

Text books on fusion research

M. Kaufmann: Plasmaphysik und Fusionsforschung“ , Springer 2013
(German only)

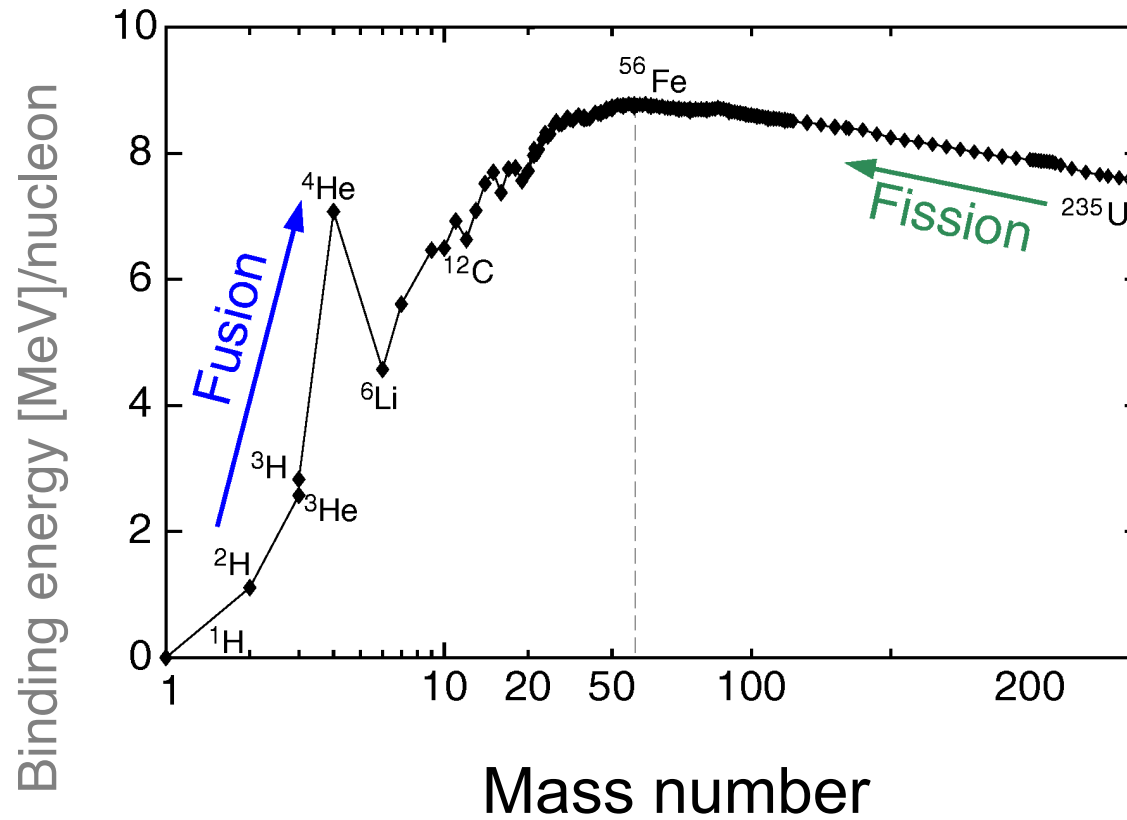
R.J.Goldston, P.H. Rutherford „Introduction to Plasma Physics“
(CRC Press English 1995, Springer German 1998)

F. Chen Introduction to Plasma Physics and Controlled Fusion (Springer
2019)

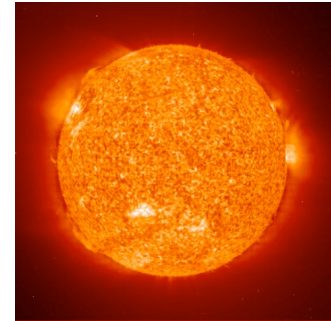
J. Freidberg. Plasma Physics and Fusion Energy (Cambridge 2008)

J. Wesson „Tokamaks“ (Oxford 2011)

Gain of energy due to nuclear fusion



How does the sun produce energy?



- **18th century: sun burns coal?**

Given the mass $M_s \sim 2 \cdot 10^{30}$ kg, sun's life time would be ~ 4600 years (but age of the earth already known to be a few billion years)

- **19th century (Helmholtz): gravitational energy?**

Sun makes use of gravitational energy, released by slow contraction: life time ~ 19 Mio years, still not enough

- **20th century (Rutherford 1923) fusion of 4 protons?**

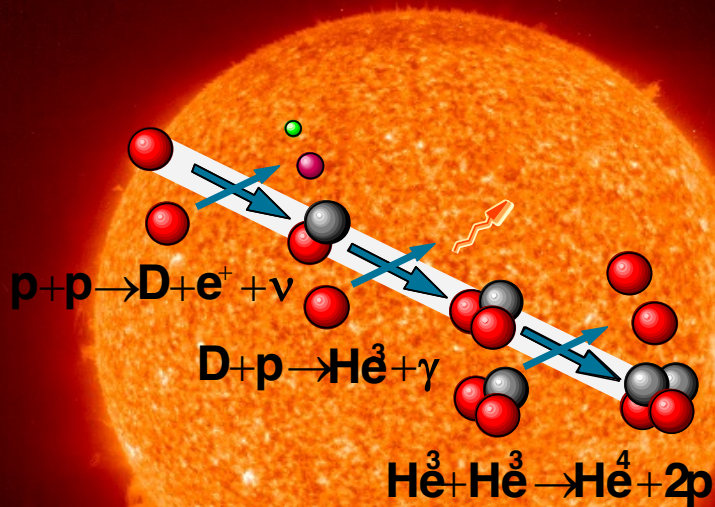
But low probability of simultaneous collision of 4 protons

Tunnel effect not yet know, thus temperature was too low given the large Coulomb repulsion

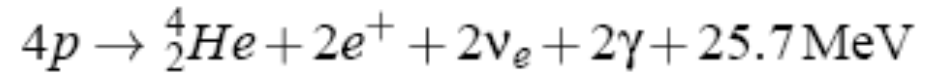
$$E_{pot} = \frac{Z_1 \cdot Z_2 \cdot e^2}{4\pi \cdot \epsilon_0 \cdot r_m} \quad \sim 400 \text{ keV for } Z=1$$

Proton chain in the sun and in small stars ($T < 2\text{keV}$)

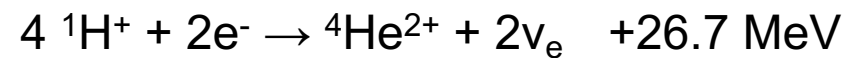
SOHO EIT, He II line, 304 Å
May 18, 1996 at 20:02



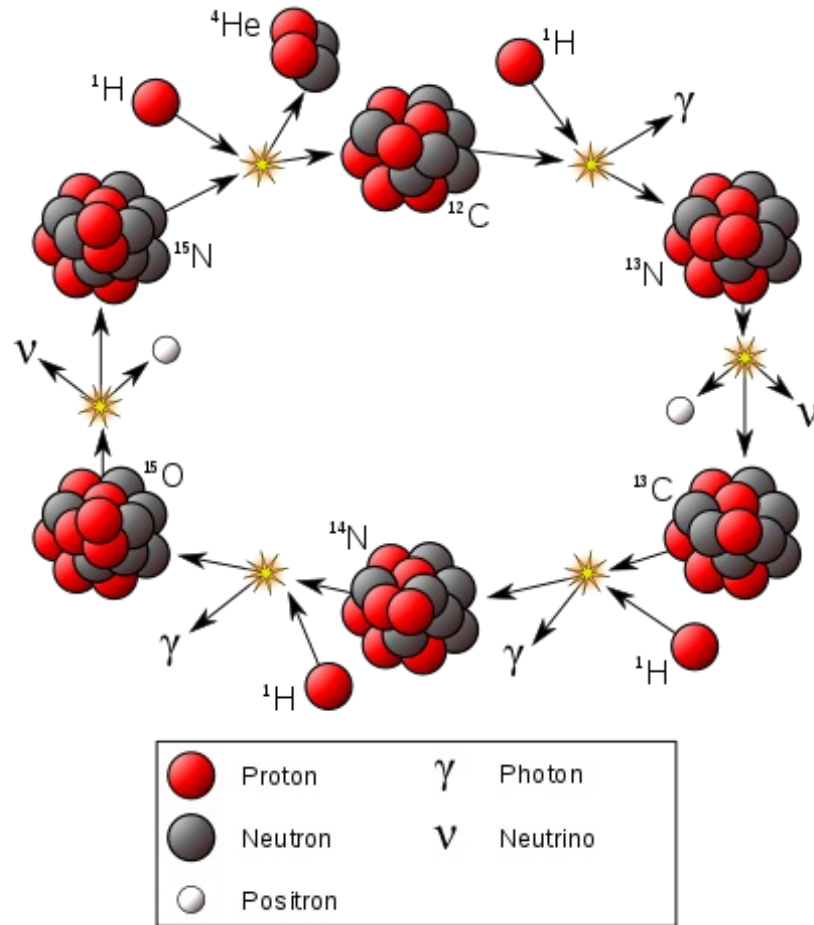
First reaction very slow, as weak interaction involved



