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Kinetic Full Wave Analyses in Inhomogeneous Plasmas Using Integral Form of Dielectric Tensor

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Outline

- What is kinetic full wave analysis?
- How to obtain integral form of dielectric tensor?
- Magnetized plasma:
  - parallel motion:
    - inhomogeneous magnetic field
  - perpendicular motion:
    - Finite Larmor radius effects
    - Bernstein wave
- Summary

Analysis of waves in inhomogeneous plasmas

- **Maxwell's equation**: $\boldsymbol{E}(\boldsymbol{r}, t)$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \qquad \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \mu_0 \sum_s \boldsymbol{J}_s - \mu_0 \boldsymbol{J}_{\rm ext}$$

  – **conductivity tensor** $\overleftrightarrow{\sigma}_s$: $\boldsymbol{J}_s = \overleftrightarrow{\sigma}_s \cdot \boldsymbol{E}$ (particle, kinetic, fluid)

- **Full wave analysis**: angular frequency ω: $\boldsymbol{E}(\boldsymbol{r}) \exp(-\mathrm{i}\,\omega t)$

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - \frac{\omega^2}{c^2} \int \mathrm{d}\boldsymbol{r}' \, \overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{E}(\boldsymbol{r}') = \mathrm{i}\,\omega \mu_0 \boldsymbol{J}(\boldsymbol{r})_{\rm ext}$$

  – **dielectric tensor**: $\overleftrightarrow{\epsilon} = \overleftrightarrow{I} + (\mathrm{i}/\omega\epsilon_0) \sum_s \overleftrightarrow{\sigma}_s$

- **Geometrical optics**: wave number $\boldsymbol{k}$: $\boldsymbol{E} \exp(\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{r} - \mathrm{i}\,\omega t)$

  – **Ray tracing**: time evolution of wave packet

$$\frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}t} = -\frac{\partial D}{\partial \boldsymbol{r}} \bigg/ \frac{\partial D}{\partial \omega}, \qquad \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{v}_{\rm g} = \frac{\partial D}{\partial \boldsymbol{k}} \bigg/ \frac{\partial D}{\partial \omega}$$

- **Dispersion rel.**: $D(\boldsymbol{k}, \omega; \boldsymbol{r}) = \det\left[(c^2/\omega^2)\boldsymbol{k} \times \boldsymbol{k} \times + \overleftrightarrow{\epsilon}(\boldsymbol{k}, \omega)\right] = 0$

Why kinetic full wave analysis is required?

- **Full wave analyses** can describe
  - **Tunneling of evanescent layer**
  - **Formation of a standing wave**
  - **Coupling with finite size antenna**
- **Kinetic effects** to be included
  - **Wave-particle resonant interaction**: Landau/cyclotron damping
  - **Finite Larmor radius effects**: Bernstein waves
  - **Thermal waves**: ion acoustic waves
- **Inhomogeneous effects** to be taken into account
  - **Density gradient sustained by sheath potential**: $n = n_0 \exp(-z/L)$
  - **Inhomogeneous magnetic field**: $B = B_0(1 + z/L)$
  - **Cyclotron resonance near the extremum of B**: $B = B_0(1 + z^2/L^2)$
- **Strong inhomogeneity** and/or **strong kinetic effects**

$$\frac{v_\mathrm{T}}{\omega} \gg L, \qquad \rho_\mathrm{c} = \frac{v_\mathrm{T}}{\omega_\mathrm{c}} \gg \lambda_\perp$$

Previous approaches of kinetic full wave analyses

- **Cold wave number approach**: no kinetic modes
  - Use $\boldsymbol{k}_{\rm cold}$ from the dispersion relation in a cold uniform plasma

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon}(\boldsymbol{r}; \boldsymbol{k}_{\rm cold}) \cdot \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\rm ext}(\boldsymbol{r})$$

- **Differential operator approach** [1]: difficult for higher order
  - Expand $\overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{k})$ with respect to $\boldsymbol{k}$ and replace $\boldsymbol{k}$ by $-\mathrm{i}\,\nabla$

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon}(\boldsymbol{r}; -\mathrm{i}\,\nabla) \cdot \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\rm ext}(\boldsymbol{r})$$

- **Spectral approach** [2]: large dense matrix has to be solved
  - Fourier transform in the direction of inhomogeneity $\boldsymbol{r}$

$$-\boldsymbol{k} \times \boldsymbol{k} \times \boldsymbol{E}(\boldsymbol{k}) - \frac{\omega^2}{c^2} \sum_{\boldsymbol{k}'} \overleftrightarrow{\epsilon}(\boldsymbol{k}, \boldsymbol{k}') \cdot \boldsymbol{E}(\boldsymbol{k}') = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\rm ext}(\boldsymbol{k})$$

- **Inverse Fourier transform of $\overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{k})$** [3]: based on uniform $\overleftrightarrow{\epsilon}$

[1] Fukuyama et al., CPR (1986), [2] Jaeger et al. PoP (2000), [3] Sauter et al., NF (1992)

How to obtain integral form of dielectric tensor?

- **Vlasov equation**:

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

- **Linearization**: angular frequency ω (complex, in general)

$$f(\boldsymbol{r}, \boldsymbol{v}, t) = f_0(\boldsymbol{r}, \boldsymbol{v}) + f(\boldsymbol{r}, \boldsymbol{v})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$$

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$$

$$\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{B}_0(\boldsymbol{r}) + \boldsymbol{B}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$$

- **Current density**: Elapsed time: $\tau = t - t'$, , $\boldsymbol{r}' = \boldsymbol{r}(t-\tau)$, $\boldsymbol{v}' = \boldsymbol{v}(t-\tau)$

$$\boldsymbol{J}(\boldsymbol{r}) = q \int \mathrm{d}\boldsymbol{v}\, \boldsymbol{v} f(\boldsymbol{r}, \boldsymbol{v})$$

$$= -\frac{q^2}{m} \int \mathrm{d}\boldsymbol{v}\, \boldsymbol{v} \int_0^\infty \mathrm{d}\tau \left[\boldsymbol{E}(\boldsymbol{r}') + \boldsymbol{v}' \times \boldsymbol{B}(\boldsymbol{r}')\right] \cdot \frac{\partial f_0(\boldsymbol{r}', \boldsymbol{v}')}{\partial \boldsymbol{v}'}\,\mathrm{e}^{\,\mathrm{i}\,\omega\tau}$$

- **Replace integral over $\boldsymbol{v}$ by integral over $\boldsymbol{r}'$**

Integral form in uniform plasmas

- **propagation in z**
- **Particle orbit**: $z = z' + v_z(t - t')$
- **Variable transformation** : $v_z = \dfrac{z - z'}{t - t'}$
- **Perturbed distribution function for Maxwellian**: $\tau = t - t'$

$$f(z, \boldsymbol{v}) = \frac{n}{(2\pi T/m)^{3/2}} \frac{q}{T} \int_0^\infty \mathrm{d}\tau\, \boldsymbol{v} \cdot \boldsymbol{E}(z')\, \mathrm{e}^{\,\mathrm{i}\,\omega\tau} \exp\left[-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2T} \right]$$

- **Current density**: variable transformation: $v_z \Rightarrow z'$

$$\boldsymbol{J}(z) = q \int \mathrm{d}\boldsymbol{v}\, \boldsymbol{v} f(z, \boldsymbol{v}) = \int \mathrm{d}z'\, \overleftrightarrow{\sigma}(z, z') \cdot \boldsymbol{E}(z')$$

- **Electric conductivity tensor**: e.g. zz component: $\tau = t - t'$

$$\sigma_{zz}(z, z') = \frac{nq^2}{\sqrt{2\pi}\, m v_{\mathrm{T}}^3} \int_0^\infty \mathrm{d}\tau \frac{(z - z')^2}{\tau^3} \exp\left[-\frac{1}{2} \frac{(z - z')^2}{v_{\mathrm{T}}^2 \tau^2} + \mathrm{i}\,\omega\tau \right]$$

Plasma dispersion kernel function (PDKF)

- **A general form of kernel function**: Inverse Fourier transform of PDF

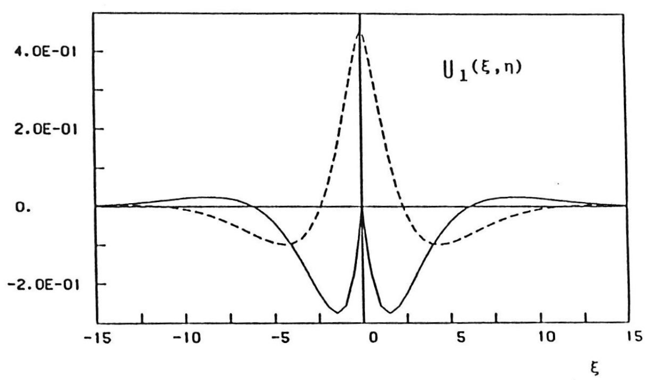
$$U_n(\xi,\eta,\nu) = \frac{\mathrm{i}}{\sqrt{2\pi}} \int_0^\infty \mathrm{d}\tau\, \tau^{n-1} \,\exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2} - \frac{1}{2}\eta^2\tau^2 + \mathrm{i}\,\nu\tau\right]$$

$$\xi = \frac{(z-z')\omega}{v_\mathrm{T}}, \qquad \eta = \sqrt{\left(\frac{v_\mathrm{T}}{\omega L}\right)^2 + \left(\frac{k_y v_\mathrm{T}}{\omega}\right)^2 + \left(\frac{\mu B_0}{2m v_\mathrm{T}\omega L_1}\right)^2}, \qquad \nu = \frac{\omega - n\omega_\mathrm{c}}{\omega}$$

