25th Topical Conference on Radio-frequency Power in Plasmas (RFPPC 2025)
from May 19-22, 2025 in Castle Hohenkammer, Germany

Spectrum Evolution and Density Limit in Self-consistent Modeling of Lower Hybrid Wave Propagation with Parametric Instabilities

Zhe Gao, Kunyu Chen, Zhihao Su, Zikai Huang, Long Zeng

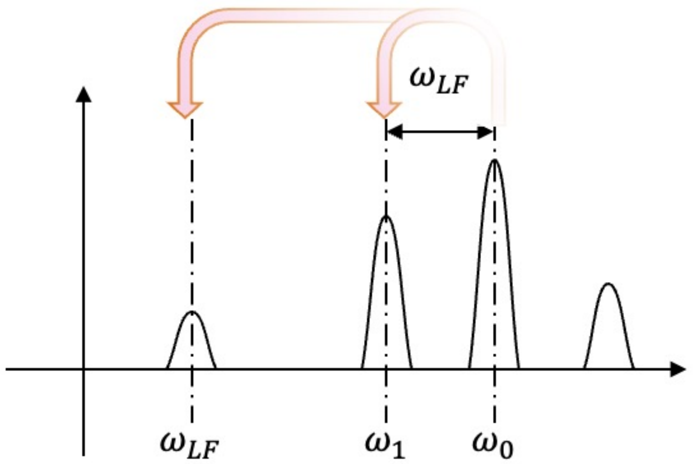
Department of Engineering Physics, Tsinghua University, Beijing 100084

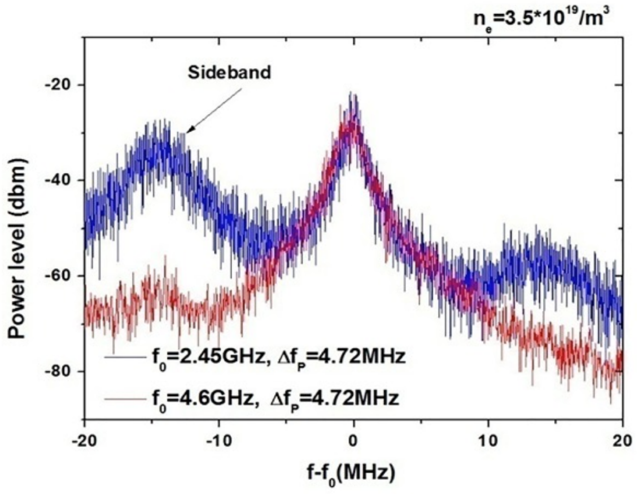
gaozhe@tsinghua.edu.cn

Tsinghua University

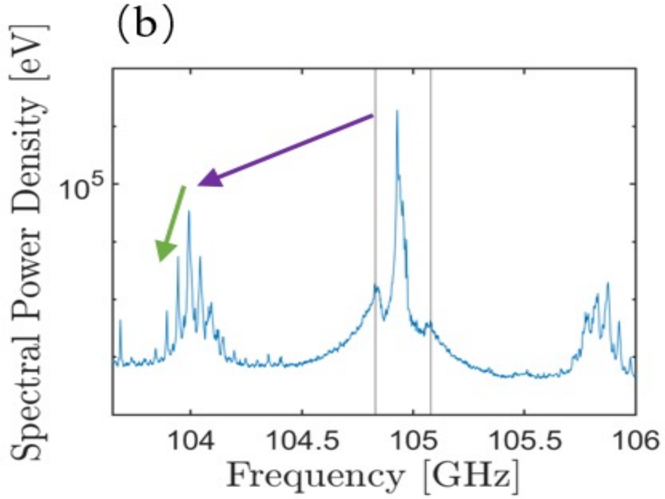
Background

- Parametric Instability (PI or PDI) is a common and important nonlinear wave-wave interaction.
- PDIs result in a degradation of efficiency of H&CD and/or unexpected power deposition. For example, PDIs at the edge may responsible for the failure of LHCD in high density plasma.





LHW@EAST MH Li 2022



ECRH@AUG Hansen 2019

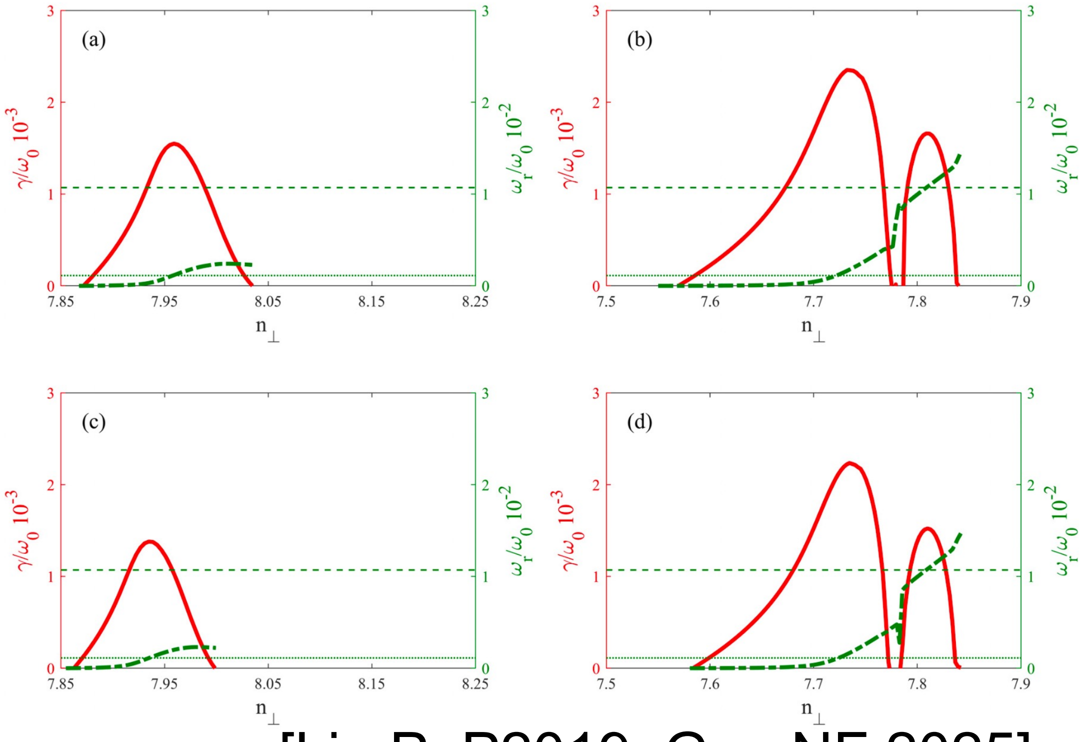
Background

- Earlier theories of parametric instability established in the 1970s, mainly focusing on stimulated scattering in laser plasma [Rosenbluth 1972, 1973, White 1974, Cohen 1976, Chen 1977, Liu 1986]
  - ✓ Usually resonant decay with only quasilinear treatment
  - ✓ Convective instability is saturated by wavenumber mismatch due to plasma inhomogeneity or finite width of the pump.
  - ✓ Usually absolute instability is excited in laser plasma, so other nonlinearities should be involved

Background

- In fusion plasma, PDI is typically displaying as quasi-mode decay, such as nonlinear ion landau damping（ion sound quasi-mode decay， ISQM） or ion cyclotron harmonic quasi-mode decay（ICQM）.
- Difficulties are coming：
  - ✓ Quasilinear coupling is not enough（A fluid-kinetic hybrid approach [Liu 1986] and a approach by integrating along an unperturbed orbit with pump field [Porkolab 1974]）
  - ✓ Mismatch of the wavenumber due to plasma inhomogeneity cannot be given by the linear dispersion relation
  - ✓ Therefore，both absolute instability and convective instability not properly calculated

