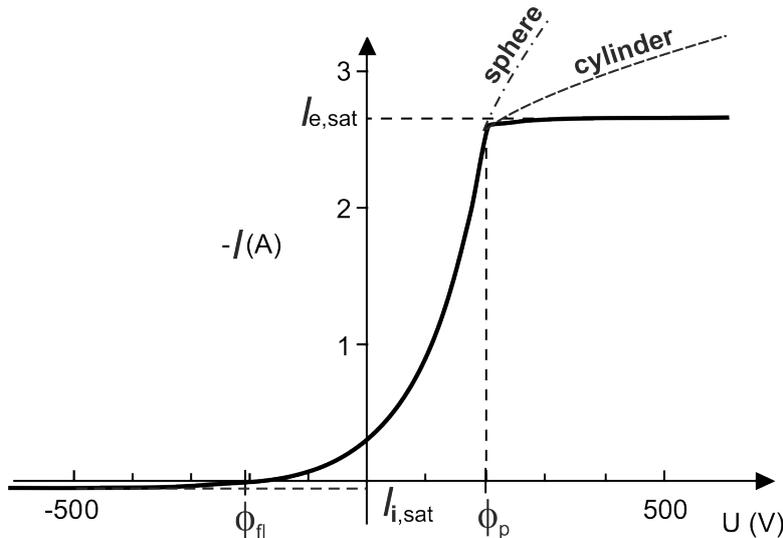


FIG. 1: Current-voltage characteristics of a Langmuir probe for different shapes of the probe head (from Ref. [23]).

For strongly negative bias voltage, the Langmuir probe will only collect an ion current which saturates at the *ion saturation current*. The electron current can be neglected for sufficiently negative bias voltage of several times $k_B T_e$. Assuming that the plasma is homogeneous, has no boundaries and contains two particle species which can each be described by a Maxwellian velocity distribution, the ion saturation current can be expressed as (e.g. Refs. [23, 24])

$$I_{i,sat} = 0.61enS \sqrt{\frac{k_B T_e}{m_i}}, \quad (6)$$

with S the probe surface. The factor 0.61 corresponds to the assumption of cold ions which results in the Bohm criterion which states that the velocity of the ions entering the sheath surrounding the probe must exceed the ion sound velocity.

The floating potential is reached, when the net current is zero. This is the potential to which the probe will charge if disconnected from the power supply. For sufficiently positive bias voltage and assuming a planar probe, only electrons will reach the probe and the corresponding current saturates at the *electron saturation current*,

$$I_{e,sat} = -enS\sqrt{\frac{k_B T_e}{2\pi m_e}}. \quad (7)$$

2.4 Plasma oscillation method

To determine the plasma density in a low temperature plasma by a Langmuir probe requires knowledge of the effective unshielded probe surface, which is a source of uncertainty and can even change over the lifetime of a probe due to film deposition or erosion. An alternative, more direct way to determine the electron density is provided by the *plasma oscillation method* [5, 6]: A thin electron beam is injected from a negatively biased filament, where thin refers to the electron density of the beam as compared to the plasma density. In the presence of such a beam, oscillations at the electron plasma frequency $f_{pe} = \omega_{pe}/2\pi$ can become unstable as a result of the two-stream instability. These oscillations can then be detected with a small antenna connected to a spectrum analyzer. According to Eq. (1) this allows the direct determination of the electron density according to

$$n = (2\pi f_{pe})^2 \frac{\epsilon_0 m_e}{e^2}, \quad (8)$$

with f_{pe} is the measured value of the electron plasma frequency.

In order for this technique to be reliable, three requirements have to be fulfilled [5]: (a) the beam density has to be low, $n_{beam}/n \ll 1$, (b) the electron collision frequency should not be too large, $\nu_e/\omega_{pe} \leq 1$, and (c) the beam energy which is set by the difference between bias and plasma potential, $E_{beam} = e(\phi_{plasma} - U_{beam})$, must be higher than the electron temperature, $E_{beam}/k_B T_e \lesssim 20$.