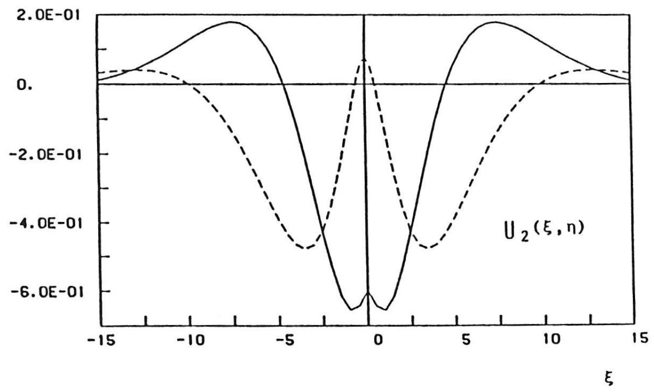
  - L: **density scale length** [$n = n_0 \exp(-z/L)$: sheath potential]
  - k_y: **wave number** in uniform direction
  - L_1: **magnetic field scale length** [$B = B_0(1 + z/L_1)$: linear]

- **Complex function**: real: dispersion, imag: dissipation

$U_1(\xi,0)$



$U_2(\xi,0)$



General formulation in magnetized plasmas

- **Magnetic field**: $\boldsymbol{B} = B(z)\,\hat{z}$, **Velocity**: $\tau = t - t'$, $\theta'_g = \theta_g - \omega_c \tau$

$$\begin{aligned} v_x &= \sqrt{2\mu B(z)/m}\cos\theta_g, \quad v'_x = \sqrt{2\mu B(z')/m}\cos\theta'_g \\ v_y &= \sqrt{2\mu B(z)/m}\sin\theta_g, \quad v'_y = \sqrt{2\mu B(z')/m}\sin\theta'_g \\ v_z &= \sqrt{2/m}\sqrt{\epsilon - \mu B(z)}, \quad v'_z = \sqrt{2/m}\sqrt{\epsilon - \mu B(z')} \end{aligned}$$

- **Perturbed distribution function**: Time dependence: $\tilde{X}(\boldsymbol{r}, t) = X(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$

$$\tilde{f}(z, \epsilon, \mu, \theta_g) = -\sqrt{\frac{2}{m}}q \int_0^\infty \mathrm{d}\tau \begin{pmatrix} \sqrt{\mu B(z')}\cos\theta'_g \\ \sqrt{\mu B(z')}\sin\theta'_g \\ \sqrt{\epsilon - \mu B(z')} \end{pmatrix} \frac{\partial f_0(\epsilon)}{\partial \epsilon} \cdot \boldsymbol{E}(x', y', z')\,\mathrm{e}^{\,\mathrm{i}\,\omega\tau}$$

- **Induced current**:

$$\boldsymbol{J}(\boldsymbol{r}) = \sum_{\pm} \int_0^\infty \mathrm{d}\mu \int_{\mu B(z)}^\infty \mathrm{d}\epsilon \int_0^{2\pi} \mathrm{d}\theta_g \frac{qB(z)}{2m^2\sqrt{\epsilon - \mu B(z)}} \begin{pmatrix} \sqrt{\mu B(z)}\cos\theta_g \\ \sqrt{\mu B(z)}\sin\theta_g \\ \sqrt{\epsilon - \mu B(z)} \end{pmatrix} \tilde{f}(z, \epsilon, \mu, \theta_g)$$

$$= \frac{n_0 q^2}{2m\pi^{3/2}T^{5/2}B_0} \sum_{\pm} \int_0^\infty \mathrm{d}\mu \int_{\mu B(z)}^\infty \mathrm{d}\epsilon \int_0^\infty \mathrm{d}\tau \int_0^{2\pi} \mathrm{d}\theta_g \frac{B(z)B(z')}{\epsilon - \mu B(z)}\,\mathrm{e}^{-\epsilon/T}\,\mathrm{e}^{\,\mathrm{i}\,\omega\tau}$$

$$\times \begin{pmatrix} \mu\sqrt{B(z)B(z')}\cos\theta_g\cos\theta'_g & \mu\sqrt{B(z)B(z')}\cos\theta_g\sin\theta'_g & \sqrt{\mu B(z)}\sqrt{\epsilon - \mu B(z')}\cos\theta_g \\ \mu\sqrt{B(z)B(z')}\sin\theta_g\cos\theta'_g & \mu\sqrt{B(z)B(z')}\sin\theta_g\sin\theta'_g & \sqrt{\mu B(z)}\sqrt{\epsilon - \mu B(z')}\sin\theta_g \\ \sqrt{\epsilon - \mu B(z)}\sqrt{\mu B(z')}\cos\theta'_g & \sqrt{\epsilon - \mu B(z)}\sqrt{\mu B(z')}\sin\theta'_g & \sqrt{\epsilon - \mu B(z)}\sqrt{\epsilon - \mu B(z')} \end{pmatrix} \cdot \boldsymbol{E}(\boldsymbol{r}')$$

Parallel motion in magnetized plasmas

- **Adiabatic motion in magnetized plasmas**:

$$\epsilon = \frac{mv_{\|}^2}{2} + \mu B, \qquad \mu = \frac{mv_{\perp}^2}{2B}$$

- **Equation of motion**: θ_g: gyration phase

$$\frac{\mathrm{d}z}{\mathrm{d}t} = v_{\|} = \pm\sqrt{\frac{2}{m}}\sqrt{\epsilon - \mu B(z)}$$

- **Linear dependence of magnetic field strength**

$$B(z) = B_0\left(1 + \frac{z}{L_1}\right)$$

- **by integrating over t**

$$z - z' = \pm\sqrt{\frac{2}{m}}\sqrt{\epsilon - \mu B(z)}\,\tau + \frac{\mu B_0}{2mL_1}\tau^2$$

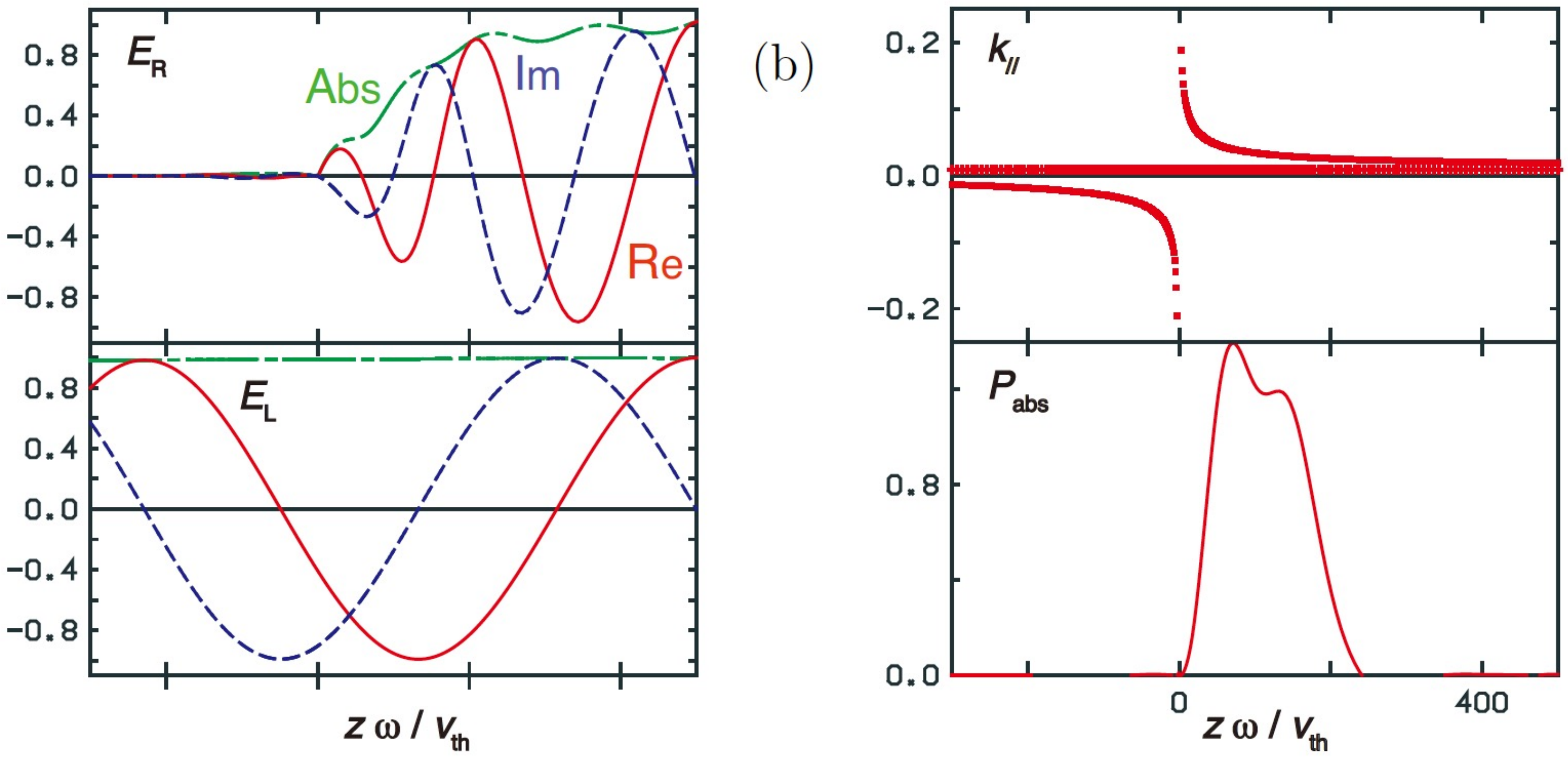
- **Integral over ϵ is transformed to that over z'**

$$\epsilon = \mu B(z) + \frac{m}{2}\left(\frac{z - z'}{\tau} - \frac{\mu B_0}{2mL_1}\tau\right)^2$$

