

# B-spline approximation of anisotropic diffusion problems: spectral symbols and anisotropic multi-iterative solvers

Marco Donatelli<sup>†</sup>, Carla Manni<sup>‡</sup>, Mariarosa Mazza<sup>†</sup>,  
Hendrik Speleers<sup>‡</sup>

<sup>†</sup>Department of Science and High Technology, University of Insubria, Italy

<sup>‡</sup>Department of Mathematics, University of Rome Tor Vergata, Italy

## Abstract

In plasma physics, tokamak reactors are magnetic toroidal confinement devices that aim to harvest energy from the fusion of small atomic nuclei heated to the plasma state. The heat diffusion in a tokamak shows an intrinsic anisotropy that generates ill-conditioning and consequently various numerical difficulties.

In this talk, we focus on the discretization of anisotropic diffusion operators with B-splines. In order to overcome the difficulties caused by the ill-conditioning, we conduct a detailed spectral study of the resulting coefficient matrices, by using the theory of generalized locally Toeplitz (GLT) sequences [1]. We exploit then the obtained spectral information to design fast anisotropic multi-iterative solvers for the corresponding linear systems combining anisotropic multigrid techniques [2] with GLT-based preconditioners for Krylov methods.

## References

- [1] C. Garoni and S. Serra-Capizzano. *Generalized Locally Toeplitz Sequences: Theory and Applications - Vol I*. Springer Monographs, 2017.
- [2] R. Fischer and T. Huckle. Multigrid methods for anisotropic BTTB systems. *Linear Algebra Appl.* **417**: 314–334, 2006.