600 Mio tons per second protons
fused to 596 Mio tons ${}^4\text{He}$



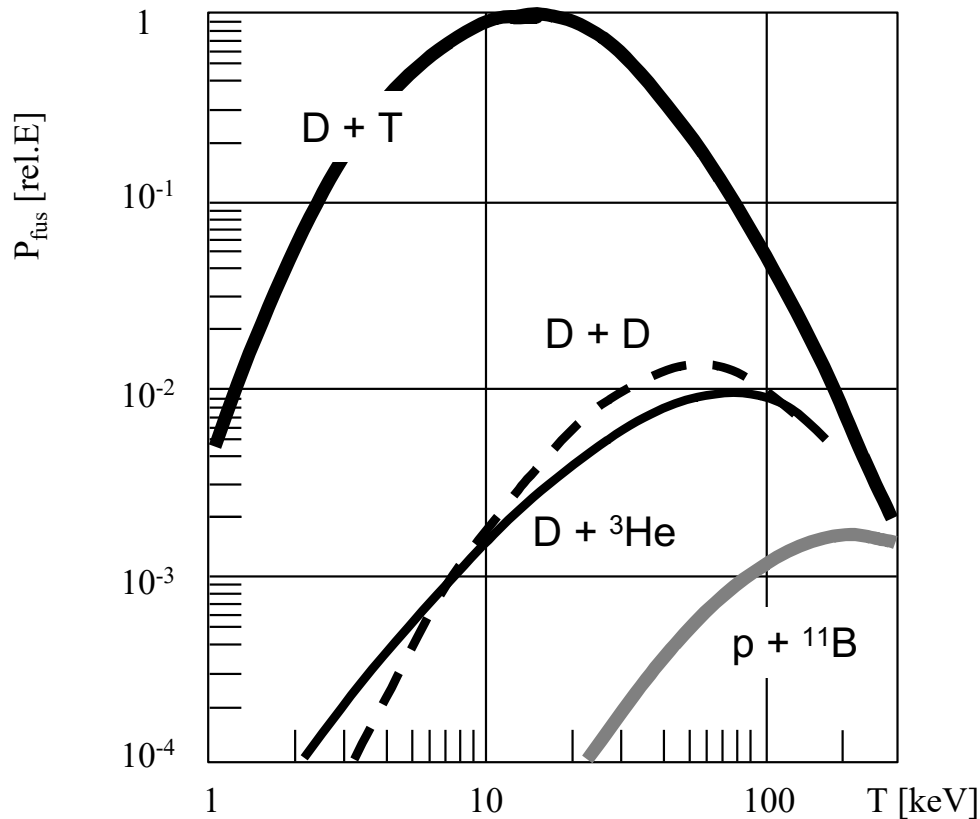
At higher temperatures fusion to heavier elements



Energy $\sim T^{20}$
(pp chain $\sim T^4$)

Bethe-Weizsäcker-cycle for hot stars ($T > 30$ Mio K), above 1.5 times the mass of the sun

Possible fusion reactions: fusion power density

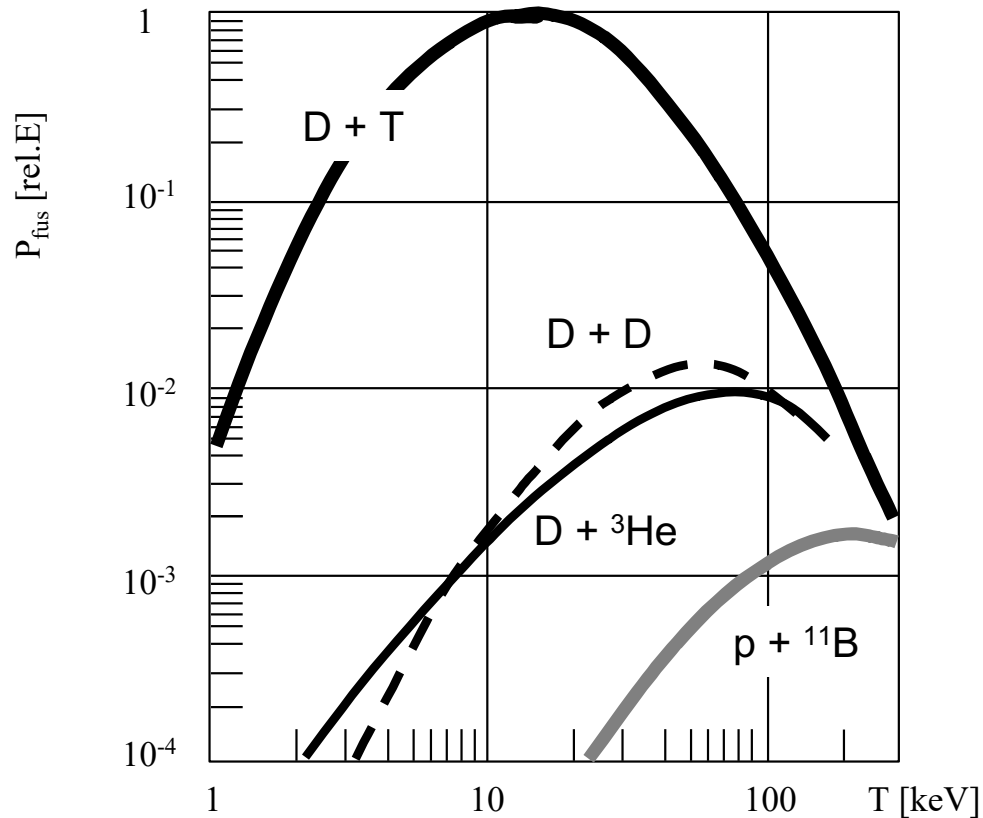


Cross section for D-T reaction about 25 orders of magnitude larger than that of proton chain

- | | | |
|-----|-----------------------|---|
| (1) | $D + D$ | $\Rightarrow {}^3\text{He} (0,8) + n (2,5)$ |
| | | $\Rightarrow T (1) + p (3)$ |
| (2) | $D + {}^3\text{He}$ | $\Rightarrow {}^4\text{He} (3,7) + p (14,7)$ |
| (3) | $D + T$ | $\Rightarrow {}^4\text{He} (3,5) + n (14)$ |
| (4) | $p + {}^{11}\text{B}$ | $\Rightarrow 3 \times {}^4\text{He} (3 \times 2,9)$ |

in parentheses amount of energy released (in MeV)

Possible fusion reactions: fusion power density

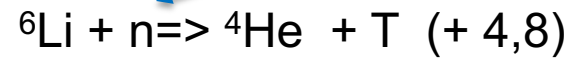
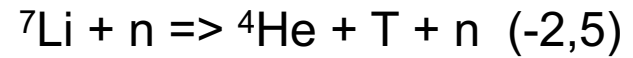
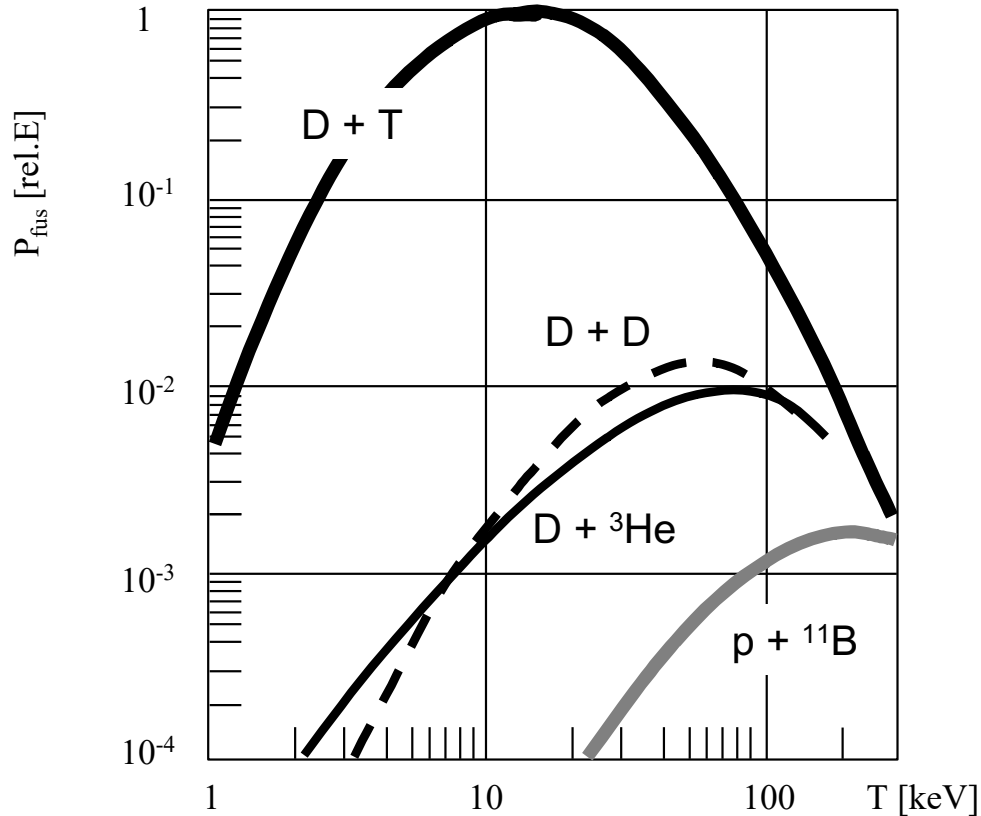


- | | | |
|-----|---------------------|---|
| (1) | D + D | \Rightarrow ^3He (0,8) + n (2,5) |
| | | \Rightarrow T (1) + p (3) |
| (2) | D + ^3He | \Rightarrow ^4He (3,7) + p (14,7) |
| (3) | D + T | \Rightarrow ^4He (3,5) + n (14) |
| (4) | p + ^{11}B | \Rightarrow 3 \times ^4He (3 \times 2,9) |

most promising

Recently more often
promised, but very unlikely

Tritium breeding

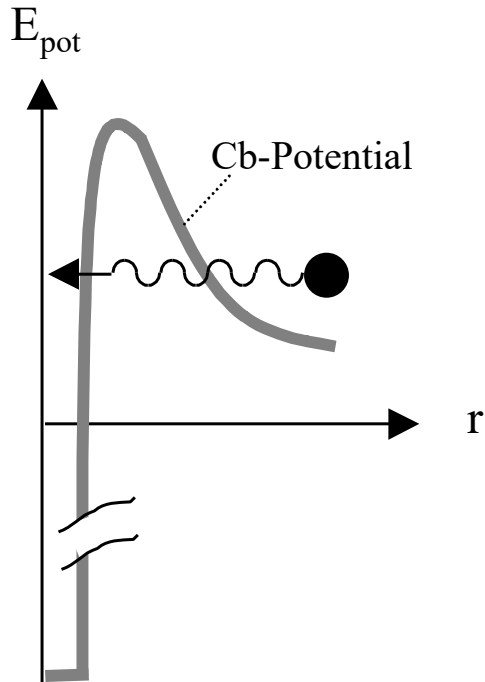


Lithium isotopes:

