# Geometric Discontinuous Galerkin Methods for Fluids and Plasmas

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Most conservative problems in fluid dynamics, plasma physics as well as many other branches of science and engineering have the form of hyperbolic conservation laws that inhibit a Lagrangian and/or Hamiltonian structure. That is their dynamical equations can be obtained from an action principle or a Poisson bracket and a Hamiltonian functional, typically the total energy of the system. Non-conservative problems are usually composed of a conservative (Lagrangian or Hamiltonian) and a dissipative part. In both cases, it is important to preserve the structure of the conservative part in the course of discretisation in order to obtain stable numerical schemes that deliver accurate and reliable simulation results.

We discuss Lagrangian and Hamiltonian structure-preserving discretisation approaches based on high-order Discontinuous Galerkin Spectral Element Methods (DGSEM). We show how these approaches relate to and generalise known energy-stable schemes based on split-forms and summation-by-parts properties. The inviscid Burgers equation and the compressible Euler equations serve as main examples. Generalisations to other important fluid and plasma systems are sketched.

A remarkable property of the proposed approach is that exact mass, momentum and energy conservation can be achieved even if the system of equations is not cast in conservative form and momentum and energy do not explicitly appear as variables.