Energy-Preserving Integrators Applied to Nonholonomic Systems

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We introduce energy-preserving integrators for nonholonomic mechanical systems. We will see that the nonholonomic dynamics is completely determined by a triple $(\mathcal{D}*, \Pi, \mathcal{H})$, where $\mathcal{D}*$ is the dual of the vector bundle determined by the nonholonomic constraints, Π is an almost-Poisson bracket (the nonholonomic bracket) and $\mathcal{H} : \mathcal{D}* \to \mathbb{R}$ is a Hamiltonian function. For this triple, we can apply energy-preserving integrators, in particular, we show that discrete gradients can be used in the numerical integration of nonholonomic dynamics. By construction, we achieve preservation of the constraints and of the energy of the nonholonomic system. Moreover, to facilitate their applicability to complex systems which cannot be easily transformed into the aforementioned almost-Poisson form, we rewrite our integrators using just the initial information of the nonholonomic system. The derived procedures are tested on several examples: A chaotic quartic nonholonomic mechanical system, the Chaplygin sleigh system, the Suslov problem and a continuous gearbox driven by an asymmetric pendulum. Their performace is compared with other standard methods in nonholonomic dynamics, and their merits verified in practice.