- 93% ${}^7\text{Li}$
- 7% ${}^6\text{Li}$

Neutron multiplier: Pb or Be

Miminal energy needed to overcome Coulomb repulsion



$$E_{pot} = \frac{Z_1 \cdot Z_2 \cdot e^2}{4\pi \cdot \epsilon_0 \cdot r_m}$$

$$r_m \sim 4 \cdot 10^{-15} \text{ m}$$

$$E_{DT} \sim 0,4 \text{ MeV}$$

(for DT)

Only after discovery of the tunnel effect fusion well processes understood:

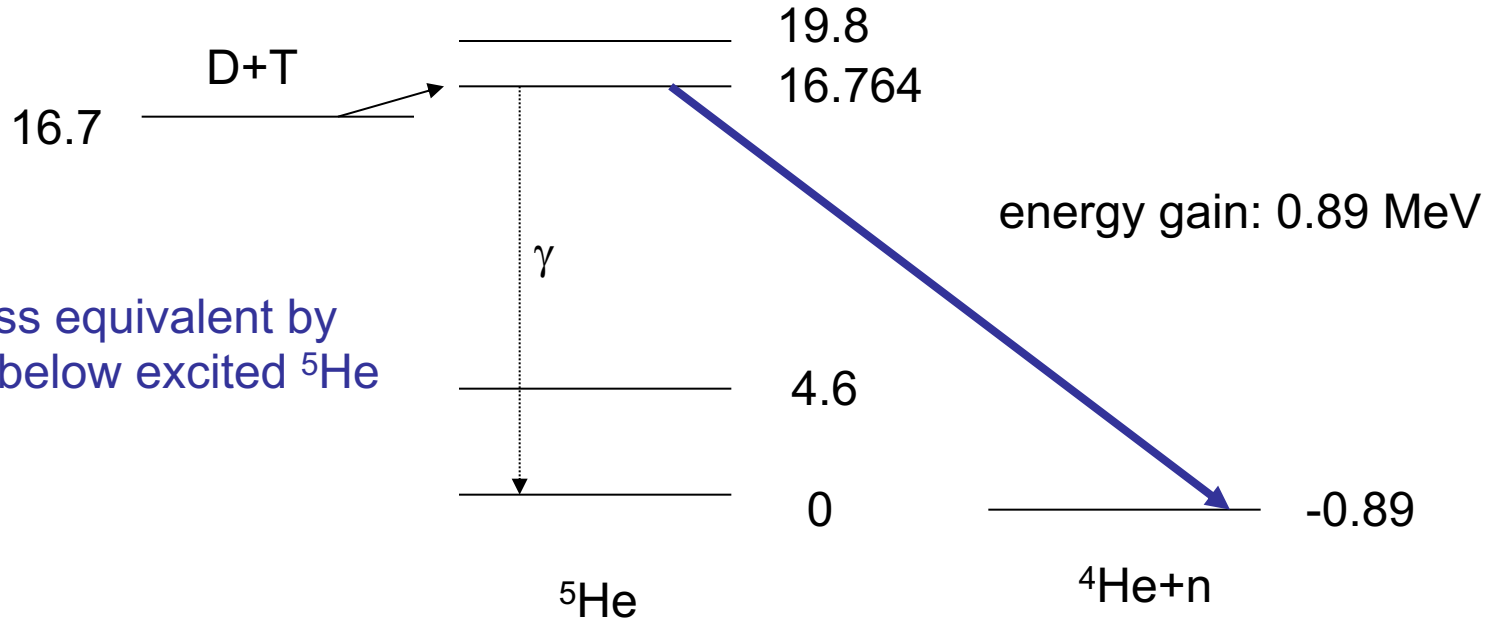
$$W_{tunnel} = const \cdot e^{-\frac{Z_1 \cdot Z_2}{v_{rel}}}$$

already significant reaction rates at 10...20 keV

Why has the reaction D-T a large cross section at relatively low temperatures?

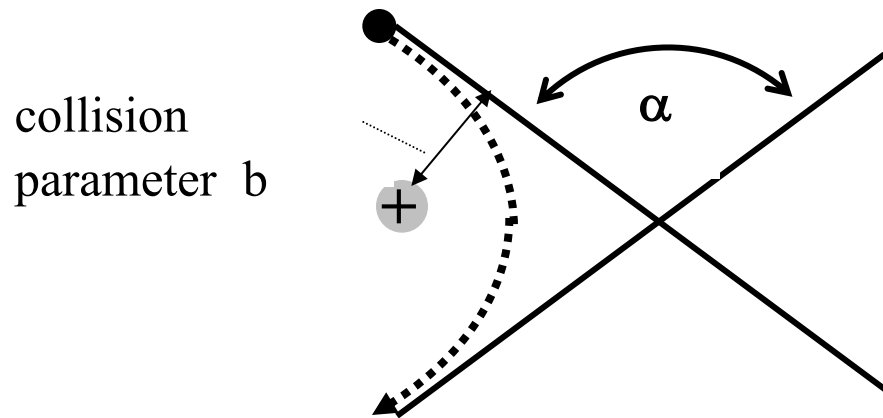
resonant mechanism!

energy level diagram of unstable ${}^5_2\text{He}$



D-T mass equivalent by 64 keV below excited ${}^5\text{He}$ state

Fusion vs. Coulomb Collisions (or why we need a thermal plasma)



collision
parameter b

example:
e-collision with ion (charge Z)

Rutherford's formula with $m_e \ll m_i$:

$$\tan(\alpha / 2) = \frac{W_{pot}}{2W_{kin}} = \frac{e \cdot Z \cdot e}{(4\pi\epsilon_0) \cdot m_e v^2 \cdot b}$$

cross section for scattering by 90° :

$$\sigma_{90} = \pi \cdot b_{90}^2 = \frac{\pi \cdot Z^2 \cdot e^4}{(4\pi\epsilon_0)^2 \cdot 4 \cdot (W_{kin})^2}$$

\Rightarrow Coulomb cross section depends strongly on particle energy : $\sim 1/W_{kin}^2$

T beam on target of deuterium ?

Assume T energy: 100 keV

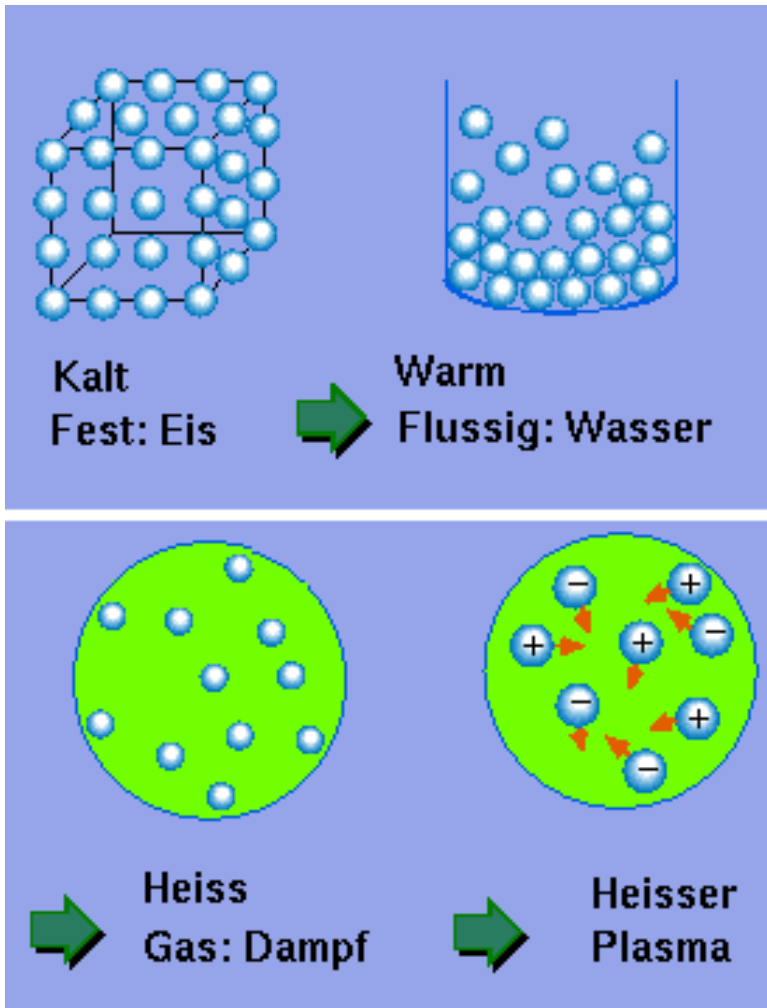
Mean free path for fusion reactions: $2.9 \cdot 10^{27} \text{ m/n[m}^{-3}\text{]}$

Mean free path for Coulomb collision: $1.9 \cdot 10^{22} \text{ m/n[m}^{-3}\text{]}$

