A conservative semi-implicit FEM-PIC approximation of Vlasov-Maxwell motivated by Klystron simulation

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The subject of this talk is twofold. First, we present a compatible finite-elements particle-in-cell (FEM-PIC) scheme for Vlasov-Maxwell equations called ChECSIM. This semi-implicit time-discretization preserves the energy and the Gauss laws. Next, we derive a new approximation of Vlasov-Maxwell equations when the system is close to the time-periodic regime, where ChECSIM fits in.

Preserving the energy and the Gauss laws at discrete level lies among the variety of long-time stability criteria for numerical approximations of Vlasov-Maxwell system. In the particular framework of compatible FEM-PIC methods, a general formulation of charge-conserving discretizations has been established [1, 2, 3], which ensures Gauss laws preservation. More generally, various schemes feature the conservation of the energy [4, 5], sometimes along with the preservation of the Gauss laws [3, 6]. Almost all of these approaches consist of fully implicit formulations, but [5] exhibits a noticeable semi-implicit scheme sparing the cost of a non-linear solver. However, the Gauss laws are not preserved in this case.

Our new method, ChECSIM, proposes to merge the latter energy-conserving semi-implicit formulation with the charge-conserving compatible FEM-PIC approximations. The new scheme shows interesting long-time stability properties with a low complexity. Its robustness also allows to take long time-steps, permitting faster executions.

The applicative goal of our method is the numerical simulation of micro-wave amplifiers such as the Klystron. In this device, the oscillations of a radio-frequency signal are sustained by the flow of an accelerated electronic beam through metallic cavities. The usual numerical tools rely on a time-harmonic formulation of Maxwell equations, allowing quick computations. However, the ever more challenging development of this technology has revealed some limits of such restrained model, and the need for efficient solvers involving the complete Maxwell equations has appeared.

This field of application has motivated a new discretization of Vlasov-Maxwell equations that we shall present in the second half of the talk. The Quas-Har formulation is specifically designed for the cases where the system tends to the time-harmonic regime, or remains close to it. It is based on a decomposition of the electromagnetic field in a time-periodic part and a noisy remainder. The periodic component is obtained by a time-harmonic Maxwell solver, while the noise-remainder is computed by a FEM-PIC approximation such as ChECSIM.

References

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