As the expected plasma frequencies lie in the GHz-range, commercial (low-price) amplifiers available for down-converted signals from satellite based television can be used.

2.5 Ion acoustic waves

Sound waves are periodic oscillations of the plasma pressure and density. In a neutral gas, these longitudinal waves propagate through binary collisions. When reducing the neutral gas pressure to the mPa range, ordinary sound waves do no longer propagate due to the long mean free path length. In a plasma, however, sound waves can propagate through the long-range Coulomb interactions of the charged particles. In the low frequency regime (low compared to the ion plasma frequency), ions and electrons oscillate in phase and the corresponding periodic density perturbations are called *ion acoustic waves* (IAWs). They were first described in 1929 [15] by Tonks and Langmuir. Today, they still play a role in plasma physics as a reliable diagnostics tool, in particular in multi-ion component plasmas [16]. The dispersion relation of the IAWs is given by [14]

$$\frac{\omega}{k} = \sqrt{\frac{k_B (T_e + \gamma_i T_i)}{m_i}} = c_s, \quad (9)$$

with γ_i the ratio of the specific heat capacities, which is $\gamma_i = 3$ for plane waves leading to one dimensional compressions of the ions. The sound speed c_s is the phase velocity of these waves. In the plasmas considered here, the cold plasma approximation $T_i = 0$ is fulfilled and the sound speed is given by

$$c_s \approx \sqrt{\frac{k_B T_e}{m_i}}. \quad (10)$$

The electron temperature can now be calculated from the sound speed.

In weakly ionized plasmas, like the one considered here, the dominant damping mechanism of ion acoustic waves occurs through collisions with neutrals. The electron-neutral collision

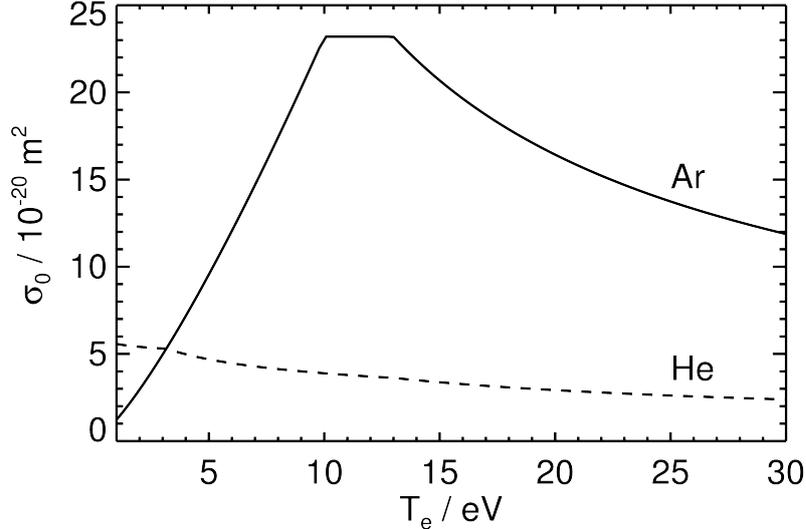


FIG. 2: Electron-neutral cross section as a function of the electron temperature (according to Ref. [17]).

frequency can be estimated as follows [18]:

$$\nu_{e0} = v_{th,e} n_0 \sigma_0, \quad (11)$$

with $v_{th,e} = \sqrt{2k_B T_e / m_e}$ the thermal velocity of the electrons, n_0 the neutral particle density and σ_0 the cross section of the electrons with the neutrals. For the low electron temperatures of a few eV expected here, the cross section σ_0 is a function of T_e (the *Ramsauer effect* needs to be considered). Figure 2 shows $\sigma_0(T_e)$ for helium and argon according to the data from Ref. [17]. The *decay length* δ of the wave, i.e. the distance the wave travels until its amplitude drops by a factor of 1/e which is also referred to as *damping length*, can be approximated by [19]

$$\delta = 2c_s / \nu_{e0}. \quad (12)$$

or, considering the wavenumber as a complex quantity $k = k_{\text{real}} + ik_{\text{imag}}$,

$$\frac{\delta}{\lambda} = \frac{1/k_{\text{imag}}}{2\pi/k_{\text{real}}} = \frac{\omega}{\pi\nu_{e0}}. \quad (13)$$

