An energy- and charge-preserving compatible FEM-PIC scheme for Vlasov-Maxwell equations

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There is a variety of long-time stability criteria for numerical approximations of Vlasov-Maxwell equations, such as the conservation of energy, of momentum, and the preservation of Gauss laws (also referred to as charge-conservation), to name but a few.
A general formulation of space-discretizations preserving some of these quatities has been established [1, 2, 3]. Various time-discretizations have also been described, that ensure these conservation laws, most of them consisting of fully implicit formulations [3, 4, 5]. Another approach [6] proposes a semi-implicit scheme that conserves energy exactly while sparing the cost of a non-linear solver. Nevertheless, Gauss laws are not preserved in this method.
We present a new scheme adapting the latter formulation to the context of compatible finite-element particle-in-cell (FEM-PIC) approximations, with the additional feature of Gauss laws preservation. Reconciling both energy-conserving approach in [6] and the charge-conserving one in [1] is not straightforward : the latter involves a time-averaged current deposition, which is a priori incompatible with the economy of a non-linear resolution.
The method we describe conserves both charge and energy, and keeps a computational complexity close to the explicit PIC algorithm. It consists of an energy-conserving prediction step followed by a Gauss-preserving correction of electric fields and an energy balance of particles' velocties. We show that such corrective step is possible when a time condition is satisfied : when necessary, a dynamic adaptation of the time-step suffices to fulfill this criterion.

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