High order implicit relaxation schemes for nonlinear hyperbolic systems

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In this work we consider the time discretization of compressible fluid models which appear in gas dynamics or plasma physics for Tokamaks. In general, for the hyperbolic system we use an explicit scheme in time. However, for some applications, the characteristic velocity of the fluid is very small compared to the fastest velocity speed. In this case, to filter the fast scales it is common to use an implicit scheme. The matrices induced by the discretization of the hyperbolic system are ill-conditioned in the regime considered and very hard to invert. In this work we propose an alternative method, based on the BGK relaxation methods. The idea is, to propose a larger and simpler model (here a BGK model [2,3]) depending of the small parameter which approximate the original system. Designing an AP scheme based on splitting method [1] for the BGK model, stable without CFL condition, we obtain at the end a very simple method avoiding matrix inversion and unconditionally stable for the initial model. This method can approximate any hyperbolic models and can be generalized to treat models including additional small diffusion terms.

References

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