

Reduced Basis Methods and Transport Mappings

Tobias Blickhan^{1,2}

¹ Max-Planck-Institut für Plasmaphysik

² Technische Universität München

Abstract

A number of applications, such as inverse problems, optimization, or real-time control, require the fast solution of many partial differential equations of similar form, varying for example in parameter values, initial conditions or geometry.

For such cases, computation time can be reduced by building surrogate or reduced models of these systems upfront to leverage the similarity of the ensuing simulations. A well established model reduction technique, the reduced basis method, represents the solution of the parametrized equations describing the system using modes (recurring features) obtained from training runs for a subset of the parameter space. However, advection-dominated problems are difficult to tackle this way: the n-width of the set of solutions to these problems is very large.

In this talk, we discuss a reduced basis approach to the Vlasov-Poisson system, discussing how its invariants can be retained in a reduced basis. Furthermore, we discuss how Linear Optimal Transportation (also known as Monge Embeddings) can be used to overcome the large n-width. In LOT, a distribution f is mapped to the optimal transport map $T_{\sigma \rightarrow f}$ that takes a fixed reference distribution σ to f . While the set of solutions $\{f_i\}_{i=1,2,\dots}$ cannot be approximated well by a small number of modes, this is possible for the set of maps $\{T_{\sigma \rightarrow f_i}\}_{i=1,2,\dots}$. This leads to a method in the spirit of snapshot remapping that is illustrated with numerical examples.