Generalized Fourier-Hermite method for the Vlasov equation

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The Vlasov equation models the evolution of a plasma in its external and self-consistent fields. It is an advection equation in phase space nonlinearly coupled to Maxwell's equations. We consider a Galerkin discretization with Fourier basis in space and generalized Hermite functions in velocity. The proposed basis allows for exact integration of the elements of Galerkin matrices and resembles the structure of the solution in velocity space.

Two special cases of generalized Hermite functions, so-called symmetrically and asymmetrically weighted Hermite bases, have been introduced in [Holloway, Spectral velocity discretizations for the Vlasov-Maxwell equations, 1996]. It has been shown that the asymmetrically weighted basis allows for exact conservation laws in discrete form [Delzanno, Multi-dimensional, fully-implicit, spectral method for the Vlasov–Maxwell equations with exact conservation laws in discrete form, 2015]. We introduce a theoretical framework for the generalized Hermite functions based on the recently developed HermiteGF theory [Kormann, Lasser, Yurova, Stable interpolation with isotropic and anisotropic Gaussians using Hermite generating function, 2019] and investigate the influence of various parameters of the basis functions on the numerical solution of the Vlasov equation.