Magnetic beach heating near ECR

First order inhomogeneous effect: $B(z) = B_0(1 + z/L_1)$

- **High field excitation**: $\omega_{pe}^2/\omega_{ce}^2 = 0.5$, $\beta = v_{the}/c = 0.01$
- **RHS circular polarization**: Absorption in strong field side of ECR
- **LHS circular polarization**: No absorption



Magnetic field strength with extremum (1)

- In tokamak configuration, the **magnetic field strength** has **extremum** along the field line.
- **Parabolic profile of magnetic field strength**: $\kappa = \pm 1$

$$B(z) = B_0 \left(1 + \frac{z}{L_1} + \kappa \frac{z^2}{L_2^2} \right)$$

- **Adiabatic motion**: $\dfrac{\mathrm{d}z}{\mathrm{d}t} = \pm \sqrt{\dfrac{2}{m}} \sqrt{\epsilon - \mu B(z)}$
  - **Trapped** ($\kappa = 1$)

$$z - z' = \pm \sqrt{\frac{2}{m}} \sqrt{\epsilon - \mu B(z)} \frac{\sin \omega_b \tau}{\omega_b} + \frac{\mu B_0}{2m} \left(\frac{1}{L_1} + \frac{2z}{L_2^2} \right) \frac{1 - \cos \omega_b \tau}{\omega_b^2}$$

  - **Untrapped** ($\kappa = -1$)

$$z - z' = \pm \sqrt{\frac{2}{m}} \sqrt{\epsilon - \mu B(z)} \frac{\sinh \omega_b \tau}{\omega_b} + \frac{\mu B_0}{2m} \left(\frac{1}{L_1} - \frac{2z}{L_2^2} \right) \frac{\cosh \omega_b \tau - 1}{\omega_b^2}$$

  where

$$\omega_b = \sqrt{\frac{2\mu B_0}{m L_2^2}}$$

Magnetic field strength with extremum (2)

- **Energy ϵ as a funtion of z'**
  - **Trapped** ($\kappa = 1$):

$$\epsilon = \mu B(z) + \frac{m}{2}\frac{\omega_b^2\tau^2}{\sin^2\omega_b\tau}\left[\frac{z-z'}{\tau} - \frac{\mu B_0}{2m}\left(\frac{1}{L_1} + \frac{2z}{L_2^2}\right)\frac{1-\cos\omega_b\tau}{\omega_b^2\tau}\right]^2$$

  - **Untrapped** ($\kappa = -1$):

$$\epsilon = \mu B(z) + \frac{m}{2}\frac{\omega_b^2\tau^2}{\sinh^2\omega_b\tau}\left[\frac{z-z'}{\tau} - \frac{\mu B_0}{2m}\left(\frac{1}{L_1} - \frac{2z}{L_2^2}\right)\frac{\cosh\omega_b\tau - 1}{\omega_b^2\tau}\right]^2$$

  - **Linear** ($\kappa = 0$):

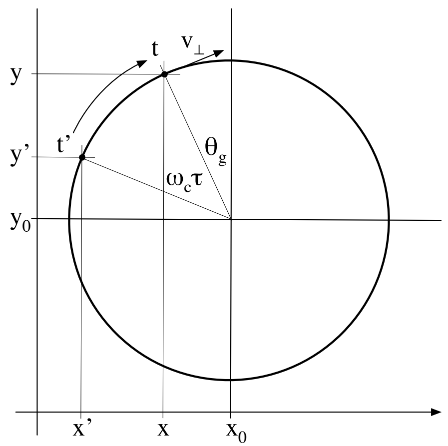
$$\epsilon = \mu B(z) + \frac{m}{2}\left[\frac{z-z'}{\tau} - \frac{\mu B_0}{2mL_1}\tau\right]^2$$

Perpendicular motion: Finite Larmor radius effects

- **In order to separate the parallel and perpendicular motions, we assume weak inhomogeneity of B**: neglecting particle drift

$$\omega_c(z') \simeq \omega_c(z) \simeq \omega_c\left(\frac{z+z'}{2}\right), \qquad \theta_g' = \theta_g - \omega_c \tau$$

- **Cyclotron motion and variable transformation** from $(v_\perp, \theta_g)$ to (x, x')



$$x = x_0 - (v_\perp/\omega_c)\sin\theta_g,$$
$$x' = x_0 - (v_\perp/\omega_c)\sin\theta_g',$$
$$y = y_0 + (v_\perp/\omega_c)\cos\theta_g,$$
$$y' = y_0 + (v_\perp/\omega_c)\cos\theta_g')$$

$$v_\perp \sin\theta_g = -\omega_c(x - x_0)$$
$$v_\perp \sin\theta_g' = -\omega_c(x' - x_0),$$
$$v_\perp \cos\theta_g = \omega_c \frac{x-x'}{2}\frac{1}{\tan\frac{1}{2}\omega_c\tau} + \omega_c\left(\frac{x+x'}{2} - x_0\right)\tan\frac{1}{2}\omega_c\tau$$
$$v_\perp \cos\theta_g' = \omega_c \frac{x-x'}{2}\frac{1}{\tan\frac{1}{2}\omega_c\tau} + \omega_c\left(\frac{x+x'}{2} - x_0\right)\tan\frac{1}{2}\omega_c\tau$$

Plasma gyro kernel function (PGKF)

- **Integral over θ_g in the kernel functions** :

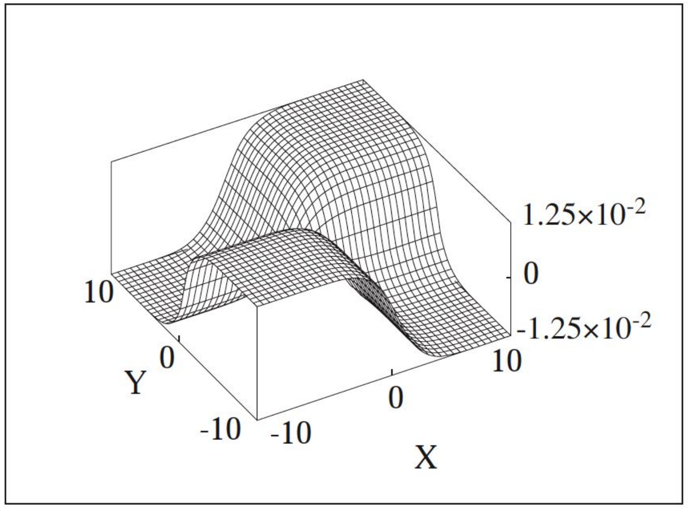
$$X = \left(\frac{x+x'}{2} - x_0\right)\frac{\omega_c}{v_T}, \quad Y = \left(\frac{x-x'}{2}\right)\frac{\omega_c}{v_T},$$

$$F_n^{(i)}(X,Y) = \frac{1}{2\pi^2}\int_0^{\pi} d\theta \left[-\frac{X^2}{1+\cos\theta_g} - \frac{Y^2}{1-\cos\theta_g}\right] f_n^{(i)}(\theta_g)$$

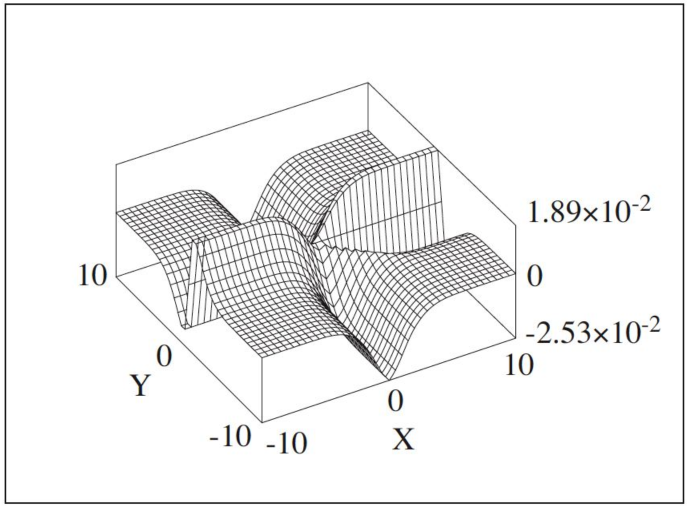
$$\mathcal{F}_n^{(ijk)}(X,Y) = \int_0^{Y} dY' \int_0^{X+Y'} dX' X'^j Y'^k F_n^{(i)}(X',Y')$$

$$f_n^{(i)}(\theta_g) = \begin{cases} \dfrac{\cos n\theta_g}{\sin\theta_g} & \text{(i=1)} \\ \sin n\theta_g & \text{(i=2)} \\ \dfrac{\sin n\theta_g}{\sin^2\theta_g} & \text{(i=3)} \\ \dfrac{\cos\theta_g \sin n\theta_g}{\sin^2\theta_g} & \text{(i=4)} \end{cases}$$