Coupling for quasi-mode decay



[Liu PoP2019, Gao NF 2025]

$$f_{sj} = f_{sj}^{\mathrm{L}} + f_{sj}^{\mathrm{NL}}$$

$$f_{sj}^{\mathrm{L}} = \mathbb{L}\left(f_{\mathrm{M}s}, \boldsymbol{E}_j\right)$$

$$f_{s,\mathrm{LF}}^{\mathrm{NL}} = \mathbb{C}\left(f_{s,0}, \boldsymbol{E}_1\right) + \mathbb{D}\left(f_{s,1}, \boldsymbol{E}_0\right)$$

$$f_{s,1}^{\mathrm{NL}} = \mathbb{E}\left(f_{s,0}, \boldsymbol{E}_{\mathrm{LF}}\right) + \mathbb{F}\left(f_{s,\mathrm{LF}}, \boldsymbol{E}_0\right)$$

QL-QL：

$$f_{s,\mathrm{LF}}^{\mathrm{QL}} = \mathbb{C}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_1\right) + \mathbb{D}\left(f_{s,1}^{\mathrm{L}}, \boldsymbol{E}_0\right)$$

$$f_{s,1}^{\mathrm{QL}} = \mathbb{E}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_{\mathrm{LF}}\right) + \mathbb{F}\left(f_{s,\mathrm{LF}}^{\mathrm{L}}, \boldsymbol{E}_0\right)$$

resonant decay： $f^{\mathrm{L}} \gg f^{\mathrm{NL}}$

QL-NL：

$$f_{s,\mathrm{LF}}^{\mathrm{NL}} \approx f_{s,\mathrm{LF}}^{\mathrm{QL}} = \mathbb{C}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_1\right) + \mathbb{D}\left(f_{s,1}^{\mathrm{L}}, \boldsymbol{E}_0\right)$$

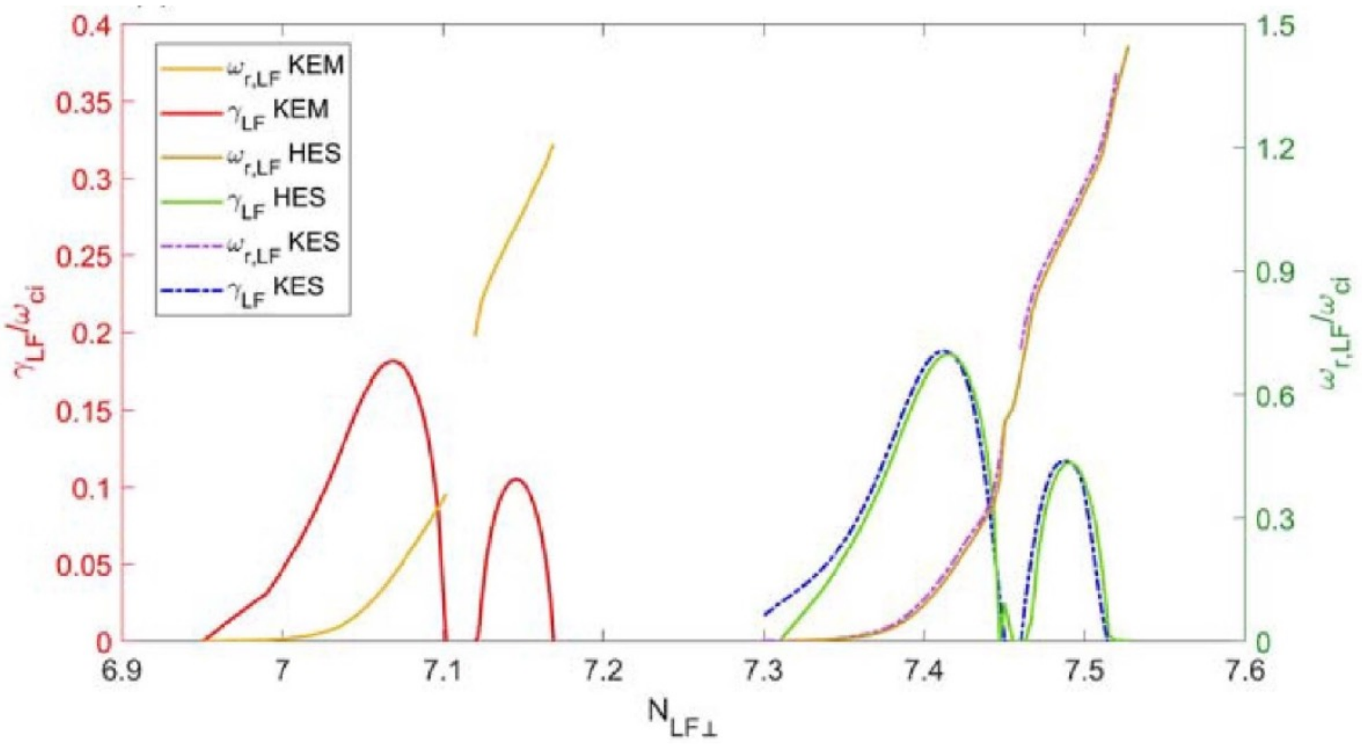
$$f_{s,1}^{\mathrm{NL}} \approx \mathbb{E}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_{\mathrm{LF}}\right) + \mathbb{F}\left(f_{s,\mathrm{LF}}^{\mathrm{L}} + f_{s,\mathrm{LF}}^{\mathrm{QL}}, \boldsymbol{E}_0\right)$$

quasi-mode decay： $f_{\mathrm{LF}}^{\mathrm{L}} \sim f_{\mathrm{LF}}^{\mathrm{NL}}$, $f_1^{\mathrm{L}} \gg f_1^{\mathrm{NL}}$

Fortunately, the coupling is mainly ES and fluid

$$\begin{bmatrix} \boldsymbol{D}_{\mathrm{LF}} & \tilde{\boldsymbol{\sigma}}^{(1)}_{\mathrm{LF}\leftarrow 1} \\ \tilde{\boldsymbol{\sigma}}^{(1)}_{1\leftarrow \mathrm{LF}} & \boldsymbol{D}_1 + \tilde{\boldsymbol{\sigma}}^{(2)}_{1\leftarrow \mathrm{LF}\leftarrow 1} \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_{\mathrm{LF}} \\ \boldsymbol{E}_1 \end{bmatrix} = \boldsymbol{0} \xrightarrow{\text{ES Limit}} \begin{vmatrix} \varepsilon_{\mathrm{LF}} & \chi^{(1)}_{\mathrm{LF}\leftarrow 1} \\ \chi^{(1)}_{1\leftarrow \mathrm{LF}} & \varepsilon_1 + \chi^{(2)}_{1\leftarrow \mathrm{LF}\leftarrow 1} \end{vmatrix} = 0 \xrightarrow{\substack{\mu_1^* = \chi^{(1)}_{1\leftarrow \mathrm{LF}}\chi^{(1)}_{\mathrm{LF}\leftarrow 1} \\ -\varepsilon_{\mathrm{LF}}\chi^{(2)}_{1\leftarrow \mathrm{LF}\leftarrow 1}}} \varepsilon_{\mathrm{LF}} = \frac{\mu_1^*}{\varepsilon_1}$$

