Degenerate Variational Integrators for Magnetic Field Line Flow and Guiding Center Trajectories

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Several of the central dynamical systems encountered in magnetized plasmas - magnetic field line flow and guiding center trajectories, in particular - are described using degenerate Lagrangians; the Legendre transform is not invertible and the Euler-Lagrange equations are only first-order differential equations. This degeneracy has severe implications for constructing variational integrators for these systems. Variational integration theory assumes the Lagranian to be non-degenerate when proving desirable properties like bounded energy error. Instead, variational integrators constructed from degenerate Lagrangians can exhibit numerical instabilities and unbounded energy error. To remedy these issues, this talk advocates a novel tactic: variational integrators for degenerate Lagrangian systems should themselves be degenerate, meaning the discrete Legendre transform is not invertible and the discrete Euler-Lagrange equations depend on a reduced set of initial conditions. These degenerate variational integrators lack the numerical instabilities of earlier constructions and exhibit the desired long-term fidelity. Numerical demonstrations of the benefits will be shown for magnetic field line flow and guiding center trajectories. These developments are intimately related to the outstanding challenge of developing symplectic integrators for Hamiltonian systems in arbitrary (non-canonical) coordinates. The algorithms presented are the first one-step discretely symplectic methods for these non-canonical Hamiltonian systems, representing exciting progress toward this long standing problem.