$\mathcal{F}_0^{(100)}$



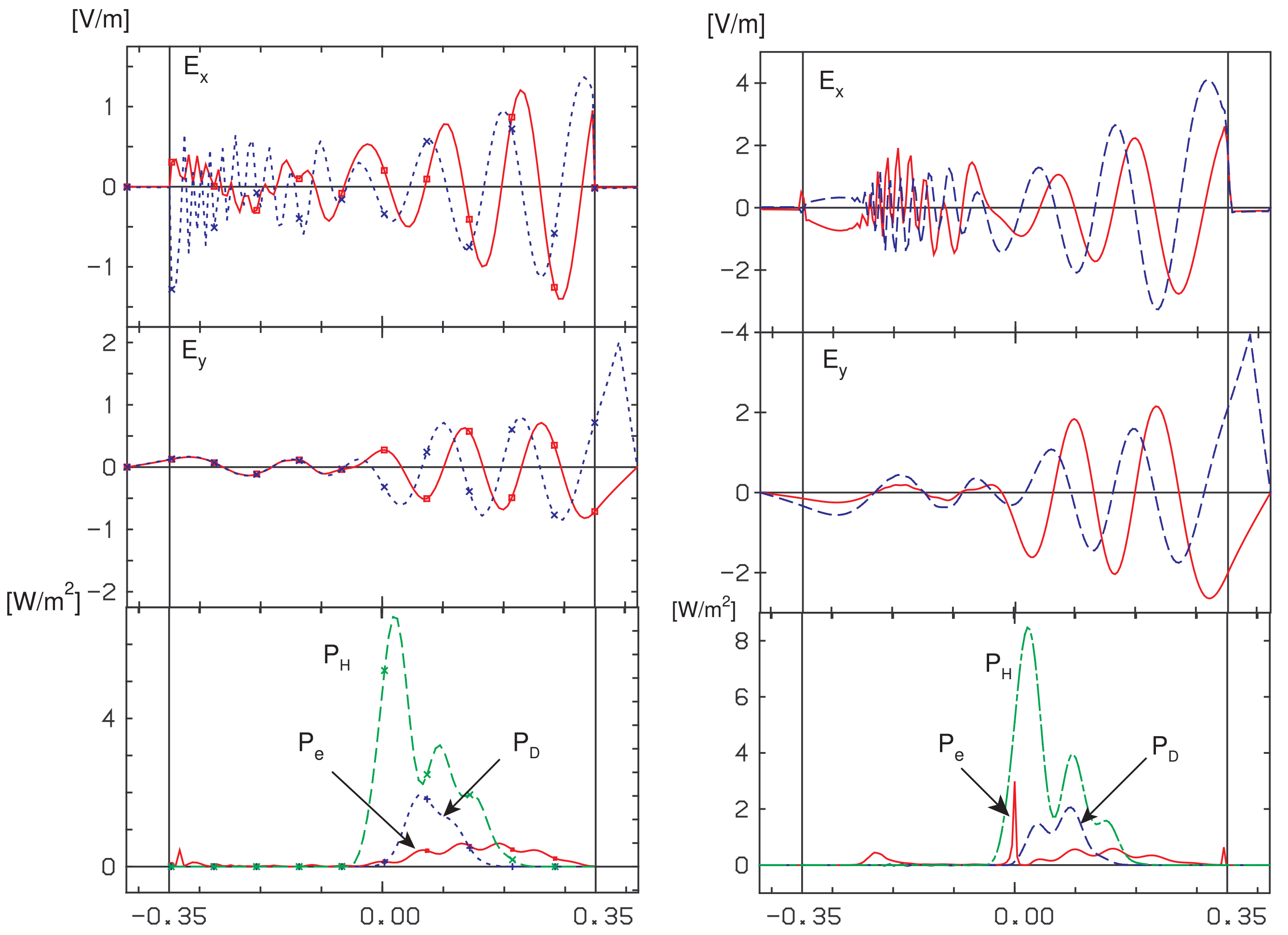
$\mathcal{F}_1^{(100)}$



FLR effects in ICRF heating (1)

ICRF minoring heating without energetic particles ($n_H/n_D = 0.1$)

Differential form   Integral form



$$R_0 = 1.31\text{m}$$

$$a = 0.35\text{m}$$

$$B_0 = 1.4\text{T}$$

$$T_{e0} = 1.5\text{keV}$$

$$T_{D0} = 1.5\text{keV}$$

$$T_{H0} = 1.5\text{keV}$$

$$n_{s0} = 10^{20}\text{m}^{-3}$$

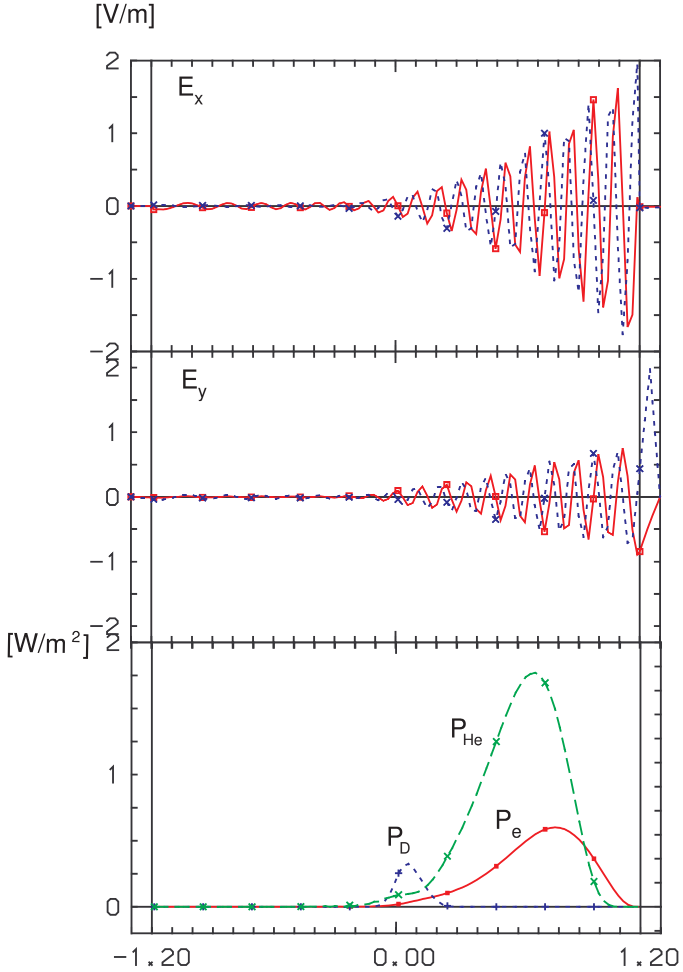
$$\omega/2\pi = 20\text{MHz}$$

Differential approach is applicable

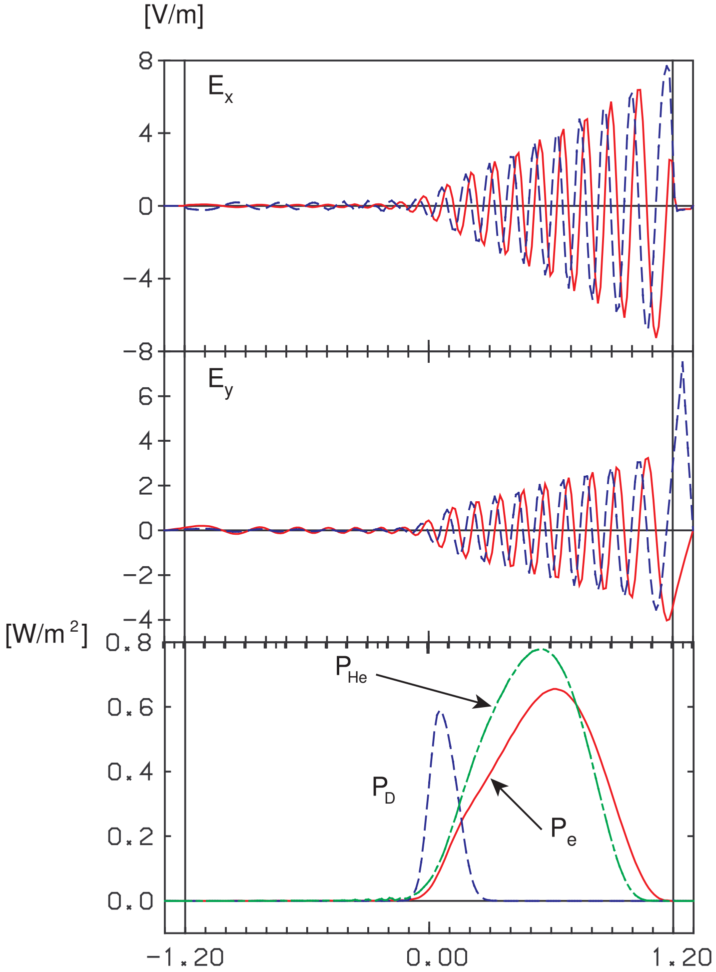
FLR effects in ICRF heating (2)

ICRF minoring heating with α-particles ($n_{\rm D} : n_{\rm He} = 0.96 : 0.02$)

Differential form



Integral form



$R_0 = 3.0\mathrm{m}$

$a = 1.2\mathrm{m}$

$B_0 = 3\mathrm{T}$

$T_{e0} = 10\mathrm{keV}$

$T_{D0} = 10\mathrm{keV}$

$T_{\alpha 0} = 3.5\mathrm{MeV}$

$n_{s0} = 10^{20}\mathrm{m}^{-3}$

$\omega/2\pi = 45\mathrm{MHz}$

Absorption by α may be over-estimated by differential approach.