same explicit form as from the fluid-kinetic hybrid approach

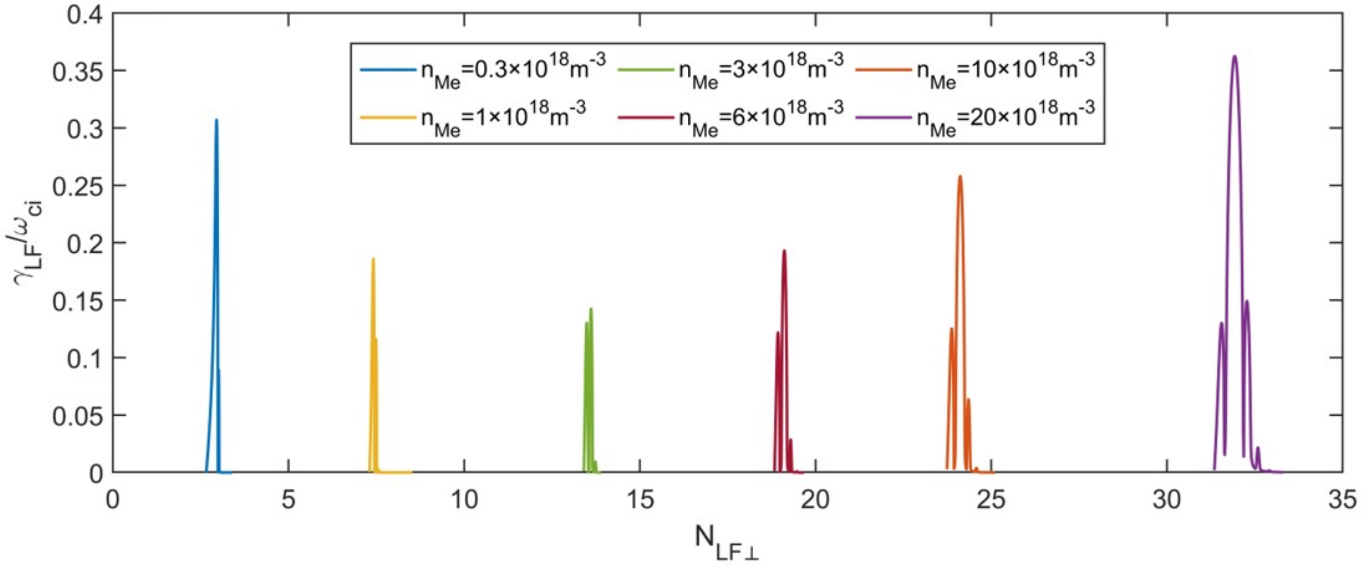


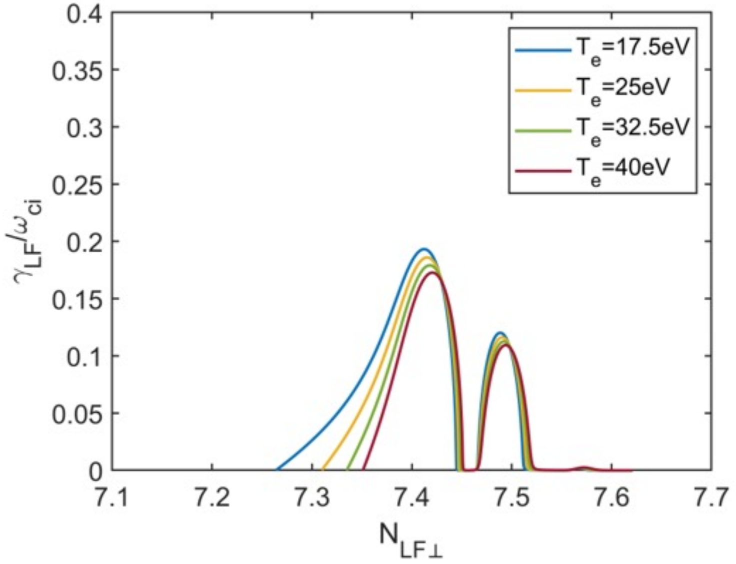
LHW@JET $N_{1z} = 3$ $\delta_1 = 90°$ [Z Gao 2025]

- Noted that kinetic effect in the linear response and EM effect in the linear dispersion relation of the pump should be kept.

Main results from the local model (with homogeneous wave and plasma)

- ISQM decay is usually suppressed by increasing plasma density
- ICQM decay is destabilized by increasing plasma density
- Both suppressed by plasma temperature and magnetic field





LHW@JET $N_{1z}=3$ $\delta_1=90°$ [Z Gao 2025]

From local to nonlocal: to characterize the inhomogeneity

PDI Equation（ES）:
$$\begin{cases} \varepsilon_{\mathrm{LF}}\phi_{\mathrm{LF}} = \alpha_{\mathrm{LF}\leftarrow 1}\phi_0^*\phi_{\mathrm{LF}} \\ \varepsilon_1\phi_1 = \alpha_{1\leftarrow \mathrm{LF}}\phi_0\phi_1 \end{cases}$$

Rosenbluth approach: for resonant decay ($\varepsilon_j \sim 0$)

$$\Phi_j = \phi_j(x,t)\exp\left[\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x} - \mathrm{i}\omega_j t + \mathrm{i}\int \Delta k_j\, x\right]$$

$$\varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}}, k_j) \to \varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}} + i\partial_t, k_j - i\partial_x)$$

$$\left[\frac{v_j}{v_{jg}} + \partial_x + \frac{1}{v_{jg}}\cdot\partial_t\right]\phi_j \exp\left[i\int \Delta k_j dx\right] = \frac{i\alpha_{i\to j}}{\partial\epsilon_j/\partial k_j}\phi_0\phi_i \exp\left[i\int(\Delta k_0 + \Delta k_i)dx\right]$$

$$\kappa = \Delta k_{\mathrm{LF}} - \Delta k_1 - \Delta k_0$$

From local to nonlocal：to characterize the inhomogeneity

PDI Equation（ES）:

$$\begin{cases} \varepsilon_{\mathrm{LF}}\phi_{\mathrm{LF}} = \alpha_{\mathrm{LF}\leftarrow 1}\phi_0^*\phi_{\mathrm{LF}} \\ \varepsilon_1\phi_1 = \alpha_{1\leftarrow \mathrm{LF}}\phi_0\phi_1 \end{cases}$$

Rosenbluth approach: for resonant decay ($\varepsilon_j \sim 0$)

$$\Phi_j = \phi_j(x,t)\exp\left[\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x} - \mathrm{i}\omega_j t + \mathrm{i}\int\Delta k_j \mathrm{d}t\right]$$

$$\varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}}, k_j) \rightarrow \varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}} + i\partial_t, k_j - i\partial_x)$$

Our approach: for quasi-mode decay ($\varepsilon_{\mathrm{LF}} \gg 1$)

$$\Phi_j = \phi_j(x,t)\exp\left[\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x} - \mathrm{i}\omega_j t\right]$$

$$\varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}}, k_j, x=0) \rightarrow \varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}} + i\partial_t, k_j - i\partial_x, x)$$

$$\left[\boxed{\varepsilon_j(\omega_j,k_j)} + \boxed{\frac{\partial\varepsilon_j}{\partial\omega_j}\cdot\mathrm{i}\partial_t - \frac{\partial\varepsilon_j}{\partial k_j}\cdot\mathrm{i}\partial_x} + \boxed{\frac{\partial\varepsilon_j}{\partial x}\cdot x}\right]\phi_j(x,t) = \alpha_{j\leftarrow i}\phi_0(x,t)\phi_i(x,t)$$

Local term

temporal and spatial evolution

Plasma inhomogeneity to replace Δk_j

Eigenvalue problem of the nonlocal equations

- Eliminating time, the nonlocal coupling equations turn to Schrödinger form:

$$\left[\frac{\partial^2}{\partial X^2}-\boxed{\left(\frac{1}{4}\left(\left(\mathrm{i}K_1+K_2\right)X+\sigma\right)^2-\lambda-\frac{\mathrm{i}K_1}{2}\right)}\right]A_1\left(X,p\right)=\boxed{A_{\mathrm{LF}}\left(X\right)|_{t=0}}$$

The Hamiltonian is non Hermitian!

