Stochastic and Spectral Particle Methods for Plasma Physics

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Particle methods are very popular for the discretization of kinetic equations like the electrostatic Vlasov–Poisson or the electromagnetic Vlasov–Maxwell systems. In plasma physics the high dimensionality (6D) of the problems raises the costs of grid based codes, favoring the mesh free transport with particles and its inherent adaptivity by following characteristics. The Particle in Cell (PIC) scheme is a Monte Carlo method that couples the particle density to a grid based field solver. This introduces an error that is comprised of three components: the time discretization error, the field discretization error (bias) and the particle noise, given as the variance of the Monte Carlo estimator. We discuss the application of stochastic methods providing a setting in which the random particle noise is quantified and reduced, which includes measures of entropy and variance propagation for the field solver also in unstructured grids. For variance reduction control variates based on parametric shape functions and spectral expansions yield a significant noise reduction. The gyroaverage operator in PIC codes is a commonly known operation such that we introduce conditional Monte-Carlo with it.For physics governed by a small number of Fourier modes the mesh free Particle in Fourier (PIF) method is presented, which conserves energy and momentum. It has a more favorable bias residing in Fourier space and exhibits different computational demands since every particle contributes to every Fourier mode. The superb conservation and stability properties of PIF are demonstrated for electrostatic Vlasov–Poisson and fully electromagnetic multi-species Vlasov–Maxwell in multiple dimensions. With the bounded Legendre and Chebyshev polynomials and the unbounded Hermite functions the concept of spectral particle methods is fully generalized.