O-X-B mode conversion of electron cyclotron waves

- **EC heating and current drive in over-dense plasmas**
  - Experimental observation of EC H&CD in a high-density plasma above the cutoff density
  - Possible mechanism is the mode-conversion to the electron Bernstein waves (EBW)
- **O-X-B mode conversion**
  - **HFS excitation**: X-mode is converted to EBW near the UHR
  - **LFS excitation**: Cutoff layer exists for both O and X modes
  - For **optimum injection angle** derived by Hansen et al. [PPCF, **27** 1077 (1985)

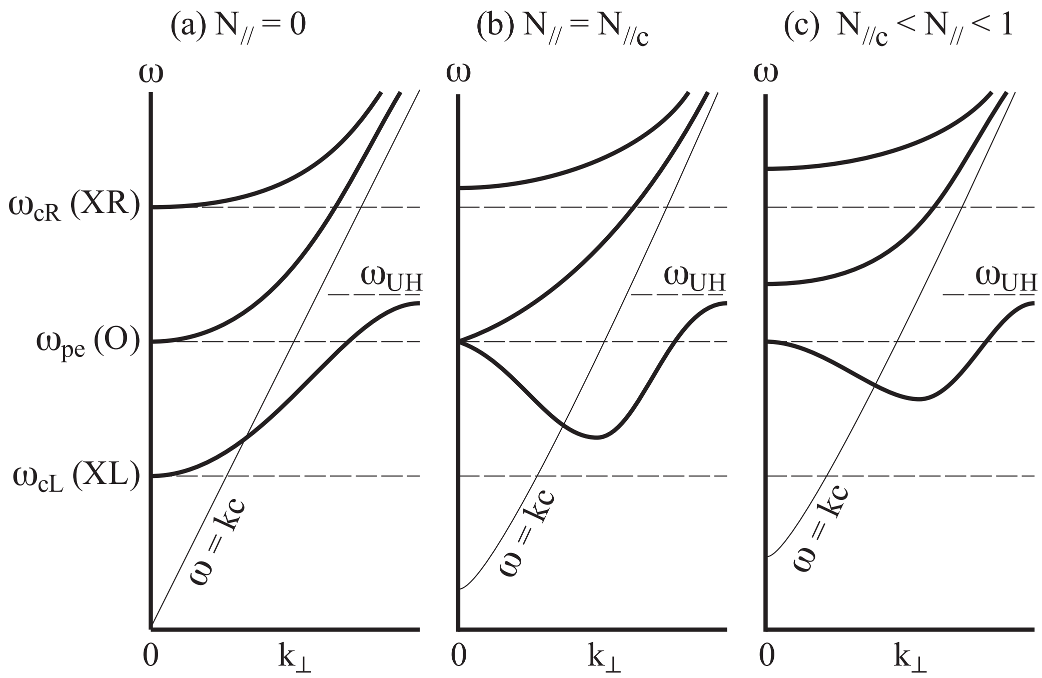
    $$N_{\parallel}^2 = \frac{|\omega_{\rm ce}|}{\omega + |\omega_{\rm ce}|},$$

    mode-conversion from O-mode to X mode occurs.
  - The X-mode is converted to EBW near the UHR, and the EBW is absorbed by EC damping near ECR.

O-X-B mode conversion of EC waves in tokamak

- **For the optimum injection angle,**
  - O-mode cutoff and X-mode cutoff are located at the same position.
  - O-X mode conversion: k changes the sign, forward to backward
- **When the injection angle is not optimum,**
  - Evanescent layer appears between the O and X cutoffs.
  - Geometrical optics cannot describe tunneling.

Dispersion relation for fixed $N_{||}$

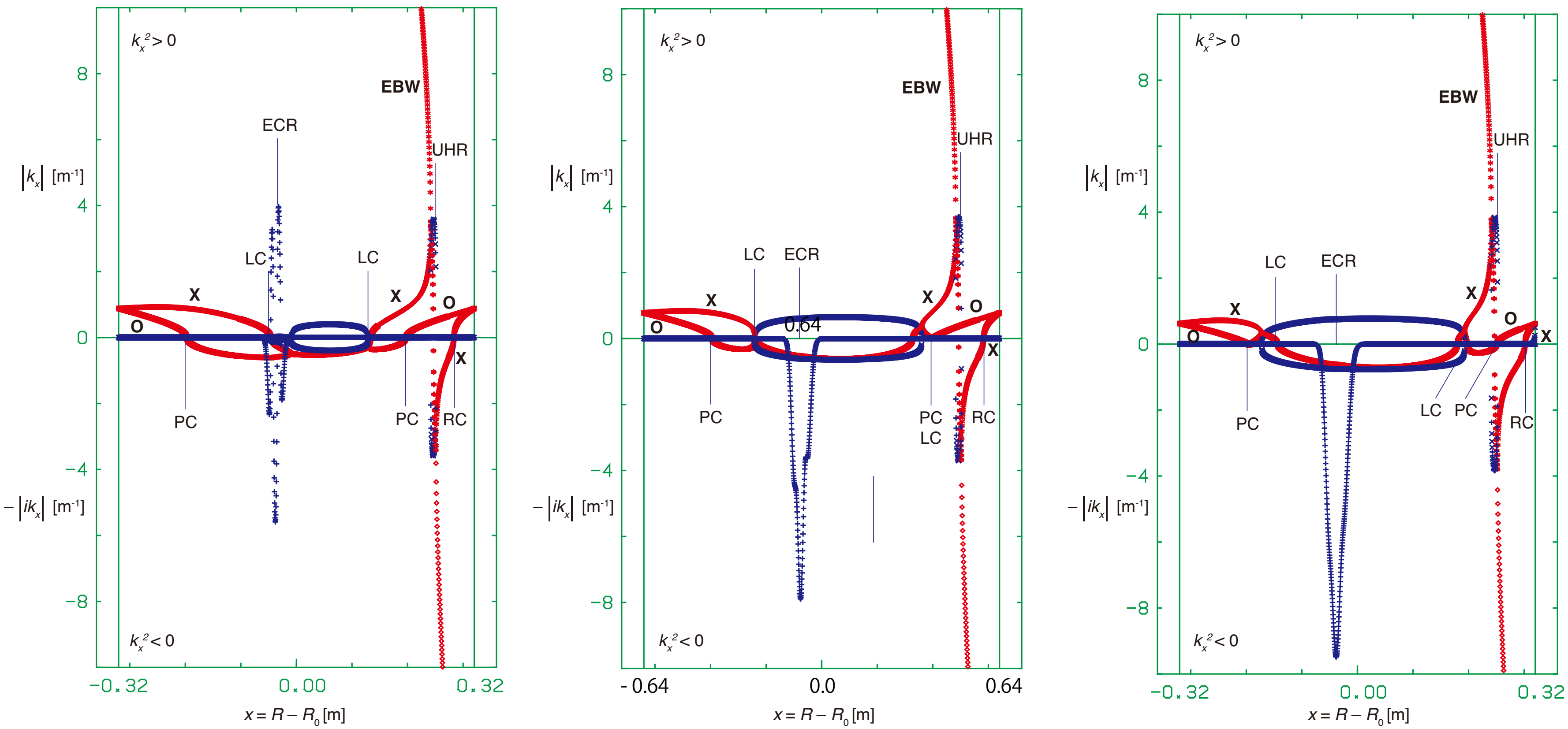


Dispersion relation: dependence on $k_{||}$

Parameters of twice-expanded Spherical Torus LATE

$R_0 = 0.44\,\mathrm{m}$, $a = 0.32\,\mathrm{m}$, $B_0 = 0.08\,\mathrm{T}$ $n_e(0) = 1.2 \times 10^{17}\,\mathrm{m}^{-3}$, $f = 2.45\,\mathrm{GHz}$