Initial value

- Dimensionless parameters: coordinate X, amplitude A, plasma inhomogeneity K_1, finite pump profile K_2, growth and/or damping rate σ (complex), coupling coefficient λ
- absolute instability: not an initial value (natural boundary condition)
- convective instability: with initial value (Laplace transform $t\rightarrow p$):

$$\phi_j\left(x,p\right)=\int_0^{\infty}\phi_j\left(x,t\right)e^{-pt}\mathrm{d}t$$

On the absolute instability

- Absolute instability: PDI can't be saturated by plasma inhomogeneity, the solutions are the superposition of time eigenmodes

$$\phi_j\left(x,t\right)=\sum_{p_k>0}\phi_{j,k}\left(x\right)e^{p_k t}$$

- Estimating the threshold of a quasi-mode decay with a WKB analysis on the complex plane for finite pump profile:

$$\lambda>\exp\left[\frac{1}{4}\mathrm{Re}\left(\sigma^2\right)\right]$$

  – Due to exponential relation between coupling coefficient and damping rate, there is no absolute instability for quasi-mode decay in MCF plasma
  – Back to resonant decay $\lambda>1+\frac{1}{4}\sigma^2$ with weak damping [White 1974]

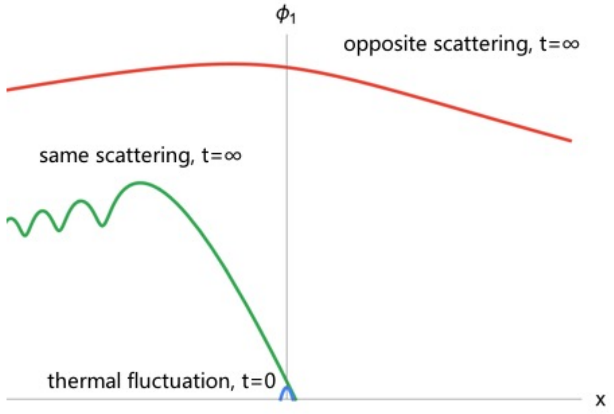
[Chen and Gao, Communications in Theoretical Physics, 2025]

On the convective instability: resonant decay

- Convective instability: PDIs are saturated by plasma inhomogeneity, the steady-state solution can be get by setting $p=0$

$$\frac{\partial \phi_j}{\partial t} = p\phi_j\left(x,p\right)+\phi_j\,|_{t=0}$$

- The solutions are different in different scattering direction and the convective amplification factor are defined as:
  - Same scattering direction ($\left|v_{1gx}v_{\mathrm{LF}gx}\right|>0$) $A=\ln\left|\frac{\phi_1(\infty)}{A^{(0)}}\right|$
  - Opposite scattering direction ($\left|v_{1gx}v_{\mathrm{LF}gx}\right|<0$) $A=\ln\left|\frac{\phi_1(0)}{A^{(0)}}\right|$



- For resonant decay, similar results obtained as in Rosenbluth with a tiny correction [Chen and Gao, PPCF 2025]

On the convective instability: quasi-mode decay

- No opposite scattering in quasi-mode decay due to strong damping.
- For same-direction scattering, the convective amplification factor is,

plasma inhomogeneity

$$A = \boxed{\int \frac{\mathrm{d}x}{v_{1gx}}}\boxed{\gamma_0} = \int \mathrm{d}t\gamma_0$$

Finite pump width, actually finite trajectory length

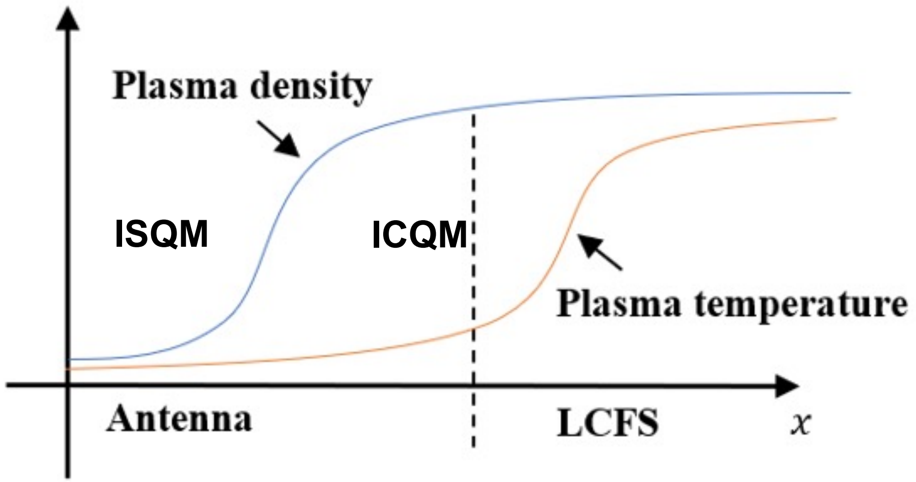
  – the PDI growth rate

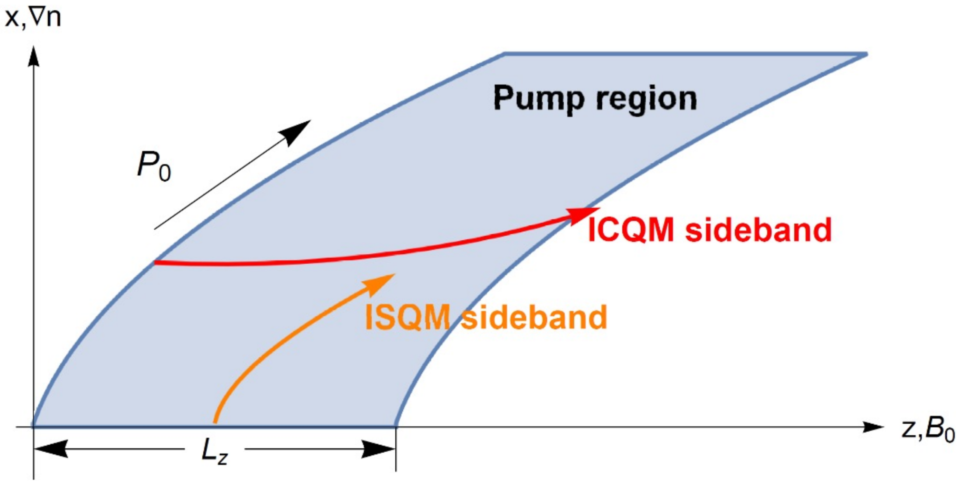
$$\gamma_0 = \mathrm{Im}\left[\frac{\alpha_{\mathrm{LF}\leftarrow 1}\alpha_{1\leftarrow \mathrm{LF}}\left|\phi_0\right|^2}{\varepsilon_{\mathrm{LF}}\left(\partial\varepsilon_1/\partial\omega_1\right)}\right] - \nu_1$$

[Chen and Gao, NF 2025]

Nonlocal model around the SOL in tokamak

- Decay channels: ISQM+ICQM
- Plasma inhomogeneity exits in x-direction (radial)
- Finite pump profile exits mainly in z-direction (toroidal) $v_{1gz} >> v_{1gy}$





PIPERS code： ray tracing + PDI

- Energy conservation equations constrained by PDIs

$$\begin{cases} \nabla\cdot \boldsymbol{P}_0(\boldsymbol{r}) + \sum\limits_{\omega_1,\boldsymbol{k}_1}\left[\nabla\cdot \boldsymbol{P}_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1) - 2\gamma_{1L}(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)U_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)\right] = 0 \\ U_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1) = U_{\text{th}}(\boldsymbol{k}_{\text{LF}})\exp(2A) \end{cases}$$

