Stochastic variational integrators for collisional Vlasov equations

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Variational integrators are derived for structure-preserving simulation of stochastic forced Hamiltonian systems. The derivation is based on a stochastic discrete Hamiltonian which approximates a type-II stochastic generating function for the stochastic flow of the Hamiltonian system. The generating function is obtained by introducing an appropriate stochastic action functional and considering a stochastic generalization of the deterministic Lagrange-d'Alembert principle. Our approach presents a general methodology to derive new structure-preserving numerical schemes. The resulting integrators satisfy a discrete version of the stochastic Lagrange-d'Alembert principle, and in the presence of symmetries, they also satisfy a discrete counterpart of Noether's theorem. Furthermore, mean-square and weak Lagrange-d'Alembert Runge-Kutta methods are proposed and tested numerically to demonstrate their superior long-time numerical stability and energy behavior compared to non-geometric methods. The Vlasov-Fokker-Planck equation is considered as one of the numerical test cases, and a new geometric approach to collisional kinetic plasmas is presented.