$k_{\|\|} = 24\,\mathrm{m}^{-1}$	$k_{\|\|} = 32\,\mathrm{m}^{-1}$	$k_{\|\|} = 40\,\mathrm{m}^{-1}$
deep X cutoff	optimum angle	shallow cutoff



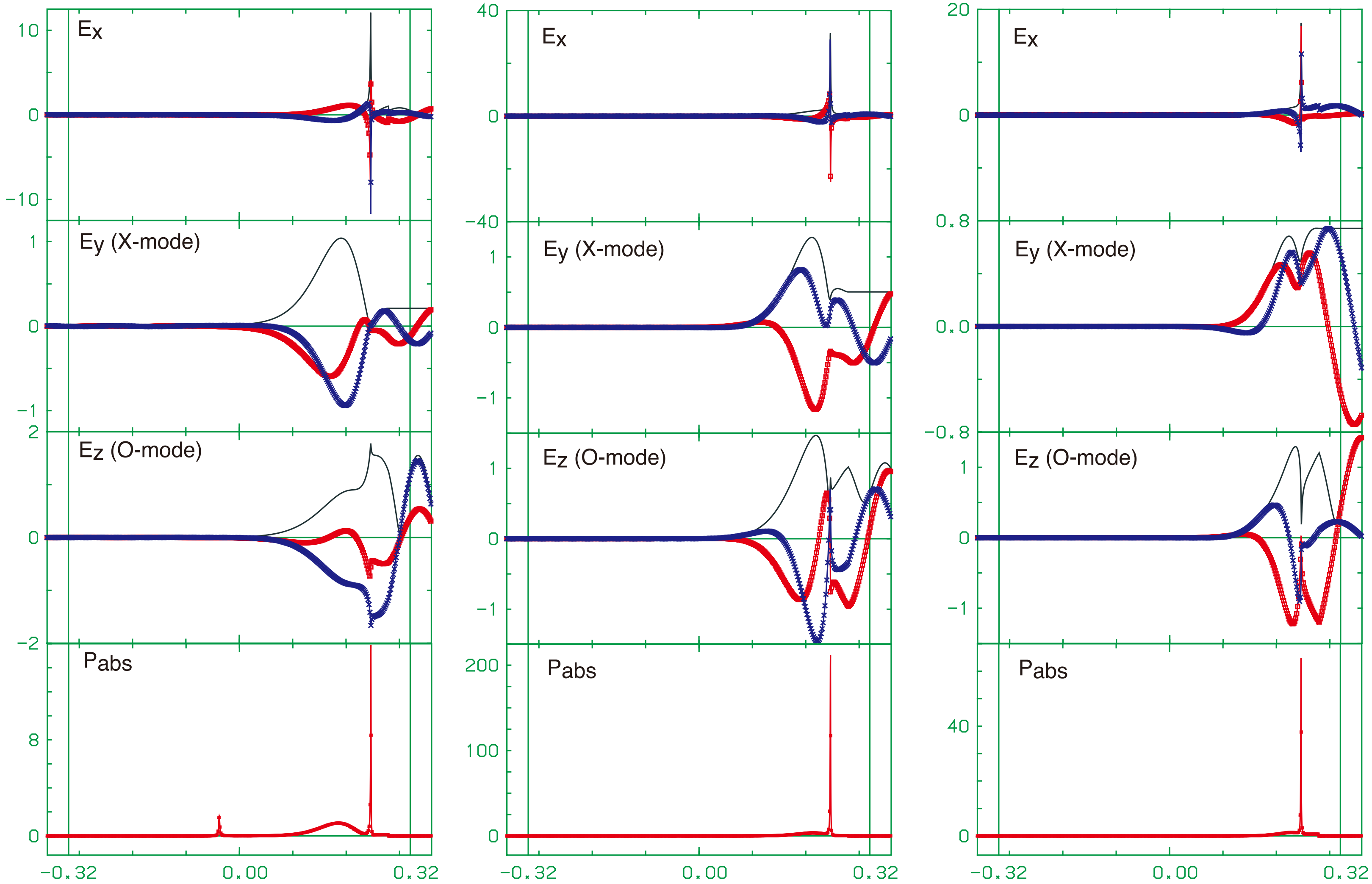
Wave structure: dependence on $k_{||}$

Cold plasma analysis with collisional absorption at UHR

$k_{||} = 24\,\mathrm{m}^{-1}$
deep X cutoff

$k_{||} = 32\,\mathrm{m}^{-1}$
optimum angle

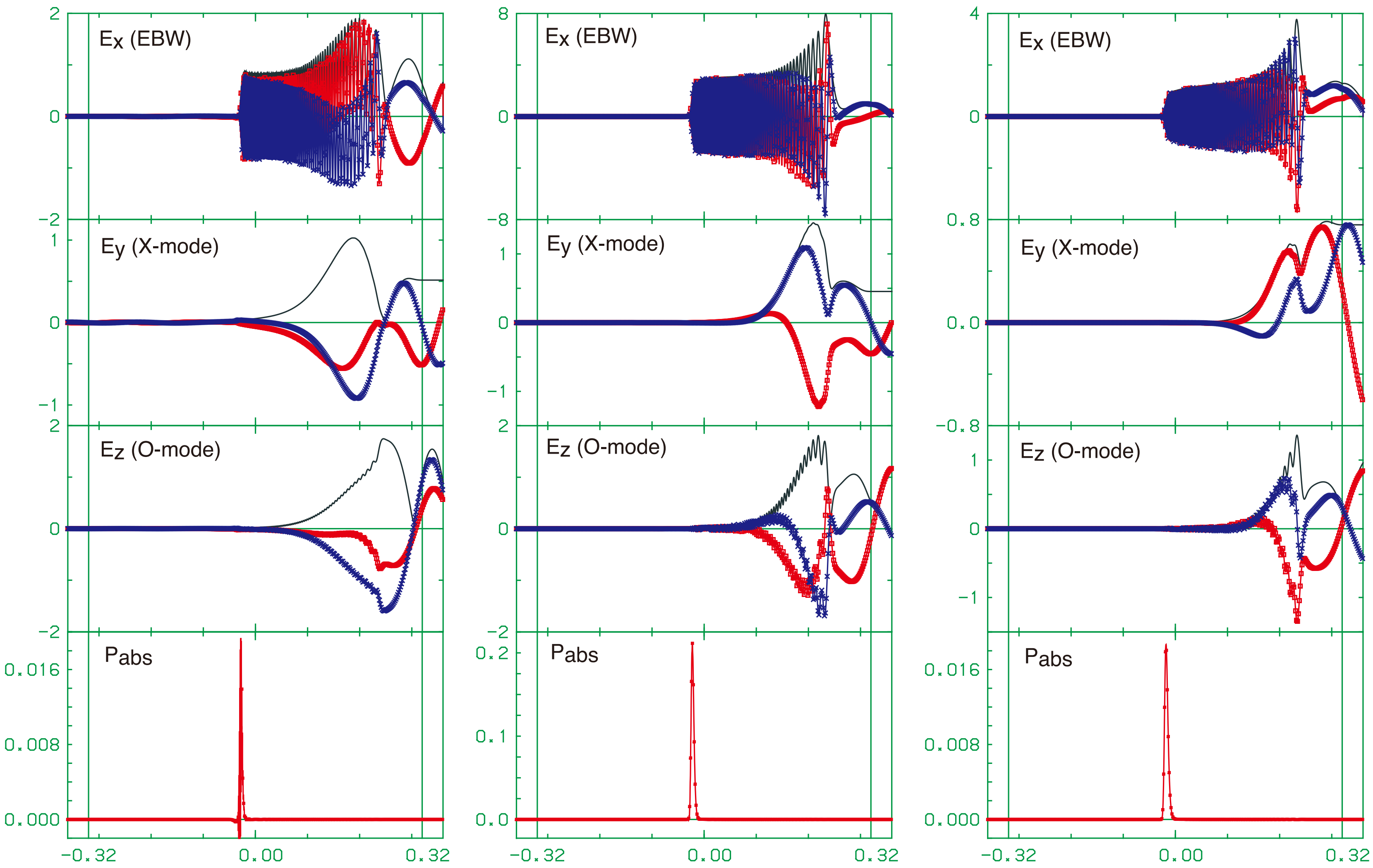
$k_{||} = 40\,\mathrm{m}^{-1}$
shallow O cutoff



Wave structure: dependence on $k_{||}$

Kinetic full wave analysis using integral form of $\overleftrightarrow{\epsilon}$

$k_{\|\|} = 24\,\mathrm{m}^{-1}$ deep X cutoff	$k_{\|\|} = 32\,\mathrm{m}^{-1}$ optimum angle	$k_{\|\|} = 40\,\mathrm{m}^{-1}$ shallow O cutoff