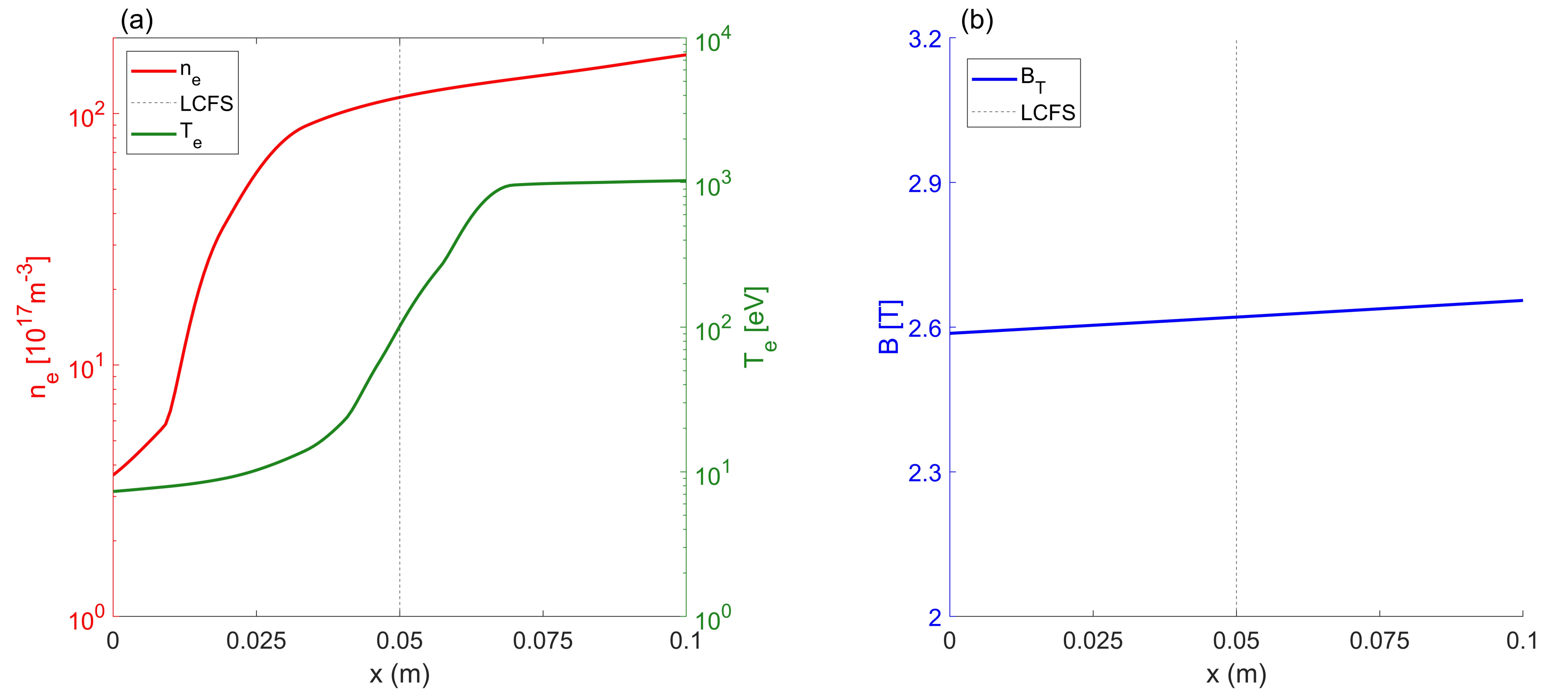
  - The pump damping and low frequency wave terms are omitted
  - The initial value of PDI is the electrostatic thermal noise

$$U_{\text{th}} = \frac{1}{2}\left(1+\frac{\omega_{\text{pe}}^2}{\omega_{\text{ce}}^2}\right)\frac{T}{1+\lambda_{\text{De}}^2 k_{\text{LF}}^2}\,\mathrm{d}\boldsymbol{k}_{\text{LF}}$$

- The convective amplification factor A is integrated along the trajectories of the sideband waves around the SOL