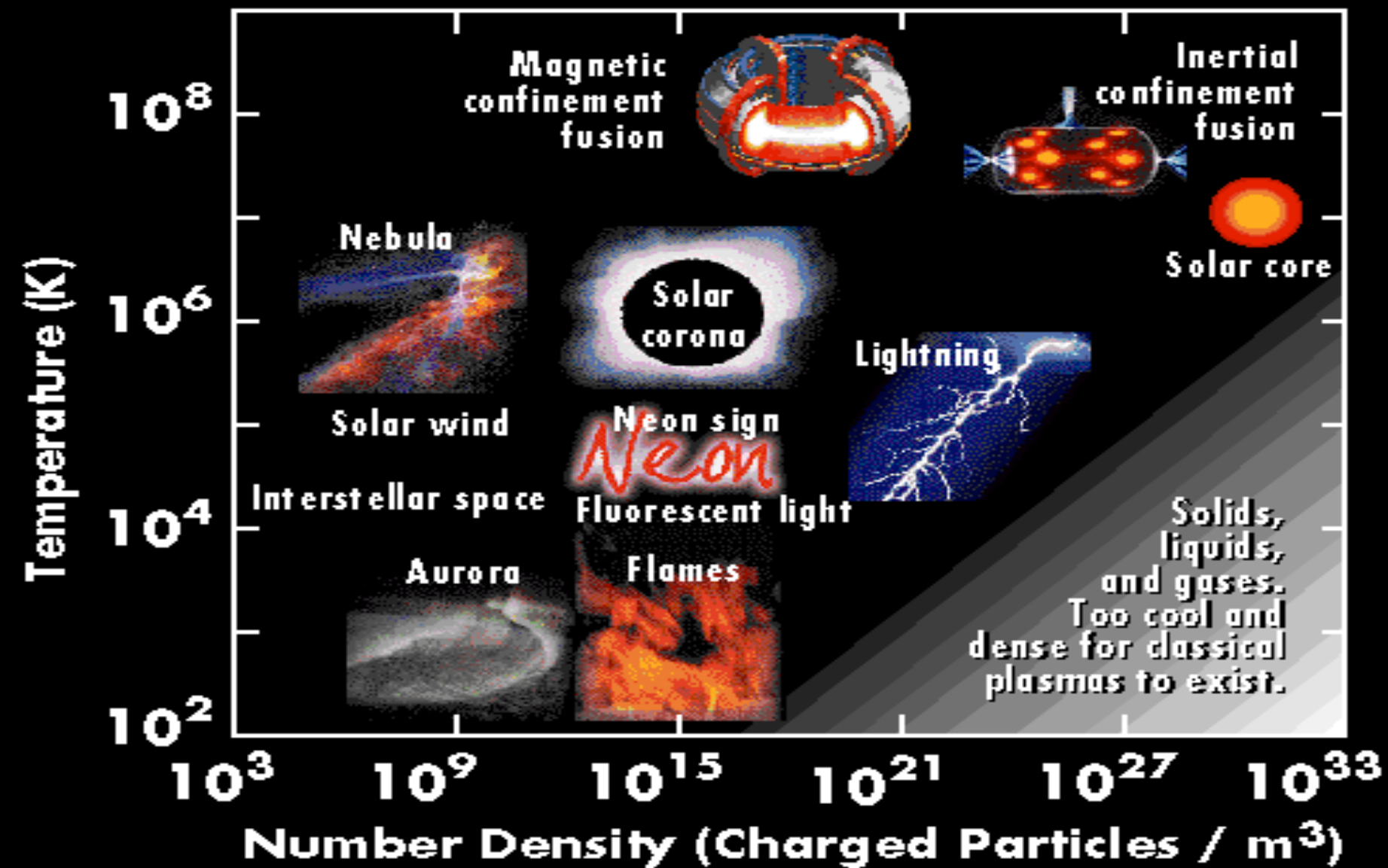
Orders of magnitude more Coulomb collisions than fusion reaction

Need to confine “thermal” plasma

Plasma, the fourth state of matter

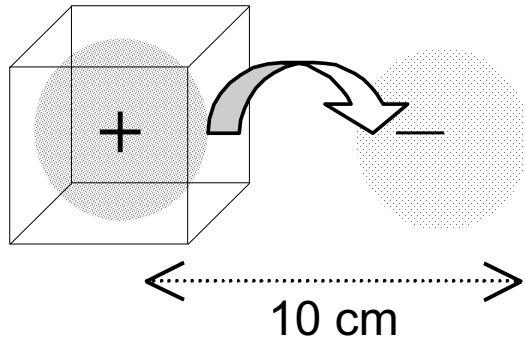


- free electrons and ions
- good electrical conductivity
- long range forces
- high thermal conductivity
- forces due to magnetic fields



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 Images courtesy of DOE fusion labs, NASA, and Steve Albers.

Quasi neutrality



Poisson equation

$$\Delta U = -\frac{1}{\epsilon_0} \rho(r) = -\frac{e}{\epsilon_0} (Zn_i(r) - n_e(r))$$

For particle densities of about 10^{16} m^{-3} a shift of all electrons by about 10 cm corresponds to a voltage of about 2 Mio V

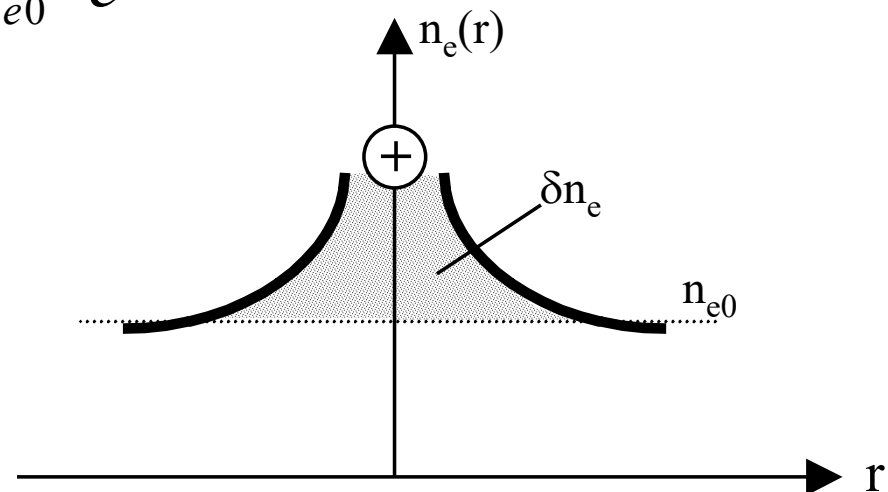
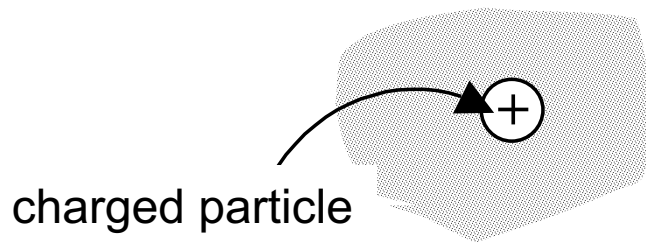
Macroscopic (> mm ... cm) charge separation in plasmas impossible!

$$n_i - n_e \approx 0$$

Screening

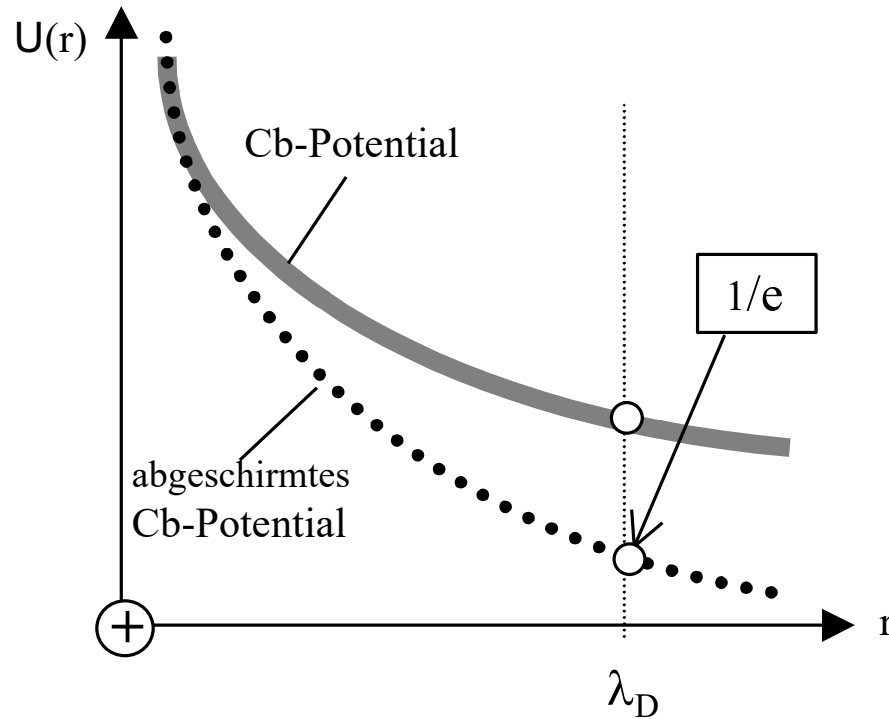
Local deviations from quasi neutrality (microscopic scales) -> re-ordering of charged particles such that electric fields are screened on macroscopic scales

$$n_e(r) = n_{e0} \cdot e^{|E_{pot} / E_{kin}|} = n_{e0} \cdot e^{\frac{e \cdot U(r)}{kT_e}}$$



Also external fields are screened

Debye Screening



$r \ll \lambda_D$: Coulomb-Potential

$r \gg \lambda_D$: outside Debye length only very small electrical

Inside Debye length charge separation possible

-> Plasma extensions has to be large compared to Debye length

$$N_e = n_e \cdot \frac{4\pi}{3} \cdot \lambda_D^3 = \left(\frac{\epsilon_0}{e^2} \right)^{3/2} \cdot \frac{kT_e^{3/2}}{\sqrt{n_e}}$$

	λ_D	N_D
fusion plasma (10 keV, 10^{20}m^{-3})	75 μm	$2 \cdot 10^8$
technical plasma (5 eV, 10^{17}m^{-3})	50 μm	$6 \cdot 10^4$
astrophysical Plasma (1 eV, 10 m^{-3})	75 km	$5 \cdot 10^{11}$
dense plasma (1,5 eV, 10^{24}m^{-3})	0,01 μm	3 <i>=limit to non-ideal plasmas</i>

1 eV = 11600 K bzw. 10 keV~100 Mio K

Plasma properties

$$\mathbf{n}_e = Z \mathbf{n}_i = \text{strict quasi neutrality}$$

$$\mathbf{L} \gg \lambda_D = \text{system size large compared to Debye length}$$

$$\mathbf{N}_{\text{tot}} \gg \mathbf{N}_D = \text{sufficient number of particles in the system}$$

To overcome the Coulomb barrier, we need a thermal plasma as the cross section for Coulomb collisions is larger than the fusion cross section (>100 times)!

