Mesh generation with isogeometric approach for 2D tokamak equilibria

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The high anisotropy in magnetized fusion plasmas requires the use of flux aligned meshes coupled with complex and realistic geometries. In this context, we are developing new numerical tools to generate block-structured meshes with regular representation in order to achieve accurate magnetohydrodynamics (MHD) simulations.

Firstly, we solve the nonlinear Grad-Shafranov equation. Then, we compute connected components of the flux graph. Such a construction is well adapted for block-structured grids. However, due to high order derivatives that appear in MHD, one may need regular grids (at least C1) in order to avoid the use of auxiliary variables. For this reason and also to treat complex geometries, we use tools that are well established in Computer Aided Design such as B-splines and NURBS [5] taking advantage of the geometric interpretation of control points. Each connected component is an image of the unit square by B-spline/NURBS mapping, which is constructed according to the Isogeometric analysis approach [4]. Finally, to enforce some constraints or grid properties, like alignment, equidistribution or orthogonality we solve optimization problem, that is formulated as a nonlinear elliptic equation [3].

We will focus in this talk on equidistributed meshes using Monge-Ampère equation with a given monitor function [1, 2], and we will present several examples to illustrate this grid generation process.

References

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