Explicit symplectic algorithms based on generating functions for charged particle dynamics

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Dynamics of a charged particle with both time-independent and time-dependent electromagnetic fields in the canonical coordinates is a Hamiltonian system, and the well-known symplectic algorithm has been regarded as the *de facto* method for numerical integration of Hamiltonian systems due to its long-term accuracy and fidelity. For long-term simulations with high efficiency, explicit symplectic algorithms are desirable. However, it is generally believed that explicit symplectic algorithms are only available for sum-separable Hamiltonians, and this restriction limits the application of explicit symplectic algorithms to charged particle dynamics. To overcome this difficulty, we combine the familiar sum-split method and a generating function method to construct second and third order explicit symplectic algorithms for dynamics of charged particle. The generating function method is designed to generate explicit symplectic algorithms for product-separable Hamiltonian with form of $H(\mathbf{p},\mathbf{x})=\mathbf{p}_i f(\mathbf{x})$ or $H(\mathbf{p},\mathbf{x})=\mathbf{x}_i g(\mathbf{p})$. Applied to the simulations of charged particle dynamics, the explicit symplectic algorithms based on generating functions demonstrate superiorities in conservation and efficiency.