Competition between collisional and cyclotron damping

- Optimum injection angle: $k_{||} = 32\,\mathrm{m}^{-1}$

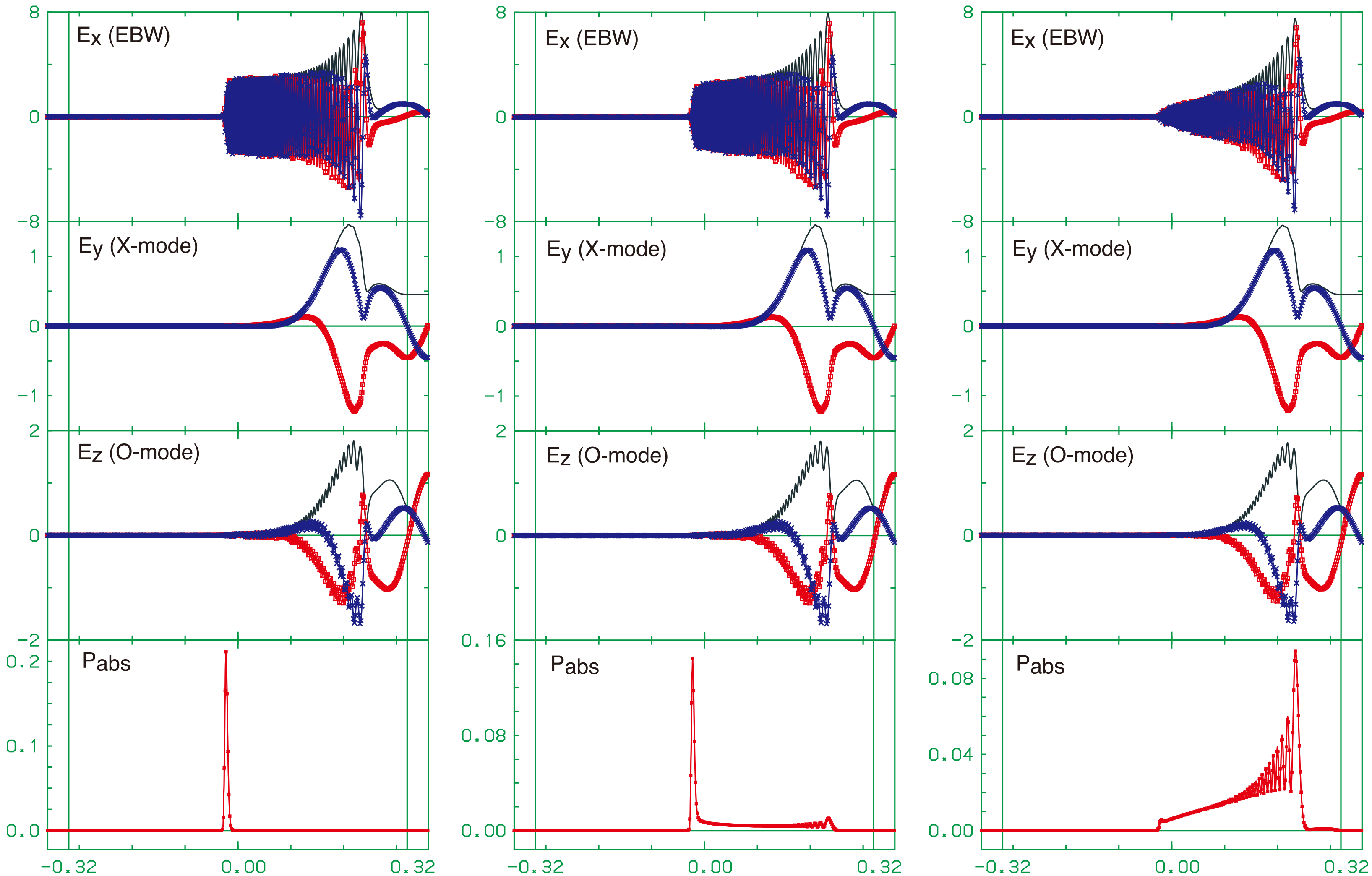
no collision | $\nu/\omega = 10^{-5}$ | $\nu/\omega = 10^{-4}$

EBW

X-mode

O-mode

P_{abs}

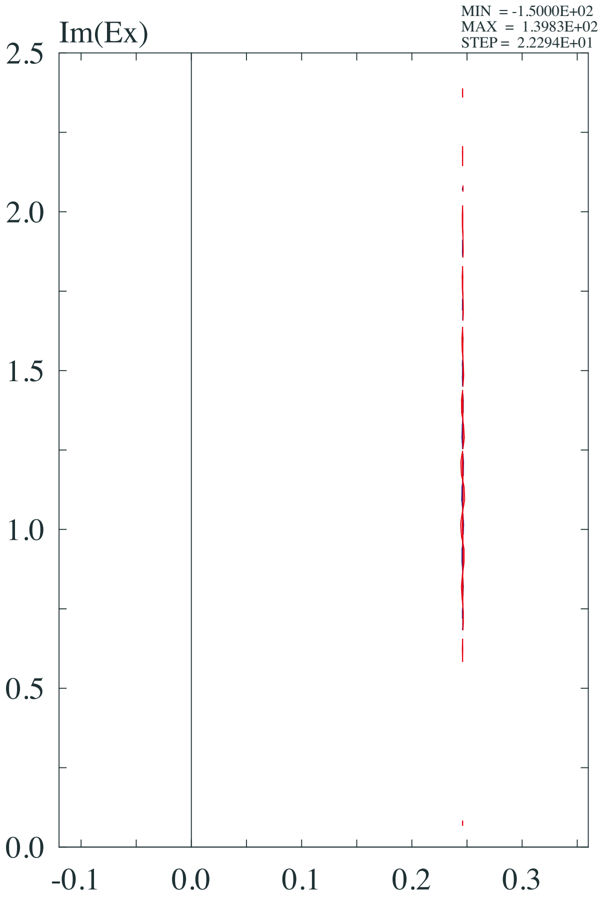


Consistent with NSTX experimental results (Diem et al. PRL, 2009)

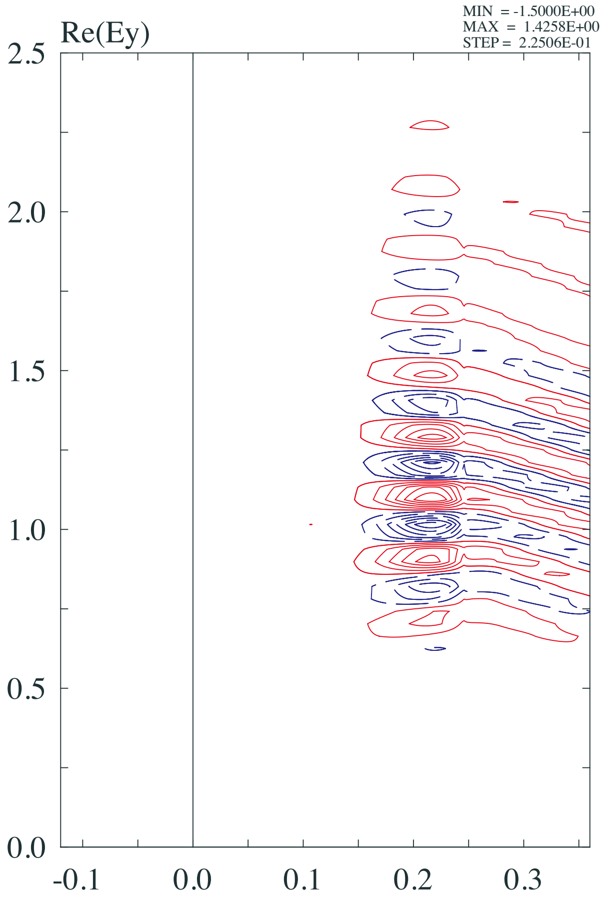
Two-dimensional analysis of O-X-B mode conversion

Cold plasma analysis with collisional absorption at UHR
Mode conversion of O-mode excited by waveguide antenna

