Solving the guiding-center model on singular mapped disk-like domains

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Abstract

A common strategy in the numerical solution of partial differential equations is to define a uniform discretization of a tensor-product multi-dimensional logical domain, which is mapped onto a physical domain through an appropriate coordinate transformation. By extending this concept to a multi-patch setting, simple and efficient numerical algorithms can be employed on relatively complex geometries. Arguably, the main drawback of such an approach is the inherent difficulty in dealing with singularities of the coordinate transformation.

We offer a comprehensive strategy for dealing with the most common situation of disk-like domains with a singularity at a unique pole, where one edge of the rectangular logical domain is collapsed into one point of the physical domain (e.g. a circle). In such geometries we present robust numerical algorithms for the solution of Vlasov-like hyperbolic equations coupled to Poisson-like elliptic equations. We describe a semi-Lagrangian advection solver that employs a new set of coordinates to integrate the characteristic equations in the whole domain, including the pole, and a finite element Poisson solver based on so-called smooth polar splines (Toshniwal et al., 2017 [1]). A self-consistent 2D guiding-center model for magnetized plasmas, equivalent to a vorticity model for incompressible inviscid Euler fluids, is then considered as a test-case coupling the two solvers. The numerical methods presented show high order rate of convergence in the space discretization parameters, uniformly across the whole domain, with no order reduction effects due to the singularity. Moreover, the techniques described may be also applied to particle-in-cell methods.

References

[1] Deepesh Toshniwal, Hendrik Speleers, René R. Hiemstra, and Thomas J.R. Hughes. Multi-degree smooth polar splines: A framework for geometric modeling and isogeometric analysis. *Computer Methods in Applied Mechanics and Engineering*, 316:1005–1061, 2017. doi:10.1016/j.cma.2016.11.009.

[2] Edoardo Zoni, Yaman Güçlü, and Eric Sonnendrücker. Solving hyperbolic-elliptic problems on singular mapped disk-like domains with the method of characteristics and spline finite elements. *In preparation.*