

# Metriplectic Integrators for the Landau Collision Operator

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We present a novel framework [1] for discretizing the nonlinear Landau and other collision operators [2, 3, 4] based on their metriplectic formulation [5]. Metriplectic dynamics extends conservative Hamiltonian dynamics by dissipative effects, featuring a bracket formulation similar to Poisson brackets. Using a Galerkin discretization for the velocity space, we transform the infinite-dimensional system into a finite-dimensional, time-continuous metriplectic system. Temporal discretization is accomplished using discrete gradients. Conservation of energy, momentum, and mass, as well as the production of entropy is demonstrated algebraically for the fully discrete system. Due to the generality of our approach, the conservation properties and the monotonic behavior of entropy are guaranteed for finite element discretizations in general, independently of the mesh configuration.

## References

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