Model Reduction of Hamiltonian Systems, Symplectic Autoencoders and Optimization on Manifolds

Structure-preserving discretizations of Hamiltonian PDEs yield high-dimensional Hamiltonian ODEs. The solution of these systems can be prohibitively expensive, which is why practitioners often employ *Model Reduction* techniques. One of these is the *Reduced Basis Method* (RBM), whose aim is the construction of a lower dimensional dynamical system that captures the high-dimensional dynamics.

The most common RBM is *Proper Orthogonal Decompostion* (POD), that finds the ideal reduced system under the assumption that the higher-dimensional system depends linearly on the lower-dimensional one. For most systems this is however not true and this is why neural network-based approaches have become popular in recent years; these are not constrained by the linear nature of the POD.

In addition, past investigations have also shown the importance of considering the Hamiltonian structure of the high-dimensional system when constructing the low-dimensional basis. The symplectic counterpart to POD is known as *Proper Symplectic Decomposition* (PSD). It has proven remarkably effective for e.g. the linear wave equation, but fails for stronly non-linear systems.

In this presentation we want to elaborate on the shortcomings of existing approaches, especially regarding the solution of non-linear Hamiltonian PDEs, and present an alternative. This will among other things involve Optimization on Manifolds, a topic that has generated a lot of interest in recent years in the Machine Learning community.