Details in poster of Zikai Huang, Monday-22

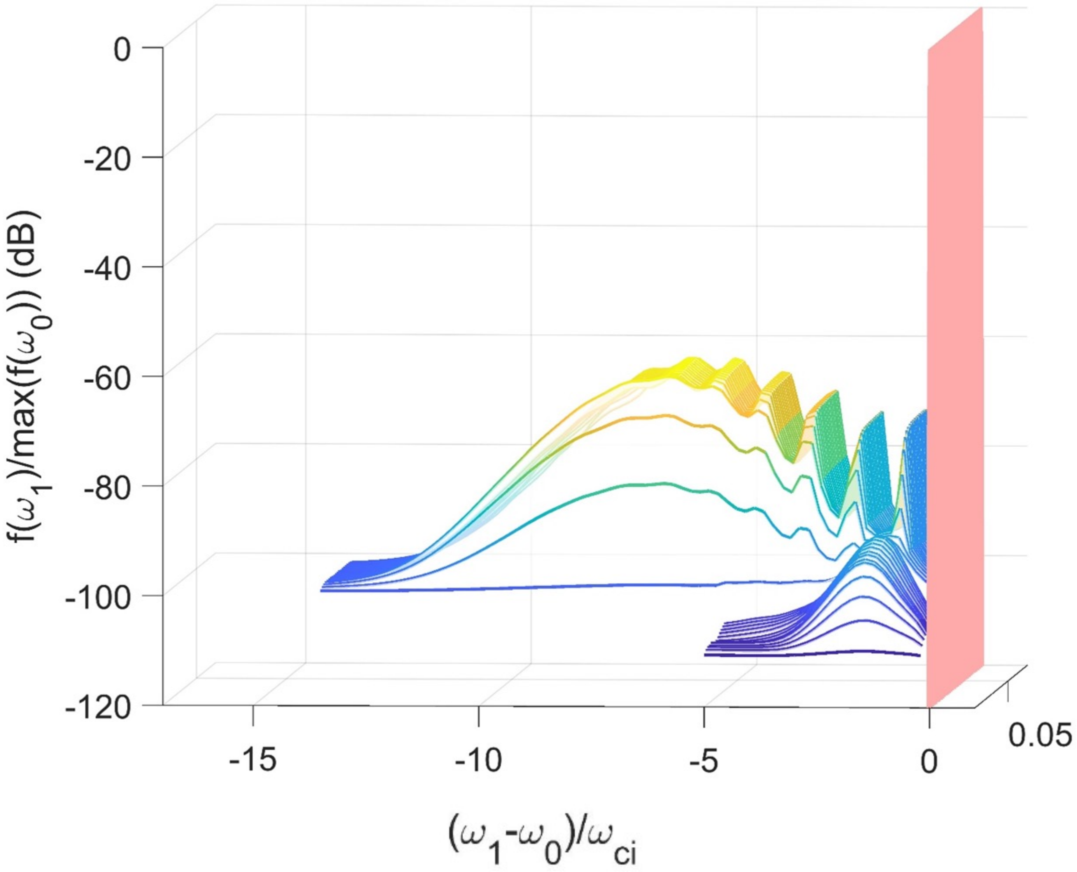
An Example for LHCD experiment

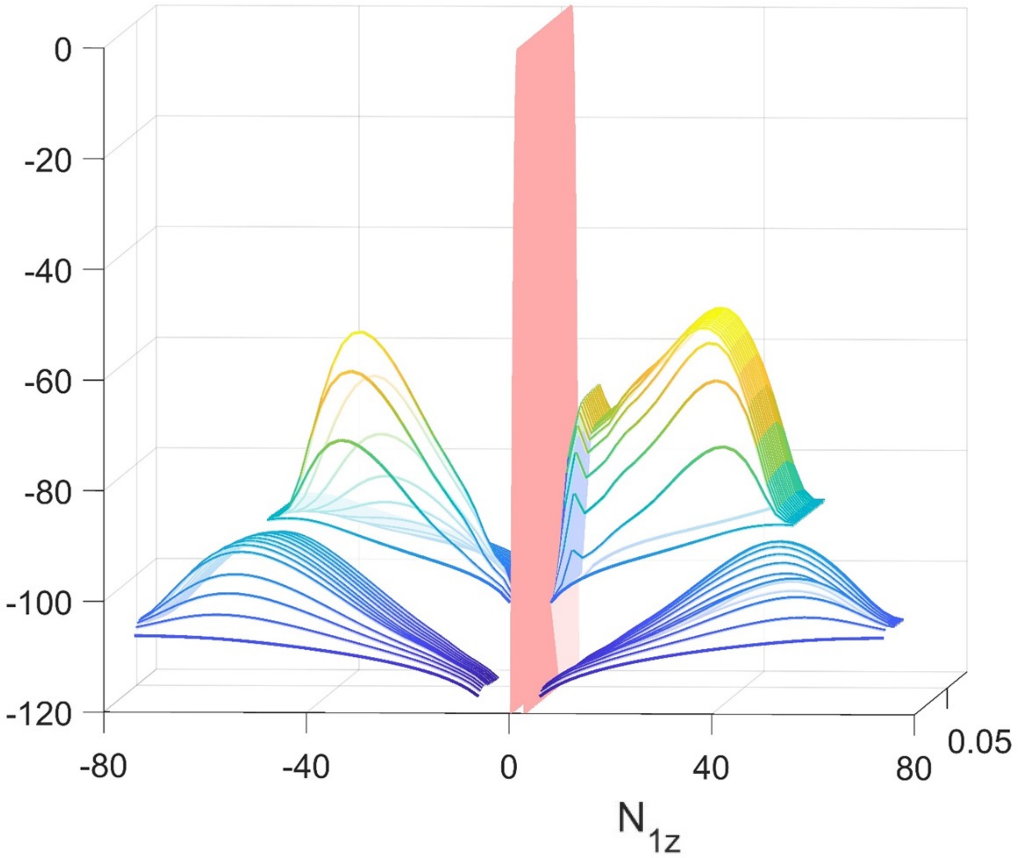


Item	Value
Frequency, f_0	3.7GHz
Parallel refractive index, n_{0z}	1.84
Large radius coordinates, R_{antenna}	3.89m
Power, P_0	2MW
The size of the antenna mouth, $L_y \times L_z$	0.884m×0.348m

[parameter profiles @JET Cesario 2006]

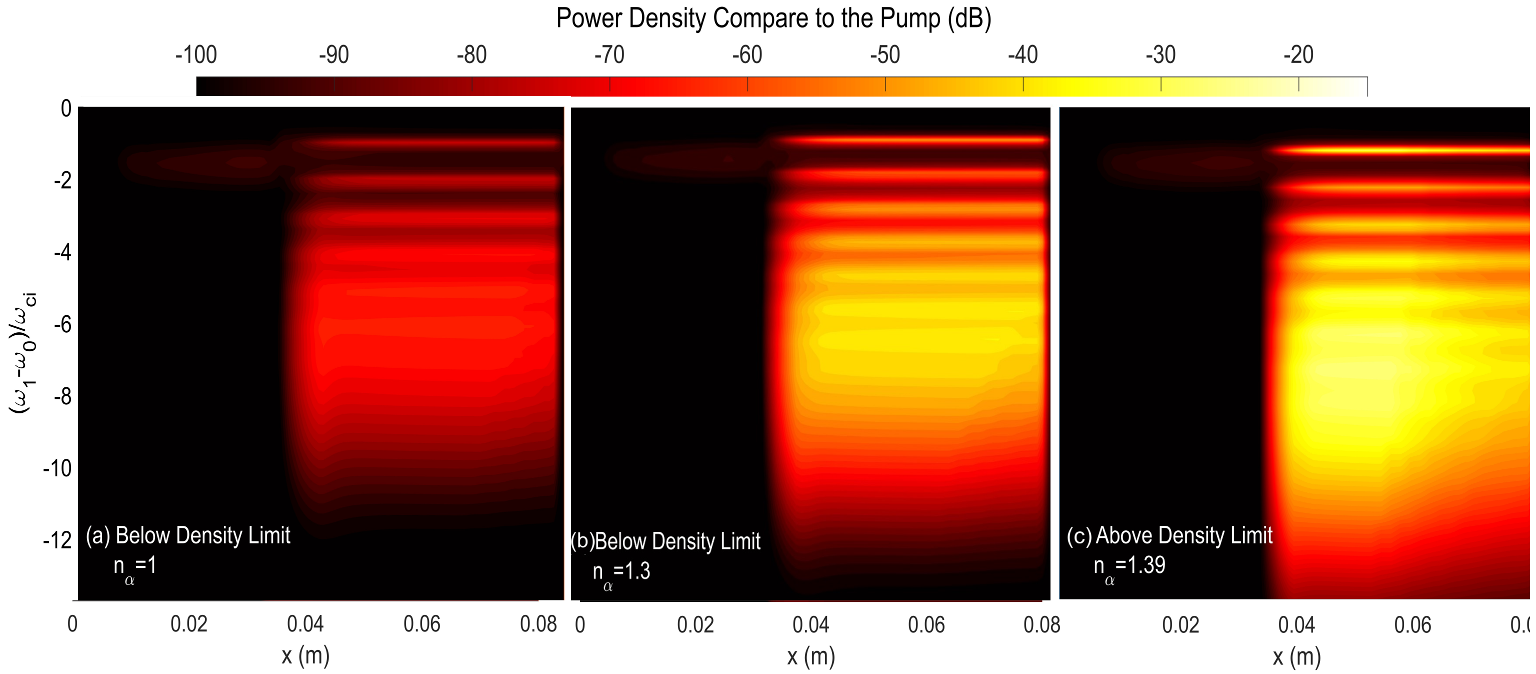
Spectrum evolution and power transfer

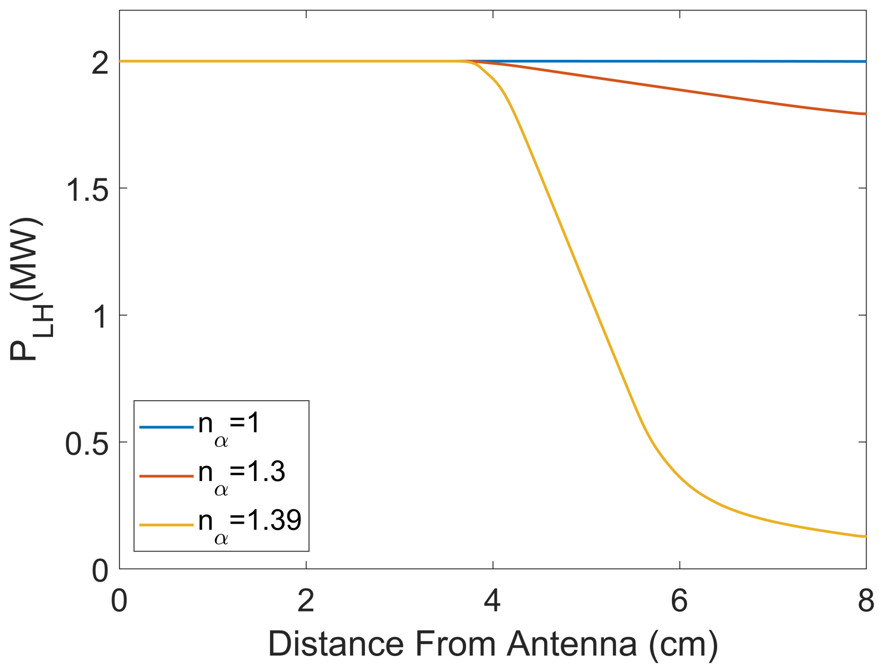




Frequency sidebands and wavenumber broadening appears
But no significant power transfer occurs when PDI is weak