Fusion cross section in a thermal plasma:

$$\langle \sigma(u) u \rangle = \frac{(m_a m_b)^{3/2}}{(2\pi kT)^3} \int d^3 v_a \int d^3 v_b \sigma(|\mathbf{v}_a - \mathbf{v}_b|) \exp\left[-\frac{m_a v_a^2}{2kT}\right] \exp\left[-\frac{m_b v_b^2}{2kT}\right]$$

Centre of mass-system: $E_{kin} = \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_b v_b^2 = \frac{1}{2} (m_a + m_b) V^2 + \frac{1}{2} m_r u^2$

$$\langle \sigma(u) u \rangle = \frac{(m_a m_b)^{3/2} (4\pi)^2}{(2\pi kT)^3} \int_0^\infty dV V^2 \exp\left[-\frac{(m_a + m_b) V^2}{2kT}\right] \int_0^\infty du \sigma(u) u^3 \exp\left[-\frac{m_r u^2}{2kT}\right]$$

$$\int x^2 e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{4a^3} \quad E_r = 1/2 m_r u^2$$

Fusion cross section in a thermal plasma:

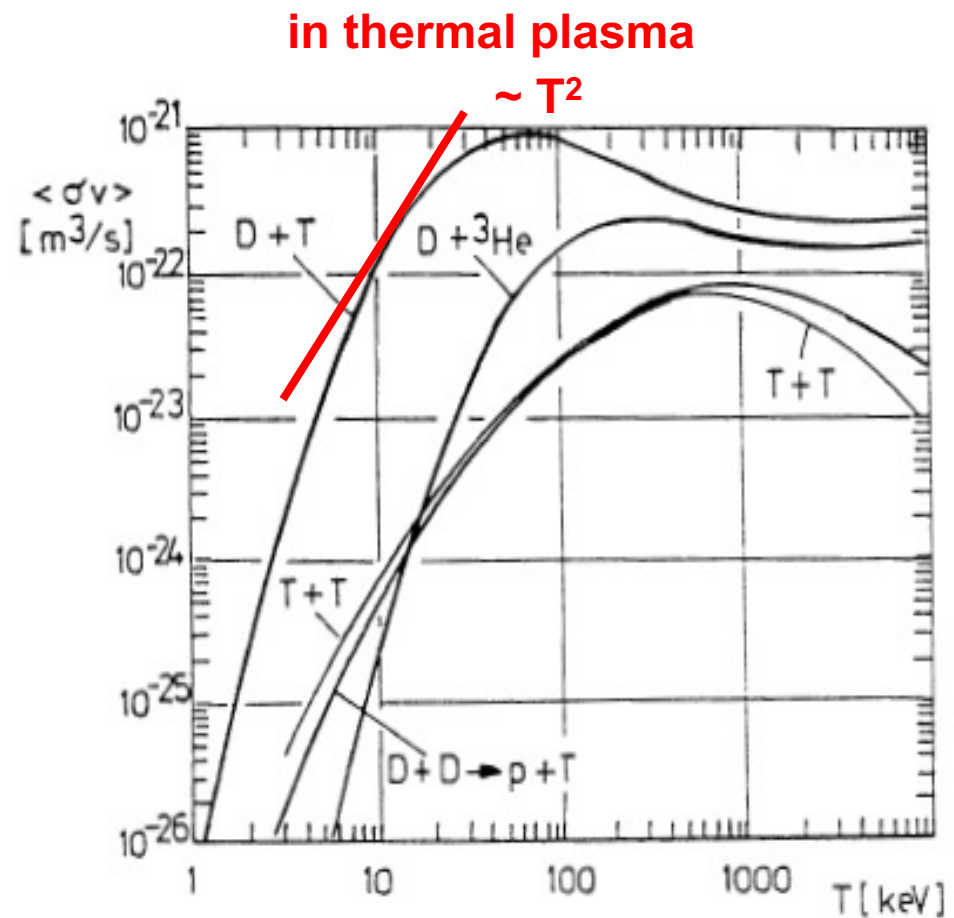
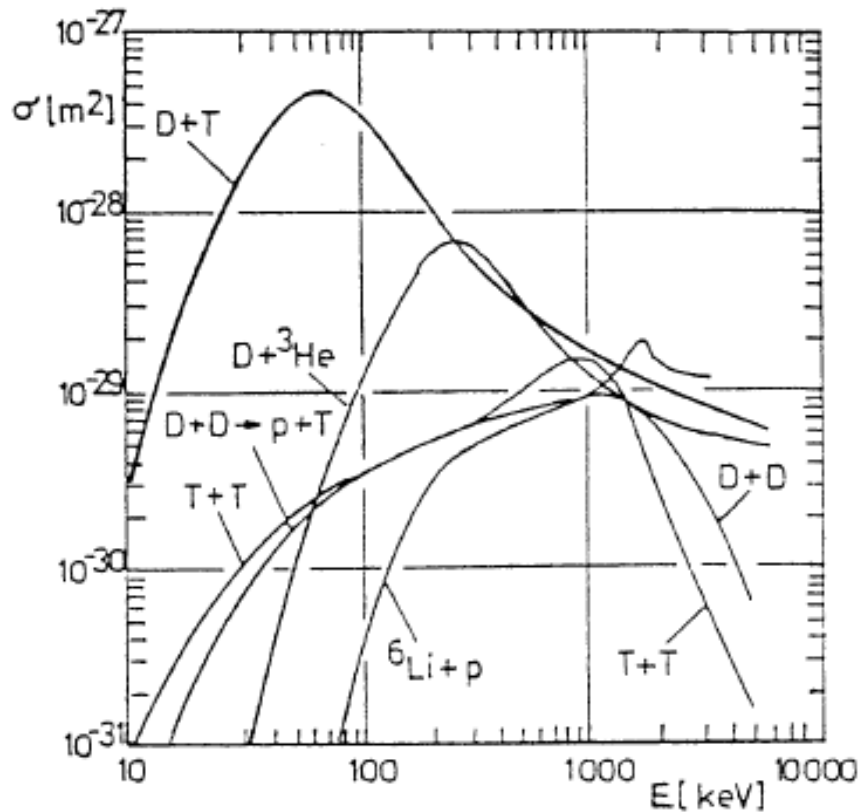
$$\langle \sigma(u) u \rangle = \frac{(m_a m_b)^{3/2} (4\pi)^2}{(2\pi kT)^3} \int_0^\infty dV V^2 \exp \left[-\frac{(m_a + m_b) V^2}{2kT} \right] \int_0^\infty du \sigma(u) u^3 \exp \left[-\frac{m_r u^2}{2kT} \right]$$

$$\int x^2 e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{4a^3} \quad E_r = 1/2 m_r u^2$$

$$\langle \sigma(u) u \rangle = \frac{4}{(2m_r \pi)^{1/2} (kT)^{3/2}} \int_0^\infty dE_r \sigma(E_r) E_r \exp \left(-\frac{E_r}{kT} \right)$$

Thermal plasma with 10 20 keV needed for gaining energy from fusion reactions

Cross section:



Deuterium-Tritium-Fusion

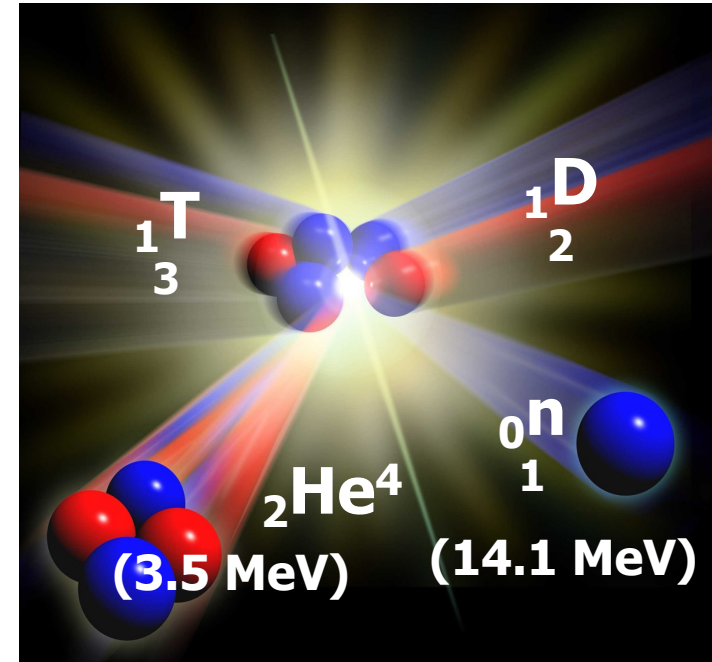
Fusion power $P_{fus} = \frac{n_1 n_2}{2} \langle \sigma v \rangle Q$

D-T fusion:

highest fusion power for $n_D:n_T = 1:1$

$$n_{DT} = n_D + n_T$$

$$P_{fus,DT} = \frac{n_{DT}^2}{4} \cdot \langle \sigma v \rangle_{DT} \cdot Q_{DT}$$



How is energy gain Q_{DT} distributed to fusion products?

Deuterium-Tritium-Fusion

How is energy gain Q_{DT} distributed to fusion products?

