

Energy-, momentum-, density-, and positivity-preserving spatio-temporal discretizations for the nonlinear Landau collision operator with exact H-theorems

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This presentation explores energy-, momentum-, density-, and positivity-preserving spatio-temporal discretizations for the nonlinear Landau collision operator. We discuss two approaches, namely direct Galerkin formulations and discretizations of the underlying infinite-dimensional metriplectic structure of the collision integral. The spatial discretizations are chosen to reproduce the time-continuous conservation laws that correspond to Casimir invariants and to guarantee the positivity of the distribution function. Both the direct and the metriplectic discretization are demonstrated to have exact H-theorems and unique, physically exact equilibrium states. Most importantly, the two approaches are shown to coincide, given the chosen Galerkin method. A temporal discretization, preserving all of the mentioned properties, is achieved with so-called discrete gradients. Hence the proposed algorithm successfully translates all properties of the infinite-dimensional time-continuous Landau collision operator to time- and space-discrete sparse-matrix equations suitable for numerical simulation.

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