The density limit observed in simulation





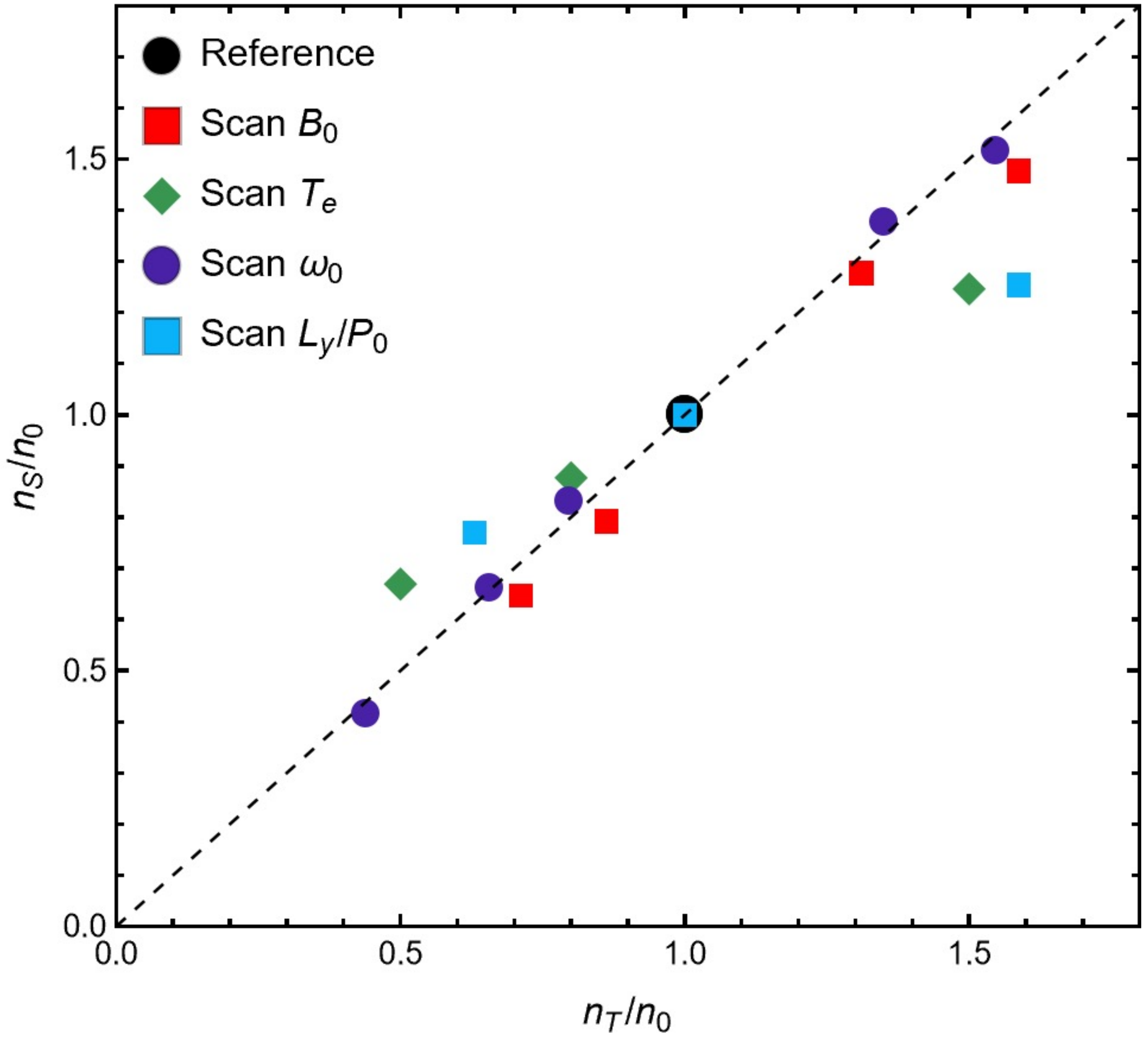
When the density increase above a limit, PDI become stronger and the pump is exhausted due to significant power transferring to sidebands

A theoretical scaling relation of the density limit

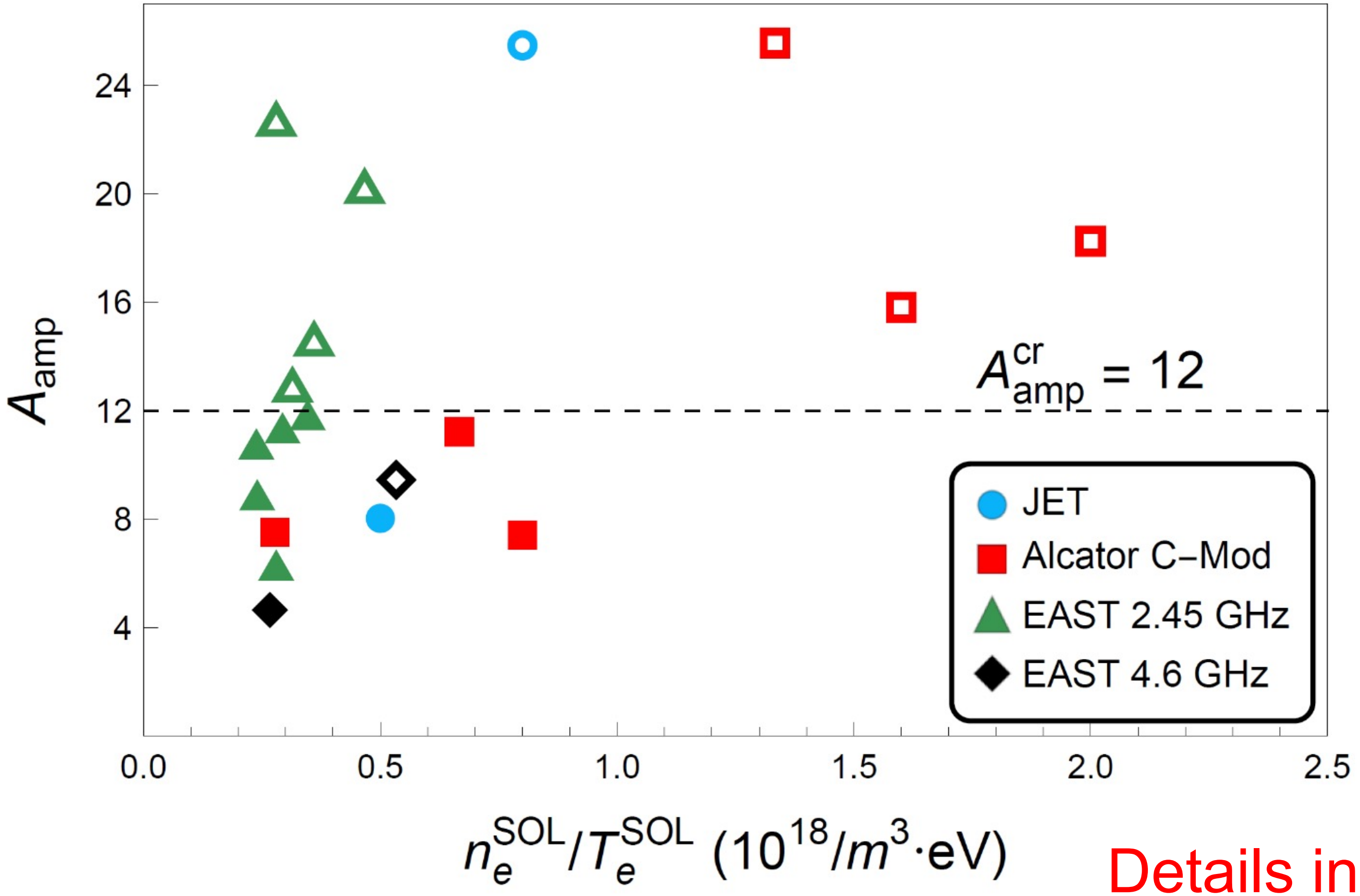
$$A_{\mathrm{amp}} \propto P_0 L_y^{-1} \omega_{LH}^3 \omega_0^{-3} B_0^{-2} T_e^{-3/2}$$