Follows from momentum and energy conservation

momentum conservation:

$$m_{He} \cdot v_{He} \approx -m_n \cdot v_n$$

Energy conservation:

$$\frac{m_{He}}{2} v_{He}^2 + \frac{m_n}{2} v_n^2 \approx Q_{DT} = 17,5 MeV$$

As momentum of the particles prior to fusion reaction is negligible

Deuterium-Tritium-Fusion: energy of fusion products

momentum conservation:

$$m_{He} \cdot V_{He} \approx -m_n \cdot V_n$$

$$m_{He}^2 v_{He}^2 = m_n^2 v_n^2$$

$$\frac{m_{He} v_{He}^2}{2} = \frac{m_n^2 v_n^2}{2 m_{He}}$$

Energy conservation:

$$\frac{m_{He}}{2} v_{He}^2 + \frac{m_n}{2} v_n^2 \approx Q_{DT} = 17,5 \text{ MeV}$$

$$\frac{m_n^2 v_n^2}{2 m_{He}} + \frac{m_n v_n^2}{2} = Q_{DT} = 17,5 \text{ MeV}$$

$$\left(1 + \frac{m_n}{m_{He}}\right) E_n = \left(1 + \frac{1}{4}\right) E_n = 17,5 \text{ MeV}$$

$$E_n = 4/5 \times 17,5 \text{ MeV} = 14.1 \text{ MeV}$$


$$E_{He} = 1/5 \times 17,5 \text{ MeV} = 3.5 \text{ MeV}$$

Energy gain is distributed to fusion products according to:

$$\frac{E_{He}}{E_n} = \frac{m_n}{m_{He}} = 1 : 4$$

Power balance – Lawson criterion

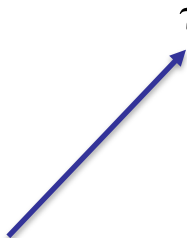
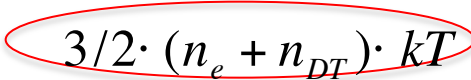
power gain (of the plasma):

$$P_{fus,charged} = \frac{n_{DT}^2}{4} \langle \sigma v \rangle_{DT} 0.2 Q_{DT}$$


Only the energy carried by the alpha particles heat the plasma
(the neutrons don't interact with the plasma)

total thermal energy of the plasma

power loss :

$$P_{loss} \equiv \frac{3/2 \cdot (n_e + n_{DT}) \cdot kT}{\tau_E}$$


τ_E : Energy confinement time (characteristic cooling down time)

Power balance – Lawson criterion

$$P_{fus,charged} = \frac{n_{DT}^2}{4} \langle \sigma v \rangle_{DT} 0.2 Q_{DT}$$

$$P_{loss} \equiv \frac{3/2 \cdot (n_e + n_{DT}) \cdot kT}{\tau_E}$$

Fusion cross section: $\langle \sigma v \rangle_{DT} \sim T^2$ (at $T \sim 10 \dots 20$ keV)

Quasi neutrality: $n_e = n_{DT}$

$$P_{fus,charged} \sim \frac{n_e^2}{4} T^2 Q_{DT}/5$$

$$P_{loss} \sim \frac{3n_e}{\tau_E} kT$$

power balance :

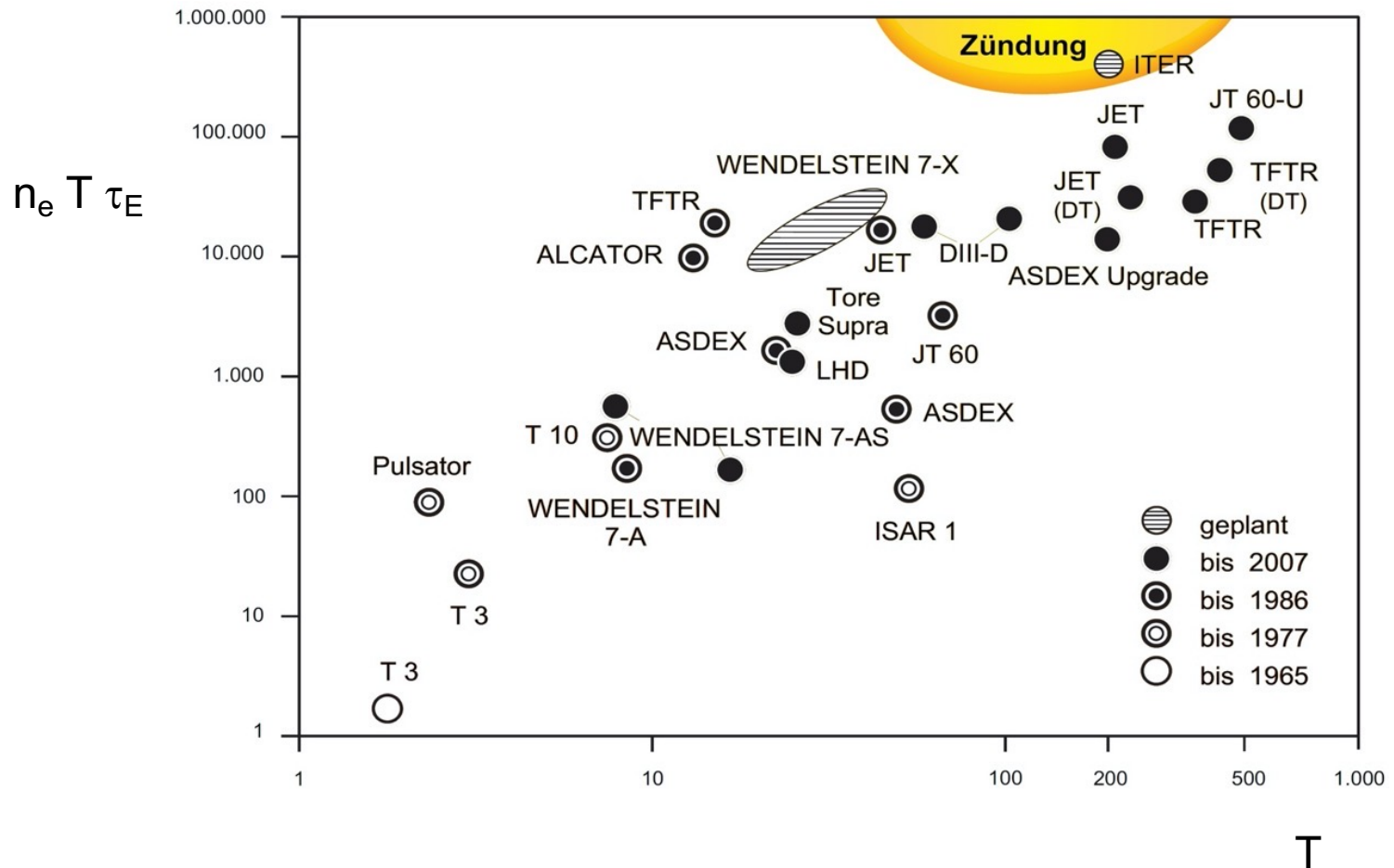
$$P_{fus,charged} = P_{loss}$$

Lawson criterion:

$$n_e \cdot T \cdot \tau_E = const$$

$$\langle n_e \rangle \cdot T_i(0) \cdot \tau_E = const$$

Figure of merit in fusion research



Inertial fusion

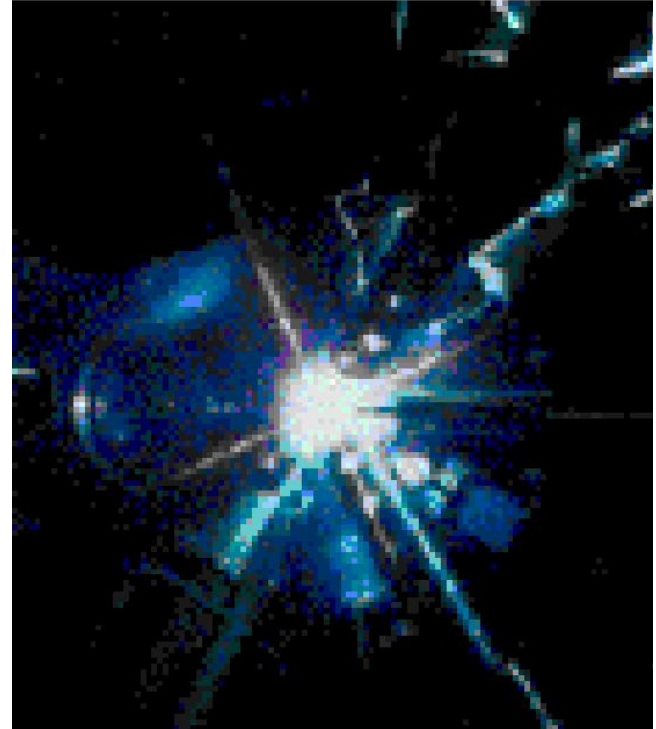
$n\tau_E$ and T are fixed, but pressure $p=nT$ is free to choose

Inertial fusion:

- Fast heating with laser or Heavy ion beam
- confinement due to inertia (ion sound wave time scale)
- Miniature explosion

n large (10^{31} m^{-3}), τ_E small (10^{-10} s)

⇒ pressure comparable to the solar core (!)



Ignition condition changes in the presence of impurities

Lawson criterion

$$n_e \cdot T \cdot \tau_E = \text{const}$$

achieved for "pure" plasmas: $n_e = n_{DT}$

- Actually what counts for fusion reaction is n_{DT}
- In reality, not only D,T ions, but at least also He, and in addition impurities due to interaction with wall materials

With impurity ions Lawson criterion will be modified by impurity ions for two reasons:



Dilution:

at same plasma pressure
 $p \sim n_e T$ less D-T ions

Radiation:

mainly bremsstrahlung
(but also line radiation)

Ignition condition: Effect of dilution

Quasi neutrality, more generally: $n_e = \sum_Z Z n_{z,i} \rightarrow 1 = \sum_Z Z n_{z,i}/n_e$

If only D and T ions: $1 = f_{DT} \quad f_{DT} \equiv \frac{n_D + n_T}{n_e}$

„dilution“ due to impurity ions:

$$1 = f_{DT} + 2f_{He} + \dots + 6f_C = f_{DT} + 2f_{He} + \sum_{i \geq 3} Z_i f_i$$

$$f_{DT} \equiv \frac{n_D + n_T}{n_e}; \quad f_{He} \equiv \frac{n_{He}}{n_e}; \quad f_C \equiv \frac{n_C}{n_e}$$

Example:

10% He, 2% C

$$1 = f_{DT} + 0.2 + 0.12$$

$$f_{DT} = 0.68$$

Ignition condition: Effect of dilution

at the same pressure less D-T-ions with significant influence on fusion power:

$$P_{fus,charged} = \frac{n_{DT}^2}{4} \langle \sigma v \rangle_{DT} Q_\alpha = \frac{f_{DT}^2 n_e^2}{4} \langle \sigma v \rangle_{DT} Q_\alpha$$