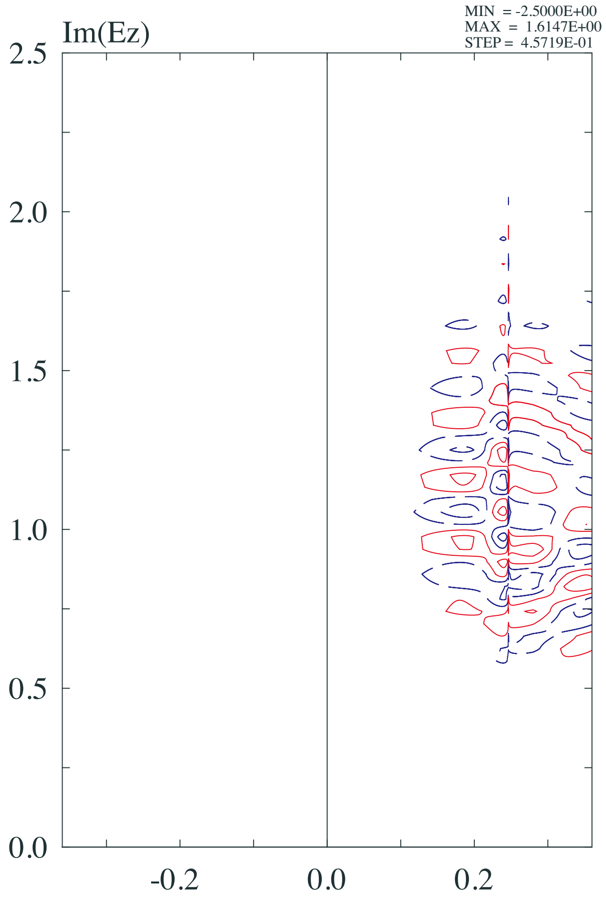
$\mathrm{Im} E_x(x, z)$
UHR



$\mathrm{Re} E_y(x, z)$
X-mode



$\mathrm{Im} E_z(x, z)$
O-mode

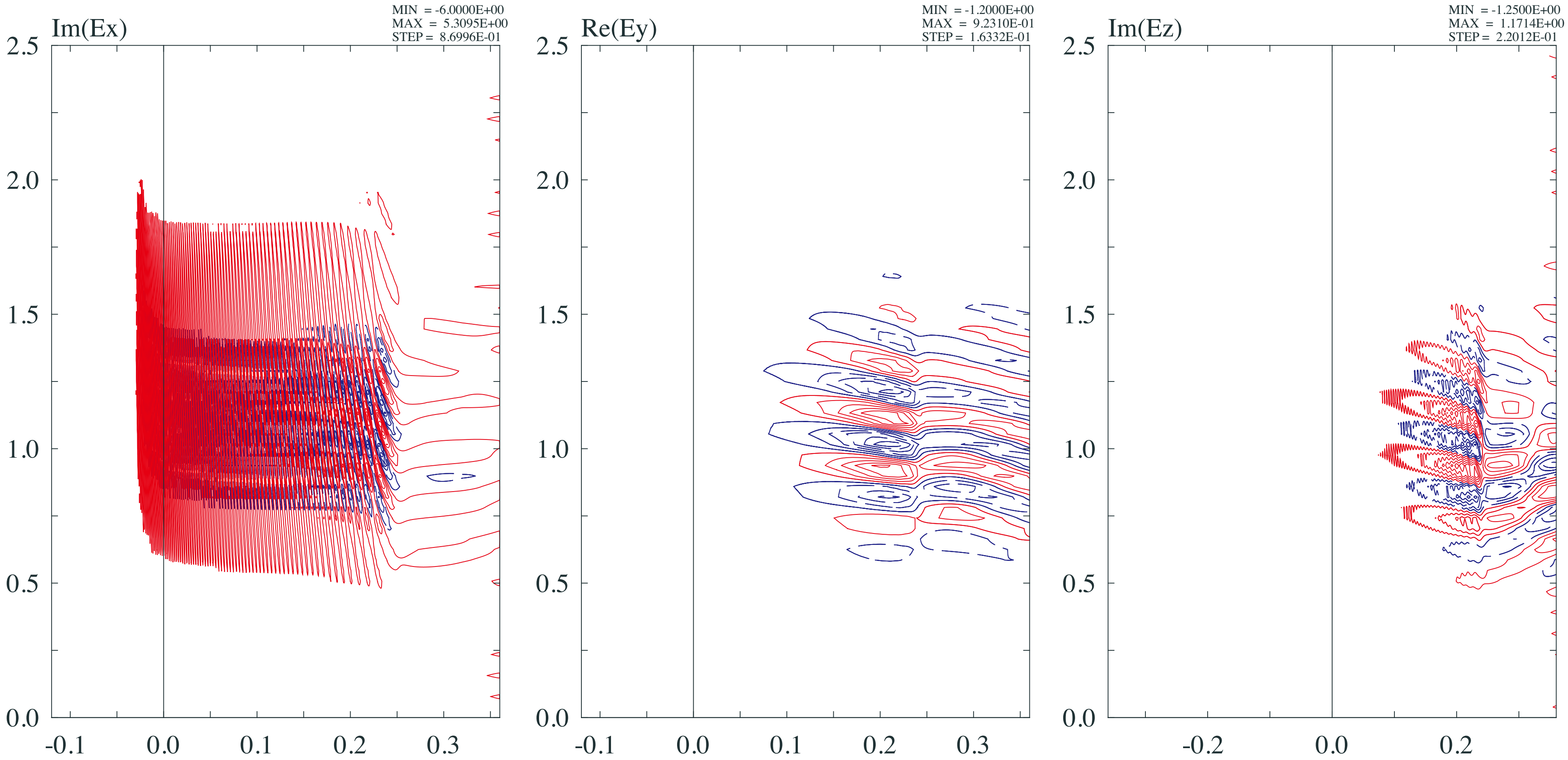


Two-dimensional analysis of O-X-B mode conversion

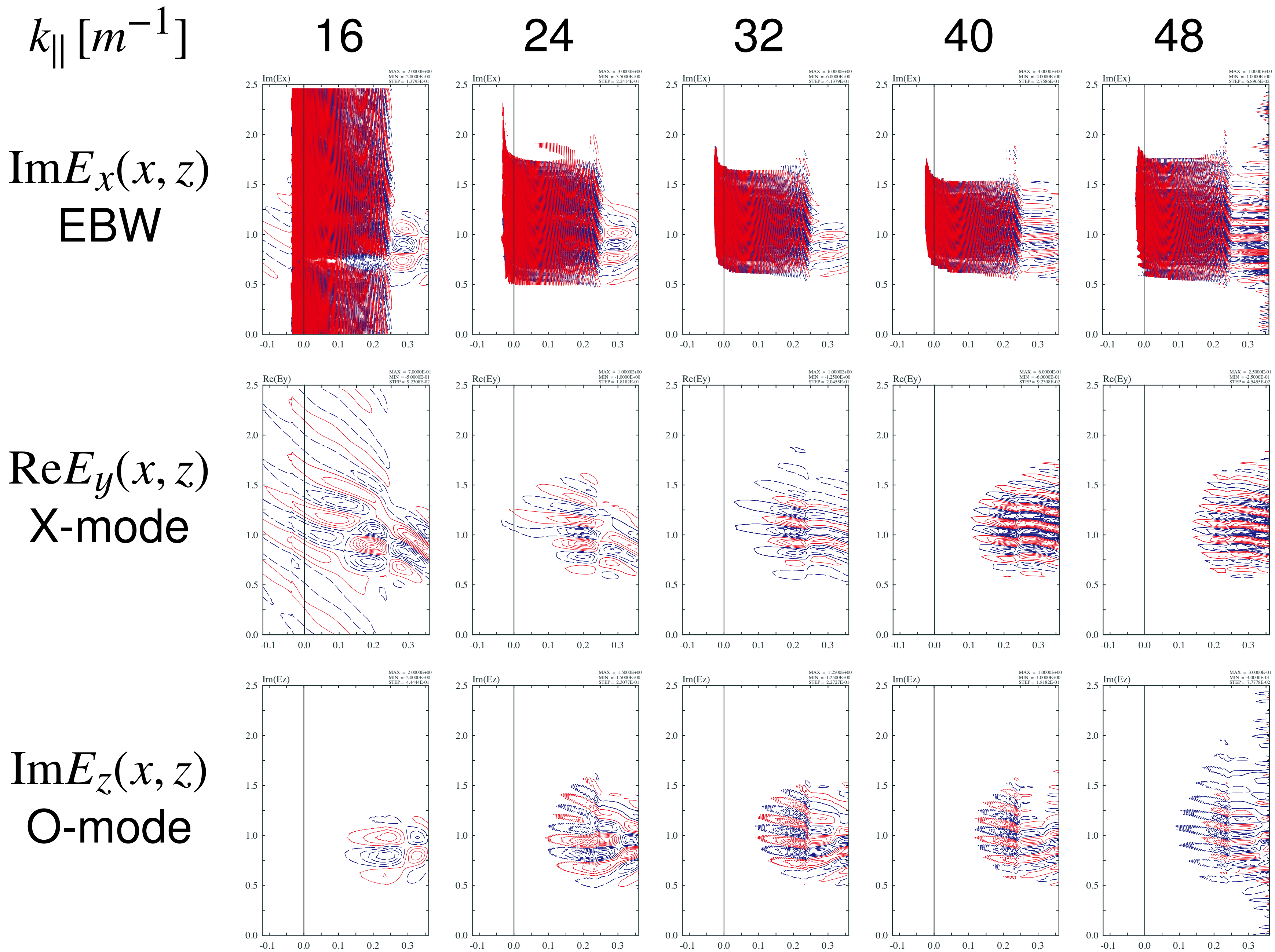
Kinetic full wave analysis using integral form of $\overleftrightarrow{\epsilon}$

Mode conversion of O-mode excited by waveguide antenna

$\mathrm{Im}E_x(x,z)$	$\mathrm{Re}E_y(x,z)$	$\mathrm{Im}E_z(x,z)$
EBW	X-mode	O-mode



Injection angle dependence of O-X-B mode conversion



Summary and works in progress

- **Kinetic full wave analysis using integral form of dielectric tensor** has been successfully applied to various inhomogeneous hot plasmas:
  - **kinetic resonance absorption**: unmagnetized, n_{e} gradient
  - **magnetic beach heating**: non-uniform magnetic field
  - **absorption of fast ions**: FLR effects
  - **Bernstein waves**: FLR effects, mode-conversion
- **Works in progress**
  - **Application to realistic 2D configuration**
    - Poloidal cross section in tokamak configuration
    - Axial plane in axisymmetric mirror configuration
  - **Arbitrary velocity distribution function**
  - **Relativistic effects**
  - **Application to the analysis of kinetic instabilities**