This means that the effect of the collisional damping will decrease with increasing wave frequency or decreasing electron-neutral collision frequency.

Another damping mechanism starts to become important when $T_i \approx T_e$ or when the wave frequency approaches the ion plasma frequency ω_{pi} , which constitutes a resonance for the ion acoustic wave: *ion Landau damping*. In this collisionless damping mechanism, ions moving slightly slower than the phase velocity of the wave can be accelerated by it. Ions moving slightly faster can transfer energy to the wave accordingly. But since in a Maxwellian plasma there are more ions moving slower than the wave, the net effect is an energy transfer from the wave to the particles and thus a damping of the wave. The damping length then scales as [19]

$$\delta \propto \exp \left\{ \frac{T_e}{T_i} \left(1 - \frac{\omega^2}{\omega_{pi}^2} \right) \right\}. \quad (14)$$

2.6 Ion acoustic shocks

A transition from the ion acoustic wave to the nonlinear shock wave can be observed when the density perturbations become larger than 10 % of the background density [19]. The propagation

speed of this collisionless shock is larger than the speed of sound. This is expressed in a Mach number M being larger than 1 (the Mach number is the ratio between the shock velocity and the sound velocity).

Those ions which move slightly slower than the shock front, can get reflected by the shock and thus accelerated to velocity slightly above that of the shock [19, 20, 21]. This results in a precursor in front of the shock, i.e. in a small bump which can be observed at the foot of the shock front. As these ions extract energy from the shock they resemble a damping mechanism. The bump is stronger for lower values of the electron temperature T_e .

In the experiment, the shock is triggered by a step excitation with the signal generator normally used for sinusoidal signals. For small amplitudes, this step excites a wave train which can be described by an Airy function, where the first two maxima are separated by approximately $16\lambda_D$. For larger excitation amplitudes the shock front steepens and the description by the Airy function is no longer valid [20].

3 The double plasma device and experimental setup

The experiment is built in a double-plasma arrangement as first used first by Taylor & Ikezi in the early 1970s for extensive studies of waves in plasmas [2, 3]. A thermionic discharge is shielded from the walls by permanent magnets in a picket-fence arrangement which ensures good confinement together with a magnetic field free region in the center of a cylindrical device. The plasma chamber is separated by a fine stainless steel mesh into two parts, a source and a target chamber. This mesh can be biased and allows to directly excite ion acoustic waves.

The experiment consists of a cylindrical vacuum chamber with an inner length of $L = 900$ mm and an inner diameter of 315 mm. A small turbo molecular pump, using a rotary vane pump as booster pump, maintains a base pressure of the order of 0.5 mPa. As working gases, argon and helium are used at pressure values in the range 1–100 mPa. The plasma is created by heated and negatively biased tungsten filaments. There is one filament in both the source and the target chamber with a length of a few cm and a wire thickness of 300 μm . The filament heating current is in the range of $I_h \approx 4\text{--}7$ A and the negative bias, required to accelerate the electrons and thus ionize the gas, has to surpass a minimum value of $U_d \gtrsim 30$ V in order to create a plasma (see Fig. 4).

The background magnetic field is produced by neodymium (NdFeB) magnets which cannot withstand high temperatures and they should not be heated above 100° C. Therefore, the NdFeB magnets are mounted on the outside of the vacuum vessel in rows of alternating polarity. This configuration is referred to as a *full-line cusp* configuration and known to lead, due to the magnetic mirror effect, to a strong increase in the confined plasma density [4]. The resulting magnetic field as calculated with the open source software FEMM [10] is shown in Fig. 3. When going from the vessel wall towards the center, the magnetic field strength quickly drops and a large region with basically no background magnetic field is created.

The vacuum chamber is separated approximately at half axial length by a fine stainless steel mesh having a wire diameter of 0.028 mm and a mesh size (grid width) of 0.224 mm. This mesh can be actively biased or kept at the floating potential. A signal generator is connected to it in order to excite ion acoustic waves.

The ion acoustic waves are investigated with a Langmuir probe. To this end the probe is biased with +40 V and connected to an oscilloscope. Thus, the density oscillations of the ion acoustic waves can be visualized. By moving the Langmuir probe, which is mounted on a manipulator, in axial direction the wavelength of the ion acoustic waves can be measured. Varying the exciting frequency at the fine mesh then gives the dispersion relation. By increasing the amplitude of the excitation signal, the transition to the non-linear regime can be directly observed.

The plasma oscillation method [5, 6] is used to directly measure the plasma density. To this end, the heating filament in the target chamber nearest to the Langmuir probe is used to inject a thin electron beam of relatively high energy by increasing the corresponding bias voltage to

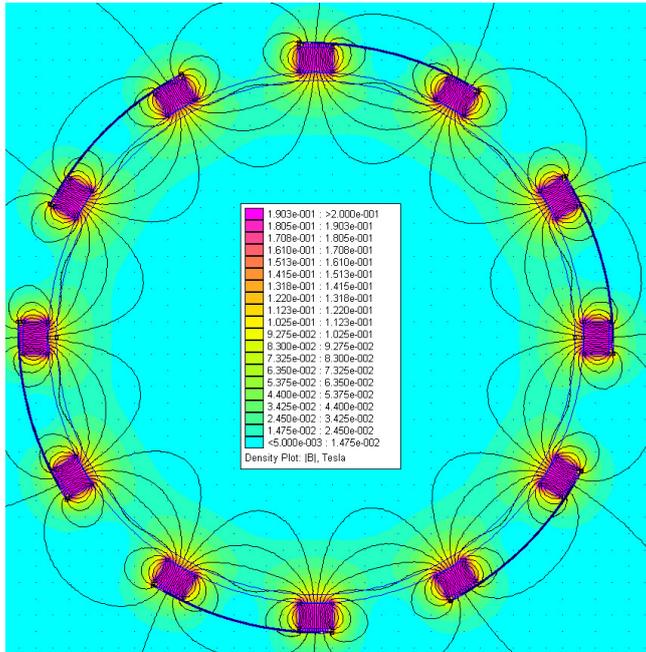


FIG. 3: Contour plot of the magnetic field strength together with the magnetic field lines in a cross section of a full-line cusp configuration; the present experiment is equipped with 8 magnets only.

$U_d \approx 100$ V and taking care that the related discharge current I_d is low. The shielded Langmuir probe positioned close to the filament and then connected to a spectrum analyzer acts as a small antenna. On the display of the spectrum analyzer, the plasma frequency oscillations can be directly seen and the plasma density measured.

Figure 4 shows the block diagram of the experimental set-up. Note that the grid, separating the source and target chamber, has no electrical contact with the vessel wall which is at ground potential. As can be seen in the diagram, the movable Langmuir probe can be used either as an antenna to detect microwave radiation in the GHz-range or as a standard Langmuir probe to obtain the current-voltage characteristics.

4 The experiments to be performed

4.1 Operating the experiment and understanding the control parameters

Familiarize yourself with the experiment and the control parameters. To this end, start investigating the discharge current I_d in the source chamber as a function of the discharge voltage U_d , of the heating current I_h , and of the neutral gas pressure p_0 . Identify the range of these parameters in which the discharge is running and then decide about the scanning range in which data will be acquired. Discuss the three obtained graphs in terms of Richardson's law and the electron-neutral ionization cross-section.