Example:
 10% He, 2% C
 $f_{DT} = 0.68$



Fusion power reduced by factor of 2

Energy balance, incl. radiation losses:

$$\frac{f_{DT}^2}{4} \cdot n_e^2 \cdot \langle \sigma v \rangle_{DT} \cdot Q_\alpha = \frac{3/2 \cdot n_e \cdot (1 + f_{DT} + f_{He} + \sum_{i \geq 3} f_i) \cdot kT}{\tau_E^*} + P_{rad}$$

τ_E^* : energy confinement time, corrected for radiation losses

He content

Energy balance:

$$\frac{f_{DT}^2}{4} \cdot n_e^2 \cdot \langle \sigma v \rangle_{DT} \cdot Q_\alpha = \frac{3/2 \cdot n_e \cdot (1 + f_{DT} + f_{He} + \sum_{i \geq 3} f_i) \cdot kT}{\tau_E^*} + P_{rad}$$

Particle balance (He):

$$\frac{f_{DT}^2}{4} \cdot n_e^2 \cdot \langle \sigma v \rangle_{DT} = \frac{n_e \cdot f_{He}}{\tau_{p(He)}}$$

„production rate “ of He

„loss rate “ of He
(transport)

He content

Combine energy and particle balance and solve for f_{He} :

$$\frac{f_{DT}^2}{4} \cdot n_e^2 \cdot \langle \sigma v \rangle_{DT} \cdot Q_\alpha = \frac{3/2 \cdot n_e \cdot (1 + f_{DT} + f_{He} + \sum_{i \geq 3} f_i) \cdot kT}{\tau_E^*} + P_{rad} \quad (1)$$

$$\frac{f_{DT}^2}{4} \cdot n_e^2 \cdot \langle \sigma v \rangle_{DT} = \frac{n_e \cdot f_{He}}{\tau_{p(He)}} \quad (2)$$

(1)/(2):

$$Q_\alpha = \frac{3}{2} \frac{(1 + f_{DT} + f_{He} \dots) kT}{\tau_E^*} \frac{\tau_p}{f_{He}} + \frac{P_{rad} \tau_p}{n_e f_{He}}$$

He content

$$Q_\alpha = \frac{3}{2} \frac{(1 + f_{DT} + f_{He} \dots) kT}{\tau_E^*} \frac{\tau_p}{f_{He}} + \frac{P_{rad} \tau_p}{n_e f_{He}}$$

Solve for f_{He} :

$$= \frac{P_{rad}}{P - P_{rad}}$$

$$f_{He} = \frac{3}{2} \frac{\tau_p}{\tau_E^*} \frac{kT}{Q_\alpha} (1 + f_{DT} + f_{He} \dots) \left[1 + \frac{P_{rad}}{n_e} \frac{2\tau_E^*}{3kT(1 + f_{DT} + f_{He} \dots)} \right]$$

With $P - P_{rad} =$

$$\frac{3/2 \cdot n_e \cdot (1 + f_{DT} + f_{He} + \sum_{i \geq 3} f_i) \cdot kT}{\tau_E^*}$$

$$f_{He} = \frac{\tau_p}{\tau_E^*} \cdot \frac{3kT}{Q} \cdot \frac{1}{\left(1 - \frac{P_{rad}}{P}\right)} \cdot \left(\frac{1 + f_{DT} + f_{He} + \sum_{i \geq 3} f_i}{2} \right)$$

He content

$$f_{He} = \frac{\tau_p}{\tau_E^*} \cdot \frac{3kT}{Q} \cdot \frac{1}{\left(1 - \frac{P_{rad}}{P}\right)} \cdot \left(\frac{1 + f_{DT} + f_{He} + \sum_{i \geq 3} f_i}{2} \right)$$

simplify ($f_{He} \ll f_{DT}$):

$$f_{He} \approx \frac{\tau_p}{\tau_E^*} \cdot \frac{3kT}{Q} \cdot \frac{1}{\left(1 - \frac{P_{rad}}{P}\right)}$$

Important: particle confinement time needs to be sufficiently short

Experience: $\frac{\tau_p}{\tau_E^*} \geq \frac{\chi_{\perp}}{D_{\perp}}; \quad \frac{\tau_p}{\tau_E^*} \approx 3$

Now include radiation losses:

Main contributions: Bremsstrahlung and line radiation

Bremsstrahlung:

$$\begin{aligned} P_{rad,brems} &= c_{brems} \cdot n_e \cdot \sum_{i \geq 1} Z_i^2 \cdot n_i \cdot \sqrt{T} \\ &= c_{brems} \cdot n_e^2 \cdot Z_{eff} \cdot \sqrt{T} \end{aligned}$$

Effective charge number:

$$\begin{aligned} Z_{eff} &\equiv \frac{\sum_i Z_i^2 \cdot n_i}{\sum_i Z_i \cdot n_i} = \sum_{i \geq 1} Z_i^2 f_i \\ &= n_e \end{aligned}$$

Now include radiation losses:

Bremsstrahlung:

$$P_{rad,brems} = c_{brems} \cdot n_e \cdot \sum_{i \geq 1} Z_i^2 \cdot n_i \cdot \sqrt{T}$$

$$= c_{brems} \cdot n_e^2 \cdot Z_{eff} \cdot \sqrt{T}$$

$$Z_{eff} \equiv \frac{\sum_i Z_i^2 \cdot n_i}{\sum_i Z_i \cdot n_i} = \sum_{i \geq 1} Z_i^2 f_i$$

Power balance (incl. radiation losses):

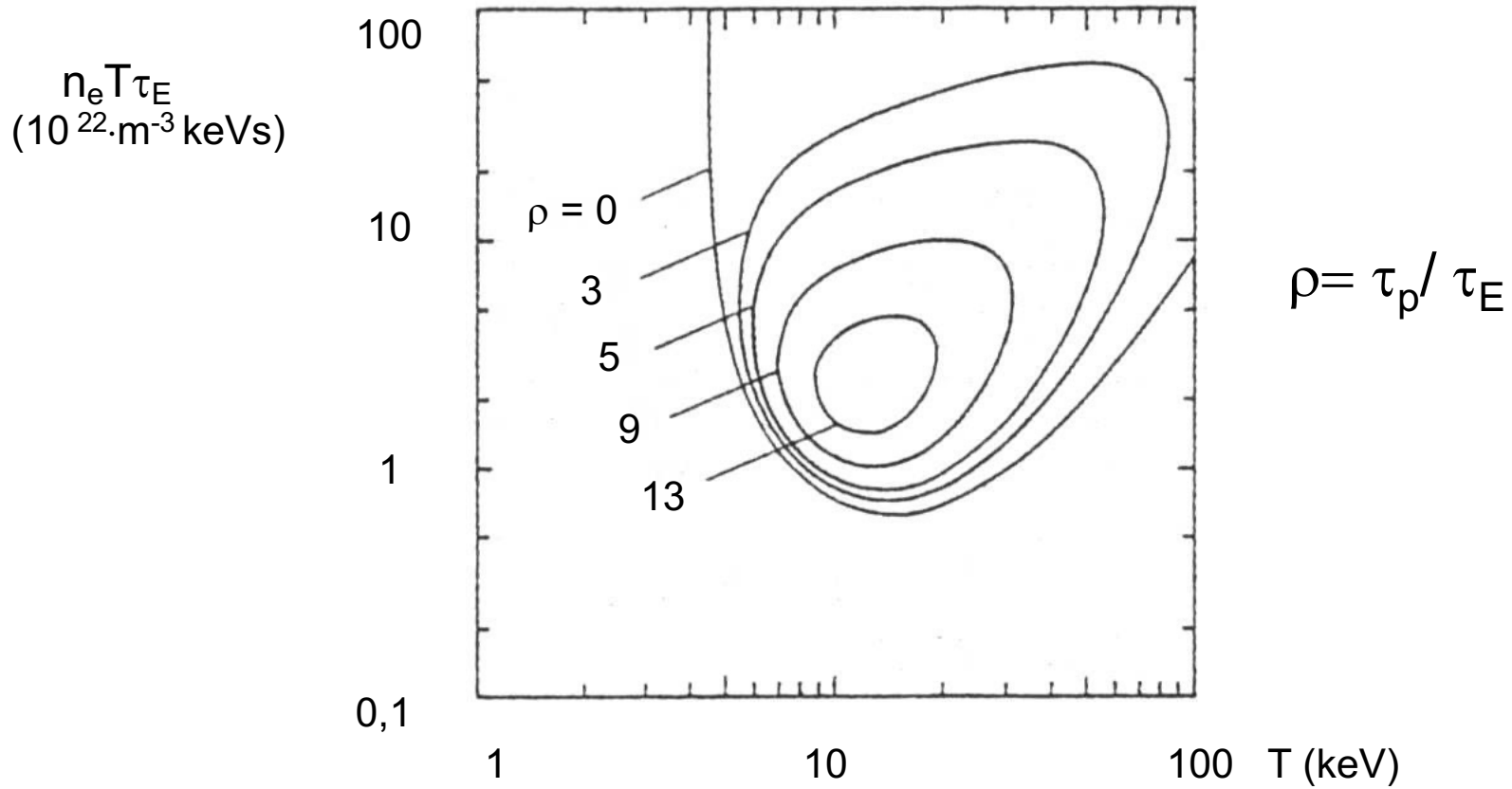
$$n_e^2 \cdot \left\{ \frac{f_{DT}^2}{4} \cdot \langle \sigma v \rangle_{DT} \cdot Q_{DT} - c_{brems} \cdot \left(f_{DT} + 4f_{He} + \sum_{i \geq 3} Z_i^2 f_i \right) \cdot \sqrt{T} \right\}$$

$\sim T^2$

$$= \frac{3/2 \cdot n_e \cdot (1 + f_{DT} + 2f_{He} + \sum_i Z_i f_i) \cdot kT}{\tau_E^*}$$