$$n_{\mathrm{PDI}} \propto P_0^{-2/3} L_y^{2/3} \omega_0^2 B_0^{4/3} T_e$$

- The simulation results agree quite well with the theoretical scaling relation



A critical convective amplification factor may be found

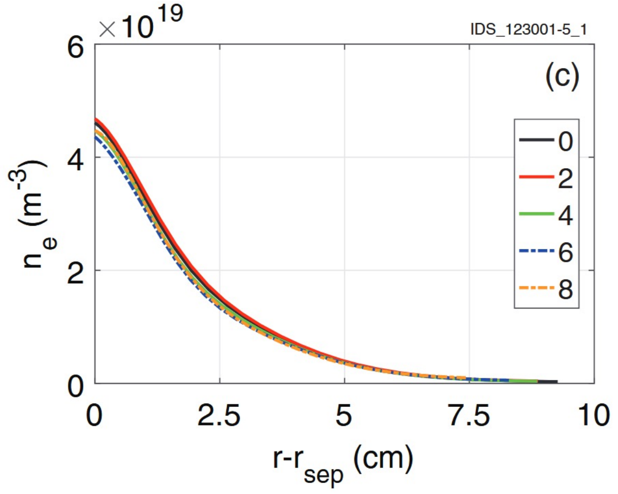


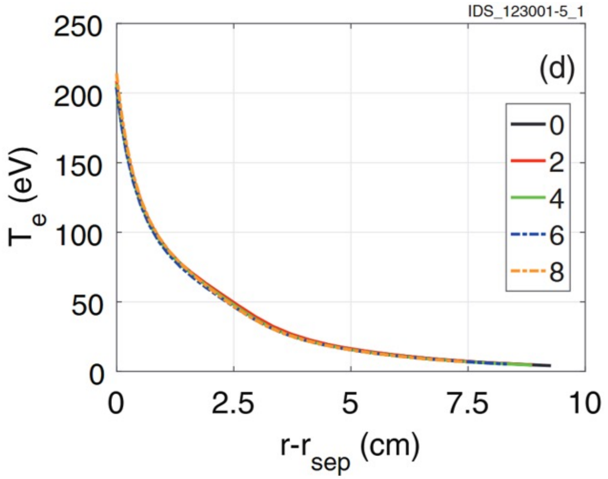
- The hollow points indicate significant degradation of LHCD efficiency
- More targeted experiments with precise diagnostics required

Details in poster of Kunyu Chen, Tuesday-13

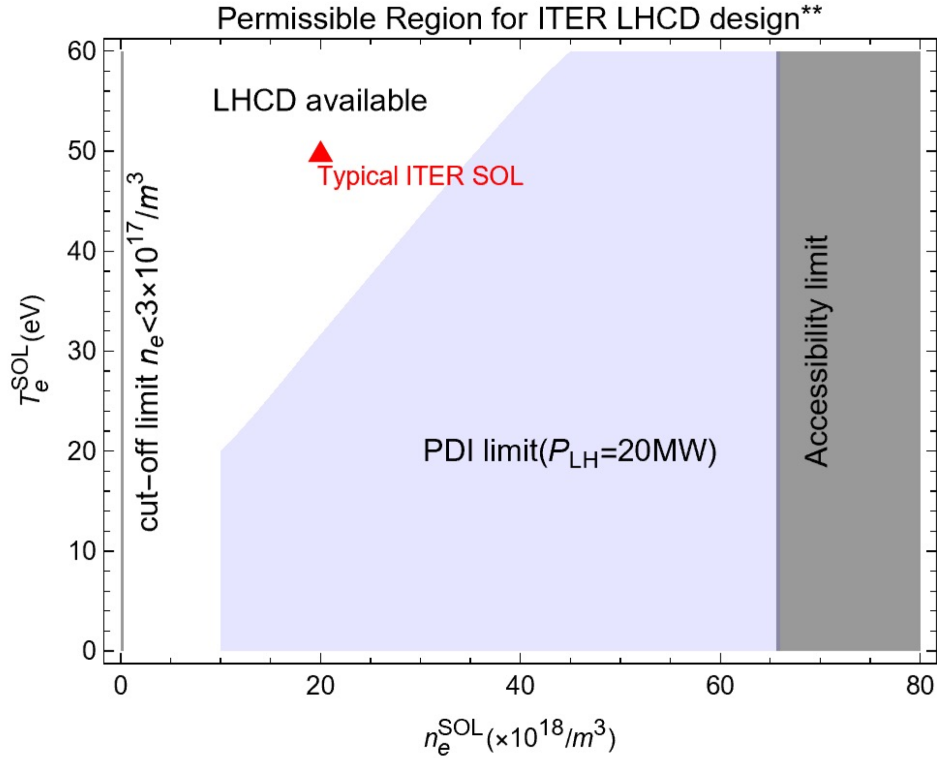
Discussion： LHCD on ITER

- According to the simulation results based on the SOL parameters simulated on ITER, the density is far from the density limit on ITER due to the high temperature around the SOL
- It may indicate that LHCD on ITER is still effective……





[Carli 2018]



Summary

- The PDI theory, especially the nonlocal theory of PDI saturation, is extended to adapt to the scenarios where non-resonant quasi-mode decays are dominant.
- A self-consistent modeling and simulation of LHWs in the SOL plasma by coupling the propagation of waves to the power transfer among waves by PDIs is performed.
- Frequency sidebands and wavenumber broadening appear due to PDIs and a cool and dense SOL leads to considerable PI growth rate and convective loss, which further results in the density limit of LHCD caused by PDIs
- A theoretical scaling relation of the density limit, which shows agreement with simulation and experimental results, which indicates that LHCD remains a promising method of driving plasma current for ITER and future fusion reactors.

ACKNOWLEDGEMENTS / REFERENCES

- Supported by NSFC, under Grant No. 12335014.
- Details can be found in following references and talks in this conference

[1] poster of Zikai Huang, Monday-22:Energy Transfer and Spectral Evolution Induced by Parametric Decay Instability During the Injection of Lower Hybrid Waves

[2] poster of Kunyu Chen, Tuesday-13 :Theoretical Scaling of the Nonlinear Density Limit of Lower Hybrid Current Drive

[3] Z. Gao, Z. H. Su, Z. R. Liu, Z. K. Huang, J. Y. Han and Z. Y. Liu, Kinetic theory of parametric instabilities in magnetized plasmas and its application to analyzing decay waves during the injection of lower hybrid waves, Nuclear Fusion 65, 046033 (2025)

[4] Kunyu Chen and Zhe Gao, Saturation of a quasi-mode parametric instability in an inhomogeneous plasma, Nuclear Fusion 65, 036020 (2025)

[5] Kunyu Chen and Zhe Gao, Analysis of the convective amplification process of a three-wave parametric instability arises from low frequency plasma fluctuation, Plasma Phys. Control. Fusion 67, 035026 (2025)

[6] Liu, Z., Gao, Z. and Zhao, A., Nonlinearity in parametric instabilities during the injection of lower hybrid waves into tokamak plasmas, Phys. Plasmas 26, 042117 (2019)

[7] Liu, ZY, Gao, Z and Zhao, AH, Kinetic theory of parametric instabilities of lower hybrid waves in tokamaks in the electromagnetic framework, PHYSICS OF PLASMAS 27,042503 (2020)