4.2 Obtaining the plasma density via the plasma oscillation method

Excite oscillations at the plasma frequency according to the description in Sec. 2.4. To this end bias the filament in the target chamber with -100 V to inject a low-density beam of energetic electrons into the target plasma. Use the Langmuir probe as antenna, connect it to an amplifier and then to the spectrum analyzer (see Fig. 4).

With the manipulator, the probe is moved as close as possible to the filament emitting the electron beam (a distance of the order of 1 cm). On the spectrum analyzer, the plasma frequency will be visible as a sharp peak, if the injected beam is thin enough. To ensure that this requirement is fulfilled, the heating current of the filament injecting the beam must be set to a small value, resulting in a small discharge current. Violating this constraint leads to a broad spectrum as different modes are excited, too [5]. In practice, the heating current and

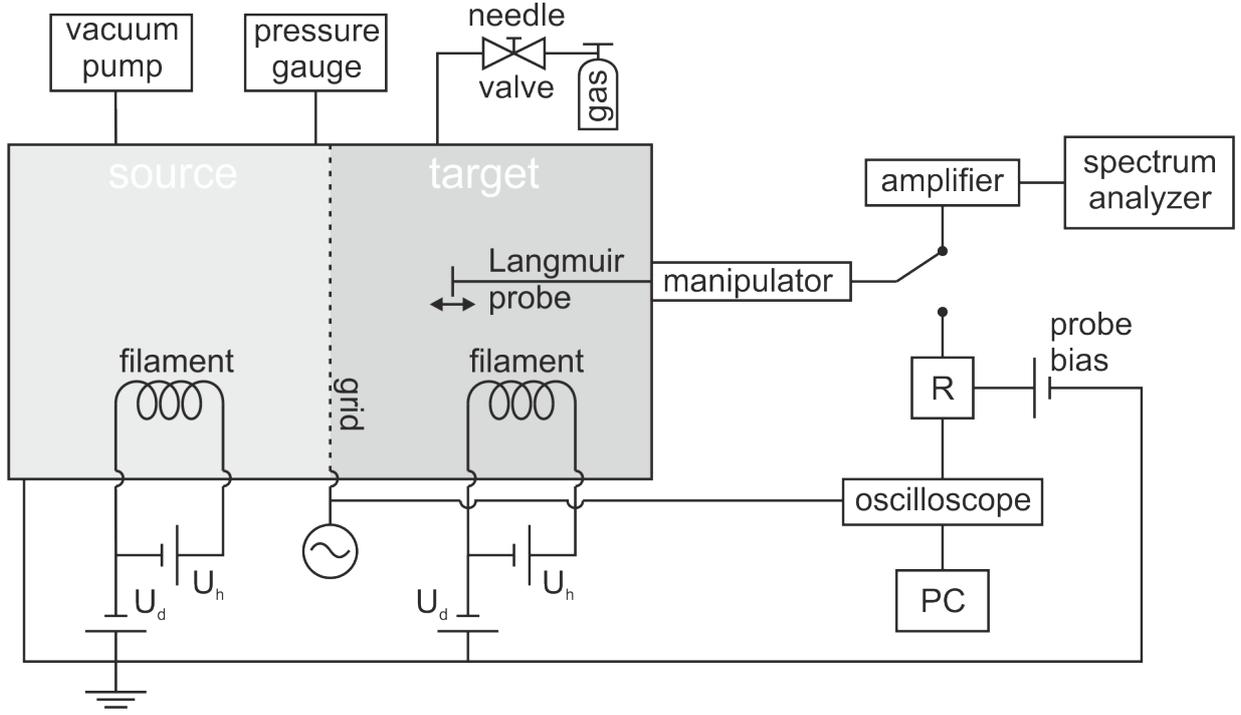


FIG. 4: Block diagram of the experimental set-up; the box labelled with a capital R (on the right hand side) contains a resistor to measure the current drawn by the Langmuir probe and a set of large and small capacitors to filter low and high frequency noise, respectively.

the resulting discharge current is kept at such a low value that a further reduction leads to the complete disappearance of the signal.