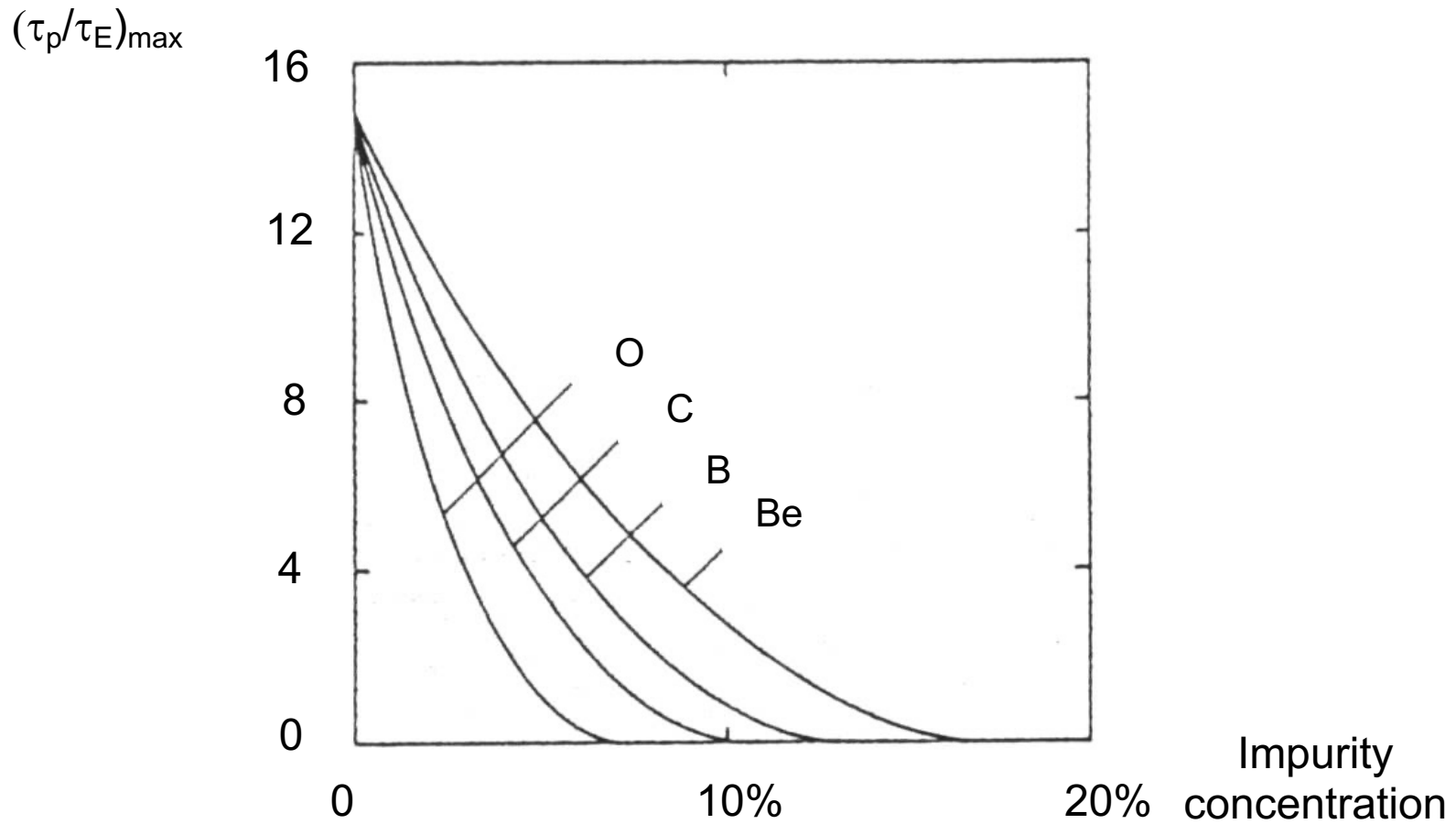
τ_E^* : energy confinement time, corrected for radiation losses

Ignition condition with helium ash:



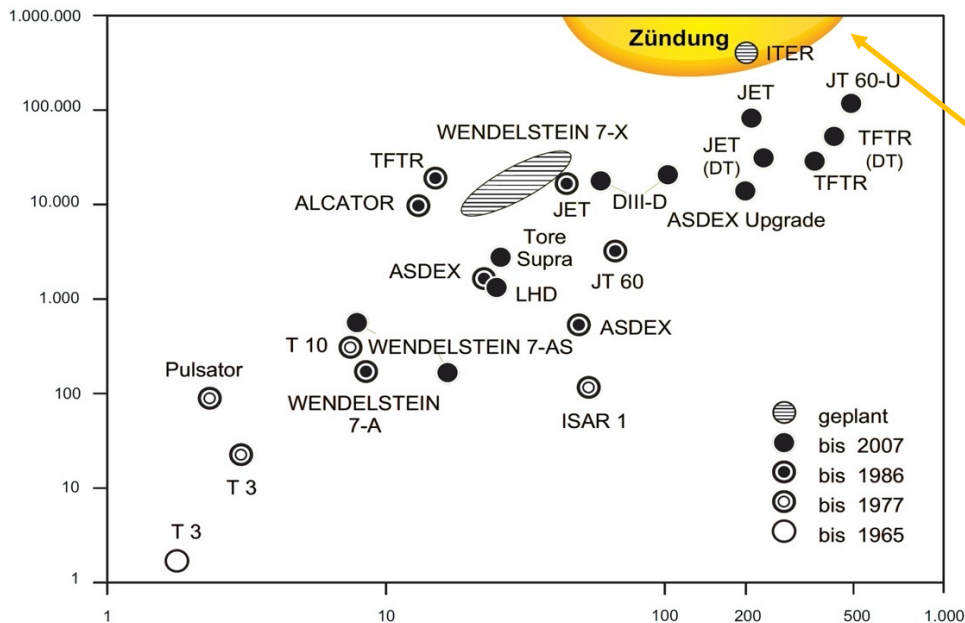
$$\frac{\tau_p}{\tau_E^*} \geq \frac{\chi_{\perp}}{D_{\perp}}; \quad \frac{\tau_p}{\tau_E^*} \approx 3$$

Other impurities:



Requirements for a reactor

$$Q \equiv \frac{P_{fus,tot}}{P_{extern}}$$



$Q = 1$ break even (definition)

$Q \geq 30..40$ minimal value for economical reactor

$Q = \infty$ fully ignited plasma

Cold fusion ?

1989 Pons and Fleischmann claimed:

- fusion of hydrogen (protium, deuterium, tritium) nuclei in palladium during electrolysis
- their explanation: lower Coulomb barrier if hydrogen in solid state
- However: not reproducible, explanation wrong

Myon-catalysed fusion

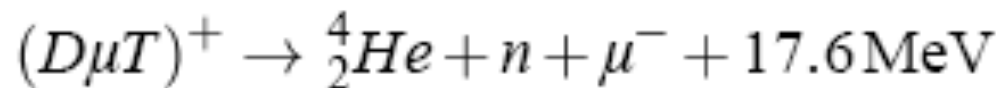
Myonic hydrogen:

Binding length: $a = a_0 \frac{m_e}{m_\mu} = 5.3 \cdot 10^{-11} m / 207 \approx 2.5 \cdot 10^{-13} m$

Add myons to D₂T₂-gas mix:

- Myons slowed down by collisions with molecules
- Formation of D₂, T₂ and D-T molecules

Probability for tunneling is increased: reaction time 10⁻¹⁰s

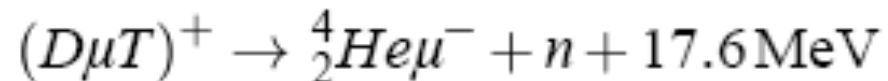


Myons: catalyst

Myon catalysed fusion

- generate myons in accelerator (3 GeV)
- Myons decay with $\sim 2 \cdot 10^{-6}$ s $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
- Slow down in D_2T_2 gas mix , time for building $D\mu T$: 10^{-9} s
- Myon could catalyse about 2000 fusion reactions

problem: competing reaction (probability: 0.6%)



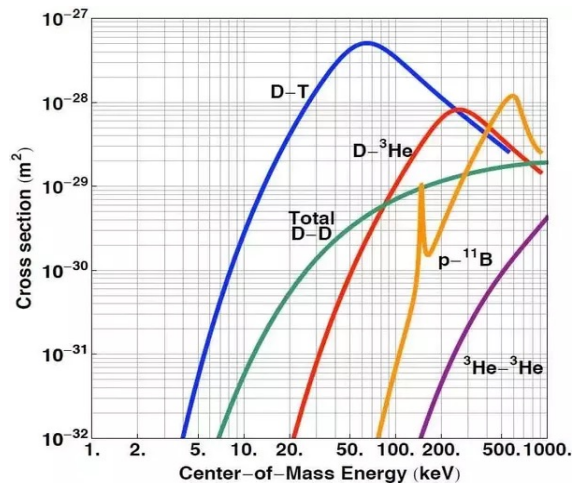
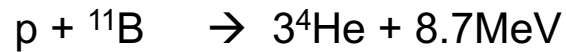
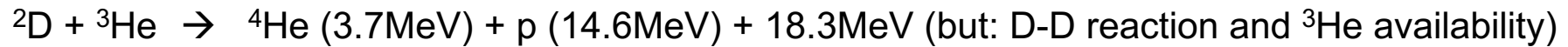
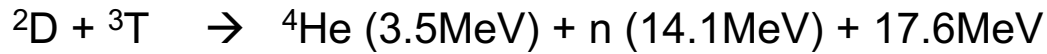
Probability that myon „survives“ N reactions: $(1 - 0.006)^N \sim 1 - 0.006 N$

One myon can theoretically catalyse only $N = 1/0.006 = 170$ fusion reactions

(in experiment: about 100)

no positive energy balance

Neutron-free fusion reactions



Parameter\Reaction	D-T	D-He³	D-D	H-B¹¹
optimum composition for maximum fusion power at given pressure (Te=Ti)	1:1	3:2	1:1	3:1
maximum fusion power density at constant pressure (rel.units)	1,00	0,02	0,04	0,0013
maximum ratio $\langle\sigma v\rangle/T^2$	1,00	0.022	0.013	0,008
burn temperature[keV] optimized for power density at given pressure	15,00	50,00	20,00	140,00
minimum required $nT\tau$ for ignition (rel.units)	1	11	16	100

also the impurity profiles play a role:

