Two-level Domain decomposition preconditioning for the high-frequency time-harmonic Maxwell equations

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This work deals with preconditioning the time-harmonic Maxwell equations with absorption, where the PDE is discretised using curl-conforming finite-element methods of fixed, arbitrary order and the preconditioner is constructed using Additive Schwarz domain decomposition methods. The theory developed here shows that if the absorption is large enough, and if the subdomain and coarse mesh diameters and overlap are chosen appropriately, then the classical two-level overlapping Additive Schwarz preconditioner (with PEC boundary conditions on the subdomains) performs optimally – in the sense that GMRES converges in a wavenumber-independent number of iterations – for the problem with absorption. Numerical experiments are given that illustrate the theory and its dependence on various parameters. These experiments motivate some extensions of the preconditioners which have better robustness for problems with less absorption, including the propagative case. Numerical experiments illustrate the performance of these on two substantial applications; the first (a problem with absorption arising from medical imaging) shows the robustness of the preconditioner against heterogeneity, and the second (scattering by a COBRA cavity) shows good scalability of the preconditioner with up to 3,000 cores.