Investigate the plasma density which you derive from the plasma frequency as a function of the same parameters as in the previous part, i.e. the discharge voltage and the heating current (in the source chamber), and the neutral gas pressure.

Since the number density of the charged particles is now known, the degree of ionization can be estimated using the ideal gas law $p_0/(k_B T_{\text{gas}}) = N/V = n_0$. Assuming the neutral gas being at room temperature, $T_{\text{gas}} \approx 300$ K.

4.3 Dispersion relation and damping of ion acoustic waves

Excite ion acoustic waves by applying a sinusoidal voltage to the stainless steel mesh and bias the Langmuir probe with +40 V to the electron saturation regime where the signal is directly linked to density modulations. Record the measured current with an oscilloscope (AC-coupled to get rid of the large DC-bias). The electron saturation regime is chosen since the signal to noise ratio is much higher than for the ion saturation current. Determine the wavelength of the ion acoustic wave by moving the probe through wave minima and maxima, starting close to the excitation grid. Measure the dispersion relation by varying the excitation frequency. Note that the wave frequency should not be too low, if the wavelength approaches the transverse dimensions of the excitation grid, the plane wave approach is violated.

Use the slope of the dispersion relation in the linear regime to estimate the sound speed (Eq. (9)) and calculate from this the electron temperature. Carry out this measurement at different values of the neutral gas pressure.

Investigate the damping of the wave by measuring the wave amplitude as a function of the distance x to the grid. The amplitude A follows roughly an exponential decay of the form

$$A(x) = A_0 e^{-x/l_d}, \quad (15)$$

where l_d is the damping length. Fit this equation to the data to determine the damping length. Study the damping length as a function of the neutral gas pressure and argue about the nature

of the damping, either collisional or collisionless Landau damping (see Sec. 2.5).

Assuming that the damping is due to collisions, insert the measured damping length l_d into Eq. (12) and use Eq. (11) to estimate the neutral particle density. How do the values compare to the ones obtained from the ideal gas law?

4.4 *Transition into the non-linear regime*

Study the transition of the ion-sound wave to the non-linear regime by increasing the amplitude of the excitation signal at the grid. Visualize the gradual steepening of the originally sinusoidal oscillation from the density perturbation measured with the Langmuir probe. Changing the excitation signal to a step-like function allows to observe a clearer shock front with the small-scale oscillations mentioned in Sec. 2.6. According to Ref. [20], these oscillations have a frequency of half of the ion plasma frequency. Compared this value with the estimation of the ion plasma frequency obtained from the dispersion relation.

5 Learning goals

The following topics should be learned from this experiment:

- General competencies
 - read up on a new topic including a little bit of literature research
 - communicate results in a written report and an oral discussion
 - cooperate (the experiment is carried out by a group of two students)
- Experimental physics
 - data analysis including error handling and curve fitting
 - vacuum generation & handling
 - usage of common laboratory equipment (oscilloscope, signal generator, spectrum analyzer)
 - research experience (students define parameter ranges to be scanned by themselves, data analysis tools are not dictated)
- Plasma physics
 - plasma definition & main properties
 - ion acoustic waves
 - plasma oscillations
 - magnetic mirror
 - electrical discharges in gases

6 Please note

- Vacuum pump: Never operate without assistant!
- Neutral gas pressure: Use values in the range of $p_0 = 1\text{--}100$ mPa.
- Langmuir probe: Handle probe system with care! During the investigation of the ion acoustic waves apply +40 V to the probe.

Acknowledgements

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