On Degenerate Lagrangians, Noncanonical Hamiltonians and Dirac Constraints

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Most systems encountered in plasma physics are Hamiltonian and therefore have a rich geometric structure, most importantly symplecticity and conservation of momentum maps. As most of these systems are formulated in noncanonical coordinates, they are not amenable to standard symplectic discretisation methods, which are popular for the integration of canonical Hamiltonian systems. Variational integrators, which can be seen as the Lagrangian equivalent to symplectic methods, seem to provide an alternative route towards the systematic derivation of structure-preserving numerical methods for such systems. However, for noncanonical Hamiltonian systems the corresponding Lagrangian is often found to be degenerate. This degeneracy gives rise to instabilities of the variational integrators which need to be overcome in order to make long-time simulations possible. In this talk we will review recent attempts to devise long-time stable structure-preserving integrators for noncanonical Hamiltonian and degenerate Lagrangian systems. The guiding-centre system will be used to exemplify the problems which arise for such systems and to demonstrate the good long-time fidelity of the newly developed integrators.